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Collective behavior of M bosonic modes interacting with a single two-level atom

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The Hamiltonian describing, without the rotating-wave approximation (RWA), the linear interaction between M bosonic modes with an Einstein spectrum and a single two-level atom is exactly and canonically transformed introducing M suitable collective independent field modes, in such a way that only one among them is coupled to the atom. Some physical consequences of this fact are analyzed and, in particular, the existence of radiation-trapping phenomena together with the possibility of atomic absorption suppression is established. The applicability of the RWA to this system is discussed and the importance of the effective-field statistics for the time evolution of the system is pointed out.

I. INTRODUCTION

The bosonic nature common to the electromagnetic field as well as to the elastic vibration field in a crystal justifies investigating theoretically radiation-matter interaction, both in solid-state physics and in quantum optics, using formally equivalent Hamiltonian models. All of these models are constructed introducing suitable hypotheses which unavoidably lead to an idealized representation of the real situation. The most studied models assume that the interaction between matter and radiation is bilinear in the field and atomic coordinates and that the internal degrees of freedom of each atom can be represented by a pseudospin operator $S=\frac{1}{2}$.

Notwithstanding the rather drastic assumption that direct processes play a dominant role, the linear Hamiltonian model, sometimes called the Dicke model in the literature,[1] is far from being trivial; on the contrary, owing to the mathematical difficulties it raises, its physical implications have not yet been completely understood. It is for this reason that analytically more tractable versions of this linear model have been considered with great attention by theorists, beginning with the completely solvable Jaynes-Cummings model,[2] which has been successively generalized in many nontrivial ways. Theoretical investigations based on these oversimplified models, however, have also gained experimental significance: In solid-state physics, for example, the theory can be related to experiments on crystals which undergo cooperative Jahn-Teller transitions or on materials containing paraelastic, paraelectric, or paramagnetic centers,[3] whereas in optics the recent development of cold-cavity techniques with Rydberg atoms[4] has opened up the possibility of testing many of its predictions. The system we consider in this paper is that of a single two-level atom which is linearly coupled to M modes of a bosonic field with an Einstein spectrum. The antiresonant contributions to the interaction term are not neglected and the coupling constants are taken as mode dependent. This model is not new in literature, having been used both in solid-state physics[5] and in quantum optics.[6–9]

The purpose of the present paper, which emphasizes the cooperative nature of the interaction between the field and the atom, is to study the dynamical behavior of the system. This aim is achieved in a physically transparent and direct way by constructing exactly and explicitly the M collective bosonic modes through which the actual interaction with the atom takes place.

Our main result is that only one among these M new collective modes is coupled to the atom and that the new coupling constant is $\sqrt{M}$ times the mean coupling constant. These facts provide us with a simple way of predicting the existence of radiation-trapping phenomena as well as of absorption suppression in the system. Moreover, we point out the influence that the presence of $M-1$ unexcited modes has on the field statistics of the excited mode and discuss its implications on the time evolution of the system.

II. HAMILTONIAN MODEL AND CANONICAL TRANSFORMATION

Our Hamiltonian model is

$$H=\hbar\omega\sum_{\mu=1}^{M}\alpha_\mu^\dagger\alpha_\mu+\hbar\omega_0 S_z+\sum_{\mu=1}^{M}\varepsilon_\mu(\alpha_\mu^\dagger+\alpha_\mu)(S_++S_-)\ . \tag{2.1}$$

Here ε_μ is the real coupling constant between the atom and the μth radiation mode which is represented by Bose operators α_μ and $\alpha_\mu^\dagger$. S_t $(t=\pm,z)$ are usual pseudospin-$\frac{1}{2}$ operators describing the internal atomic degrees of freedom. It is not restrictive to choose ε_μ as a real c number because all phase factors can be canonically eliminated. Let us introduce the following canonical transformation:[10]

$$U=U_2U_3\cdots U_m\cdots U_M,\quad 2\le m\le M \tag{2.2}$$

where

$$U_m=\exp[\gamma_m(\alpha_1^\dagger\alpha_m-\alpha_m^\dagger\alpha_1)]\ . \tag{2.3}$$

