Comparison between dynamical theories and metastable states in regular and glassy mean-field spin models with underlying first-order-like phase transitions

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A comparison of dynamical theories and metastable states in regular and glassy (mean-field) spin models with underlying first-order-like phase transitions is made. We emphasize that for the glassy case, fluctuations probe the region in phase space where metastable states exist. This is contrasted to the situation for regular spin models. We physically interpret our results in terms of a dynamical instability of the paramagnetic phase in the glassy models. For the glassy case we conclude that as soon as metastable states exist, the mean-field model exhibits an ergodic to nonergodic transition where full ensemble averages are no longer equal to time averages.

I. INTRODUCTION

In three previous papers[1,2] (hereafter denoted by papers I) we studied the relaxational dynamics in two mean-field (MF) spin-glass (SG) models with discontinuous Edwards-Anderson (EA) order parameter[3] $q_{\rm EA}$ at the glass transition temperature. In particular we considered a soft-spin version of the p-spin ($p>2$) interactions SG model[1] and a soft-spin version of a p-state Potts glass (PG) model.[2] To be concrete in this paper we treat only Pott's models,[4] although, as we indicate below, our arguments seem very general.

The main results of papers I, and a subsequent paper by Kirkpatrick and Wolynes[5] (hereafter denoted by paper II), can be stated as follows. The dynamical theory for the equilibrium two-spin site-diagonal time autocorrelation function predicts a continuous slowing down and a freezing at a temperature which we denote by T_A. As $T \rightarrow T_A^+$ the relaxation time diverges according to a power-law singularity, and at T_A the EA order parameter jumps discontinuously from zero to a finite value. The transition at T_A can be related to static or equilibrium theory. In paper II it was shown that the freezing at T_A locates a thermodynamically metastable state in the sense that the (component[3]) average free energy of the frozen state is of $O(N)$, where N is the number of lattice sites, higher than a continuation of the paramagnetic state free energy. The notation T_A was introduced to signify that in non-mean-field models the long-time dynamics is expected to be governed by activated transport between the possible metastable states. It was also shown that at T_A there are an extensive or macroscopic number of possible metastable states with the same $q_{\rm EA}$. Due to the existence of a macroscopic number of metastable states, it was shown that the canonical free energy F_c was not the physically meaningful free energy. Furthermore, F_c was not equal to the average free energy $\bar{F}$ of the metastable states. Instead, F_c was shown to equal the paramagnetic-state free energy. This indicates that conventional arguments which do not allow metastable states in exact equilibrium statistical mechanics do not apply to the PG case because the extensive number of metastable states make up for their small Boltzmann weight. Although the above conclusions were drawn based on an explicit calculation for the infinite range Potts model, the implications are more general. The generality follows from the following: (a) there be a macroscopic number of solutions to the variational equation with each carrying approximately equal weight and (b) they be grouped in such a way that the probability of transition between these groups is small at least on the experimental time scale. It seems reasonable that both these will be satisfied by generic random systems and consequently $\bar{F}$ is the physically meaningful free energy. At a temperature $T_K < T_A$, the number of metastable states becomes nonextensive and at T_K the SG model undergoes a true equilibrium transition. Below T_K the distinction between $\bar{F}$ and F_c is no longer necessary at least in the thermodynamic limit. In analogy with the structural glass problem, T_K was identified as the Kauzmann temperature.[6] Recently, some of the key results contained in paper II have been obtained from a purely dynamical approach by incorporating the memory of initial condition into the equation of motion.[7]

In a regular-nonrandom-mean-field spin models with first-order phase transitions, dynamical theories can also locate metastable states. There are, however, several crucial differences between the pure system and the SG models discussed above. First, the number of metastable states is nonextensive and the canonical free energy is physically meaningful, and consequently F_c is equal to the (component) average free energy. Second, in regular models, the parts of phase space which correspond to

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metastable states can only be probed if there are external nonequilibrium perturbations that are of $O(N)$. Thus in the absence of an external field, metastable states in pure systems belong to a disjoint part of phase space and the canonical and average free energy is higher [by $O(N)$] than the stable state. As the temperature is approached where metastable states first exist, both equilibrium static fluctuations and equilibrium time correlation functions are well behaved. We also note that once a system is in a metastable state it will stay there forever because nucleation out of the metastable state is impossible in mean-field models.[8-11] It is also for this reason that metastable states can be described precisely within mean-field models.[9]

It is the purpose of this paper to address the question of how a dynamical theory for *fluctuations* can locate metastable states in SG models that are in some sense far [$\sim O(N)$] away. It is known[12] that static fluctuations are finite as $T\to T_A^+$ so that one cannot argue that usual thermal fluctuations cause the transition. However, we show below that equilibrium time correlations and their fluctuations are singular as $T\to T_A^+$. Further, we will argue that the transition at T_A is, in a well-defined technical sense, an ergodic to nonergodic transition where time averages no longer equal complete ensemble averages.[13] We stress that although the phase transition at T_A is caused by dynamical effects it is also possible[5] to discuss this glass transition using purely static or statistical mechanical methods. The transition is therefore both a dynamical and thermodynamic phase transition.

The fundamental reason that metastable states can be located by fluctuation effects in random systems is that the symmetry of the disordered paramagnetic phase and the glassy phases is identical. This should be contrasted with the difference in symmetry between the paramagnetic and the macroscopically ordered phases. It should be noted that these remarks are fairly general and also hold good for both the spin-glass problem (which is addressed here) as well as the structural glass problem. In the latter case the global symmetry of the liquid phase and the glassy phase are the same while that of the crystalline phase is different.

There are several implications of our results. First, the transition at T_A is an unusual example of a dynamic blocking transition.[14] The phase transition occurs because the paramagnetic state effectively becomes dynamically unstable and not because a different thermodynamic state has a lower free energy. Secondly, conventional statistical mechanics can be used for $T<T_A$ if restricted phase-space averages, or restricted saddle-pont evaluations, are used so that one focuses on a metastable state with finite $q_{\rm EA}$. Such ideas are common[6] in glassy physics in general but in these mean-field SG models the dynamical calculations force this interpretation of statistical mechanics.

The plan of this paper is as follows. In Sec. II we define the soft-spin Potts models and give some results from previous work. For completeness, in Sec. III we briefly discuss the relaxational dynamics of a soft-spin ferromagnetic Potts model and illustrate the points for the regular model discussed above. In Sec. IV we consider the soft-spin PG model. The main new contribution is the calculation of the dynamical PG susceptibility for $T\to T_A^+$. We conclude the paper in Sec. V with some further discussion.

II. THE POTTS MODELS AND SOME PREVIOUS RESULTS

A. The model

Our motivation for considering Langevin or relaxational dynamics for Potts models as opposed to some