In (2.3) γ_m is a free parameter to be fixed later. The unitary operators U_m satisfy the following relations:

$$[U_m,S_t]=\left[U_m,\ \sum_{\mu=1}^{M}\alpha_\mu^\dagger\alpha_\mu\right]=0\ , \tag{2.4}$$

$$\begin{aligned}U_m^\dagger\alpha_1U_m&=\alpha_1\cos\gamma_m+\alpha_m\sin\gamma_m\ ,\\ U_m^\dagger\alpha_mU_m&=\alpha_m\cos\gamma_m-\alpha_1\sin\gamma_m\ .\end{aligned} \tag{2.5}$$

Using (2.4) to transform H yields

$$\begin{aligned}U^\dagger HU\equiv\tilde H=\hbar\omega\sum_{\mu=1}^{M}\alpha_\mu^\dagger\alpha_\mu+\hbar\omega_0S_z\\ +U^\dagger\sum_{\mu=1}^{M}\varepsilon_\mu(\alpha_\mu^\dagger+\alpha_\mu)U(S_++S_-)\ .\end{aligned} \tag{2.6}$$

To obtain an explicit form for the second term on the right-hand side of (2.6) we proceed as follows:

$$\begin{aligned}U^\dagger\sum_{\mu=1}^{M}\varepsilon_\mu\alpha_\mu U=U_M^\dagger U_{M-1}^\dagger\cdots U_3^\dagger\left[U_2^\dagger(\varepsilon_1\alpha_1+\varepsilon_2\alpha_2)U_2\right.\\ \left.+\sum_{\mu=3}^{M}\varepsilon_\mu\alpha_\mu\right]\\ \times U_3\cdots U_{M-1}U_M\ ,\end{aligned} \tag{2.7}$$

where (2.2) and (2.3) have been used. From (2.5) we easily obtain

$$\begin{aligned}U_2^\dagger(\varepsilon_1\alpha_1+\varepsilon_2\alpha_2)U_2=(\varepsilon_1\cos\gamma_2-\varepsilon_2\sin\gamma_2)\alpha_1\\ +(\varepsilon_2\cos\gamma_2+\varepsilon_1\sin\gamma_2)\alpha_2\ .\end{aligned} \tag{2.8}$$

Putting $\gamma_2=-\arctan(\varepsilon_2/\varepsilon_1)$, (2.8) takes the form

$$U_2^\dagger(\varepsilon_1\alpha_1+\varepsilon_2\alpha_2)U_2=\varepsilon_1^{(2)}\alpha_1\ , \tag{2.9}$$

where

$$\varepsilon_1^{(2)}=(\varepsilon_1^2+\varepsilon_2^2)^{1/2}.$$

It is easy to generalize this procedure to yield, from (2.7),

$$U^\dagger\sum_{\mu=1}^{M}\varepsilon_\mu\alpha_\mu U=\varepsilon_1^{(M)}\alpha_1\ , \tag{2.10}$$

with

$$\gamma_m=-\arctan(\varepsilon_m/\varepsilon_1^{(m-1)})\ , \tag{2.11}$$

$$\varepsilon_1^{(m)}=\left[\sum_{\mu=1}^{m}\varepsilon_\mu^2\right]^{1/2}\ . \tag{2.12}$$

From (2.12) we have, in particular,

$$\varepsilon_1^{(M)}=\left[\sum_{\mu=1}^{M}\varepsilon_\mu^2\right]^{1/2}\ . \tag{2.13}$$

Inserting (2.10) in (2.6) we obtain

$$\tilde H=\hbar\omega\sum_{\mu=1}^{M}\alpha_\mu^\dagger\alpha_\mu+\hbar\omega_0S_z+\varepsilon_1^{(M)}(\alpha_1^\dagger+\alpha_1)(S_++S_-)\ . \tag{2.14}$$

This is our central result. From (2.14) we see that the action of U on H introduces, in place of the M true field modes described by the untransformed Bose operators of (2.1), M collective field coordinates described by the new Bose operators in (2.14). The meaning of this result can be best appreciated by considering

$$\begin{aligned}U\alpha_1^\dagger U^\dagger&=\sum_{\mu=1}^{M}\frac{\varepsilon_\mu}{\varepsilon_1^{(M)}}\alpha_\mu^\dagger\ ,\\ U\alpha_j^\dagger U^\dagger&=\frac{\varepsilon_1^{(j-1)}}{\varepsilon_1^{(j)}}\alpha_j^\dagger-\sum_{s=1}^{j-1}\frac{\varepsilon_j\varepsilon_s}{\varepsilon_1^{(j)}\varepsilon_1^{(j-1)}}\alpha_s^\dagger,\quad j>1\ .\end{aligned} \tag{2.15}$$

III. PHYSICAL MEANING OF $\tilde H$

Formally $\tilde H$ represents two independent subsystems: (i) a simple single-mode, single-atom system consisting of a fictitious radiation mode with the same frequency as the original bosonic modes of (2.1), linearly coupled to the (old) two-level atom with an effective coupling constant given by (2.13); and (ii) $M-1$ fictitious radiation modes described by the new Bose operators $\alpha_2,\alpha_3,\ldots,\alpha_M$, decoupled from each other and from the first subsystem. The appearance of these collective field coordinates has a transparent physical meaning: when M modes of a bosonic field with an Einstein spectrum interact with a two-level atom, they must be considered as a single quantum system and should not be treated as independent. In other words, the canonical transformation of H accomplished by U clearly shows that the two-level atom induces among the field modes coherence properties which are responsible for the collective behavior of the field subsystem of (2.1). This result, although obtained for a different system, is intimately related to that of Dicke[11] who was the first to point out the cooperative nature of N two-level atoms with a single bosonic mode. Recently, using a mathematical approach very similar to that presented here, we have discussed some of the features of such an atomic collective behavior.[12] We remark that if we modify (2.1) by simply getting rid of counterrotating contributions in its interaction term, obtaining in this way a model describing exactly the action of circularly polarized light for instance upon a magnetic sublevel,[9] our approach is still applicable. In fact, if we had written (2.1) in this form from the beginning we would have again obtained (2.13) with γ_m and $\varepsilon_1^{(m)}$ still given by (2.11) and (2.12), respectively, but with the counterrotating contributions absent from the interaction term. Moreover, it is interesting to note that as this simplified version of (2.1) can also be obtained by doing the so-called rotating-wave approximation (RWA) our before-mentioned results suggest that a certain caution must be exerted in adopting such an approximation in (2.1). In fact, our exact treatment of H by U clearly evidences that the correct parameter to be used in establishing the applicability of the RWA to (2.1) is $\varepsilon_1^{(M)}/\hbar\omega$ (the detuning $|\omega-\omega_0|$ is the same for H and $\tilde H$) which, according to (2.13), scales with M as $\sqrt{M}$ (at least in the homogeneous case $\varepsilon_\mu=\varepsilon$). Thus it may happen that, as a consequence of the increased value of the effective coupling constant, the single-atom, single-mode subsystem present in H cannot be treated in the RWA. This implies that the justification of the RWA version of (2.1), on the mere basis of the smallness of $\sum_\mu \varepsilon_\mu / M\hbar\omega$, is incomplete and it might give incorrect results.

Exact analytical eigenvalues and eigenvectors of $\tilde{H}$ for arbitrary values of the parameters are not known, essentially because the simple one-fermion, one-boson linear model has been solved without approximations only in its RWA version. In this case, the eigenstates and eigenvectors of (2.1) were given by Quattropani[7] many years ago using a long algebraic method. In what follows we will not reobtain these results, although we mention that, using U together with the well-known unitary operator[13] which diagonalizes the Jaynes-Cummings system, they can be easily obtained in a rather compact and elegant form. For the single-atom, single-mode linear system some general exact properties as well as approximations of the associated eigenvalues and eigenvectors in various coupling regimes have been given.[14] Swain[15] has presented formally exact continued fraction expressions, whose usefulness is limited only by the fact that results can be extracted from them only numerically. Although we are not able to give here exact explicit solutions of (2.1), however, we can predict some of its interesting physical properties that can be directly related to the structure of $\tilde{H}$ and, more precisely, to the circumstance that the atom exchanges energy with the field through only one of the collective modes. In Secs. IV–VI we shall discuss three different effects: (a) radiation trapping, (b) atomic absorption suppression, and (c) field-statistics modifications.

IV. RADIATION TRAPPING

Suppose that at $t=0$ the system has been prepared in a state with an arbitrarily assigned energy distribution in the (old) field modes and with the atom in its ground state. This should lead, in general, to an exchange of energy between the field and atom. We have shown in (2.14), however, that such an exchange is governed by a single-atom, single-mode effective Hamiltonian. It is then natural to take into account in detail how the energy is distributed among the field collective modes. In general, a certain amount of this energy is initially present in the $M-1$ uncoupled collective modes: this energy will remain rigorously trapped in these modes both in the sense that it shall not be exchanged with the atom and also in the sense that no intermode (new) energy transfer shall take place. It is interesting to note that trapping phenomena are present for any $M>1$ and that we may conceive extreme initial conditions for which radiation trapping is total or completely absent. To illustrate this last point we consider an initial situation corresponding to a ground-state atom in the presence of only one excitation conveniently distributed in the old modes of the field. This state, described by $|\psi\rangle$, is chosen to be of the form

$$|\psi\rangle = \sum_{\mu=1}^{M} \frac{\varepsilon_\mu}{\varepsilon_1^{(M)}} |0_1,\ldots,1_\mu,\ldots,0_M,-\rangle , \tag{4.1}$$

with $S_z|\psi\rangle = -\frac{1}{2}|\psi\rangle$,

$$\alpha_{\mu'}^\dagger \alpha_\mu |0_0,\ldots,1_\mu,\ldots,0_M,-\rangle = \delta_{\mu\mu'} |0_1,\ldots,1_\mu,\ldots,0_M,-\rangle .$$

To express $|\psi\rangle$ in the new representation we apply $U^\dagger$, obtaining

$$U^\dagger|\psi\rangle = \sum_{\mu=1}^{M} \frac{\varepsilon_\mu}{\varepsilon_1^{(M)}} U^\dagger \alpha_\mu^\dagger U U^\dagger |0_1,\ldots,0_\mu,\ldots,0_M,-\rangle . \tag{4.2}$$

Since

$$U^\dagger|0_1,\ldots,0_\mu,\ldots,0_M,-\rangle = |0_1,\ldots,0_\mu,\ldots,0_M,-\rangle , \tag{4.3}$$

using the properties of $U^\dagger$, (4.2) can be cast in the form

$$\begin{aligned} U^\dagger|\psi\rangle &= U^\dagger \sum_{\mu=1}^{M} \frac{\varepsilon_\mu}{\varepsilon_1^{(M)}} \alpha_\mu^\dagger U |0_1,\ldots,0_\mu,\ldots,0_M,-\rangle \\ &= \alpha_1^\dagger |0_1,\ldots,0_\mu,\ldots,0_M,-\rangle \\ &= |1_1,\ldots,0_\mu,\ldots,0_M,-\rangle . \end{aligned} \tag{4.4}$$

(4.4) means that only the atomic coupled collective mode is excited (in its one-excitation Fock state) so that no energy trapping will be observed. The dynamics of the system will be dominated by the effective coupling constant $\varepsilon_1^{(M)}$ and, in particular, for the RWA version, we should observe Rabi oscillations in the energy exchanges at a frequency $\Omega_M = \varepsilon_1^{(M)}/\hbar$. To illustrate the total radiation trapping we consider, for simplicity, an initial state $|\varphi\rangle$ of the form

$$\begin{aligned} |\varphi\rangle = \exp&\left[-\eta \frac{\varepsilon_2}{\varepsilon_1^{(2)}} (\alpha_1^\dagger - \alpha_1)\right] \\ &\times \exp\left[\frac{\varepsilon_1}{\varepsilon_1^{(2)}} (\alpha_2^\dagger - \alpha_2)\right] |0_1,\ldots,0_\mu,\ldots,0_M,-\rangle , \end{aligned} \tag{4.5}$$

where α_1 and α_2 are old field coordinates and η is a real nonzero number. In the new representation (4.5) becomes

$$\begin{aligned} U^\dagger|\varphi\rangle = U^\dagger \exp&\left[-\eta \frac{\varepsilon_2}{\varepsilon_1^{(2)}} (\alpha_1^\dagger - \alpha_1)\right] \\ &\times \exp\left[\eta \frac{\varepsilon_1}{\varepsilon_1^{(2)}} (\alpha_2^\dagger - \alpha_2)\right] U U^\dagger \\ &\times |0_1,\ldots,0_\mu,\ldots,0_M,-\rangle . \end{aligned} \tag{4.6}$$

Using (4.3) and the transforming properties of U, (4.6) can be put in the following form:

$$U^\dagger|\varphi\rangle = |0_1,\eta,0_3,\ldots,0_M,-\rangle . \tag{4.7}$$

(4.7) illustrates the meaning of radiation trapping in our context, which is characterized by the absence of excitations in the coupled collective mode associated with an initial state such as (4.5), where the energy expectation value is $\hbar\omega\eta^2 - \frac{1}{2}\hbar\omega_0 \neq 0$. As a consequence of this energy trapping, atomic time development becomes independent of the excitation number in the system at $t=0$. In our example this means that the atomic dynamics is uninfluenced by the value of η chosen in (4.5). Time evolution of the two-level variables depends on the effective coupling regime, and it is rather complicated because of the presence of virtual processes. In the RWA model, total radiation trapping, in particular, implies exact decoupling between the atom and field.

V. ATOMIC ABSORPTION SUPPRESSION

While energy trapping is independent of M, atomic absorption suppression, on the contrary, becomes more and more effective as M increases. This effect in our model is related to the existence of initial conditions for which the atom is in its ground state and the probability amplitude of the collective coupled-mode excitation decreases to zero when M increases. Again we prove our statement by giving an explicit example in which the initial state $|c\rangle$ is

$$\begin{aligned}|c\rangle &= |\eta,0_2,\dots,0_M,-\rangle\\ &= \exp[\eta(\alpha_1^\dagger-\alpha_1)]\,|0_1,\dots,0_\mu,\dots,0_M,-\rangle\ ,\end{aligned}\tag{5.1}$$

where α_1 is an old field coordinate. Transforming $|c\rangle$ by $U^\dagger$ yields

$$\begin{aligned}U^\dagger|c\rangle &= U^\dagger e^{\eta(\alpha_1^\dagger-\alpha_1)}UU^\dagger\,|0_1,0_2,\dots,0_M,-\rangle\\ &= e^{A}e^{B}\,|0_1,0_2,\dots,0_M,-\rangle\ ,\end{aligned}\tag{5.2}$$

where

$$\begin{aligned}A &= \eta\frac{\varepsilon_1}{\varepsilon_1^{(M)}}(\alpha_1^\dagger-\alpha_1)\equiv\delta(\alpha_1^\dagger-\alpha_1)\ ,\\ B &= -\eta\varepsilon_1\sum_{l=2}^{M}\frac{\varepsilon_1}{\varepsilon_1^{(l)}\varepsilon_1^{(l-1)}}(\alpha_l^\dagger-\alpha_l)\ .\end{aligned}\tag{5.3}$$

We thus see that when M increases δ decreases towards a positive value which vanishes only for a homogeneous or quasihomogeneous distribution of the coupling coefficients. This means that the presence of many modes, even if in their vacuum state at $t=0$, changes in a significant way the atomic absorption of radiation with respect to the single-atom, single-mode case, leading, in particular, to its asymptotic suppression if virtual exchanges are totally ignored.

VI. FIELD-STATISTICS MODIFICATION

The importance of and the role played by the collective behavior of the M modes in their interaction with the atom can be further appreciated considering the fact that the statistical properties of the field associated with its collective coupled mode are, in general, very different from those associated with the old modes. Some examples may help to clarify our point. If initially the system is in a state such as $|c\rangle$, given by (5.1), the coherent distribution of bosons in the first old field mode is transferred as such to the coupled mode [with η substituted by $\eta\varepsilon_1/\varepsilon_1^{(M)}$; see (5.3)]. In this case the cooperative effect which originates from the presence of $M-1$ unexcited modes at $t=0$ has only the consequence of scaling η by the factor $\varepsilon_1/\varepsilon_1^{(M)}$. Thus time evolution of the atomic dynamical variables is characterized by a behavior qualitatively similar to the single-atom, single-mode case. In RWA, in particular, periodic spontaneous collapse and revival[16] of the atomic inversion should exist starting from this condition. If, on the contrary, at $t=0$ the field is supposed to be in a Fock state (with n quanta in the first old mode and with the other modes in vacuum), while the two-level atom is in its ground state, then the statistical distribution of the quanta in the coupled mode is greatly modified with respect to the case $M=1$. In fact, we have

$$\begin{aligned}U^\dagger|n_1,0_2,0_3,\dots,0_M,-\rangle = \sum_{n_1+n_2+\cdots+n_M=n}&\sqrt{n!/n_1!n_2!\cdots n_M!}\left[\frac{\varepsilon_1}{\varepsilon_1^{(M)}}\right]^{n_1}\\ &\times\prod_{s=2}^{M}\left[-\frac{\varepsilon_1\varepsilon_s}{\varepsilon_1^{(s)}\varepsilon_1^{(s-1)}}\right]^{n_s}|n_1,n_2,\dots,n_M,-\rangle\ ,\end{aligned}\tag{6.1}$$

where the meaning of the symbols is obvious. (6.1) shows significant changes in the statistical distribution of bosons induced by cooperativity. Moreover, it indicates that the dynamical response of the system must be related to the boson distribution effectively present at $t=0$ in the coupled mode only. If we look at (4.3) from this point of view we may say that the field vacuum state has statistical properties which are not modified by U. A physical consequence of this fact is that the spontaneous emission of the atom is completely governed by the coupled subsystem of $\tilde{H}$; in the RWA, in particular, Rabi oscillations[8] occur with a frequency $\Omega_M=\varepsilon_1^{(M)}/\hbar$.

VII. CONCLUSION

We wish to conclude this paper with two remarks. The first concerns the possibility of experimental observation of such effects. Meystre *et al.*[6] have described a *Gedanken experiment,* where an atom placed in a Fabry-Perot interferometer is irradiated by an electromagnetic field. Assuming that the scattering on the atom is elastic, they have shown that, as in our model, only modes with the same frequency of the electromagnetic field are taken into account. Our model may also have relevance to those experiments where the tridimensional character of the radiation field cannot be ignored.[8] The second remark concerns the significance of our results to a weakly dispersive model. It seems reasonable also in this case to assume that during spontaneous decay the atom interacts mainly with a collective mode, which is a suitable linear combination of old modes of the field with frequencies lying in a band such that modes amplitudes are very little dephased during the decay. The condensation from many modes to a single mode has been discussed in the literature.

As an example we indicate the work of Lang *et al.*:[17] these authors, considering the theory of an optical maser, based on a model of the cavity having a semitransparent wall as one of the mirrors, have stressed that the narrowness of the laser linewidth could be regarded as a consequence of a locking phenomenon between the many modes of the universe corresponding to each of the Fox-Li–type quasimode.

Other examples are the paper of Ernst *et al.*[18] and those of Bonifacio *et al.*,[19] in which these authors have shown the combined role of atomic collective behavior and of cooperativity between field modes in the emission of N two-level atoms in a quasimode.

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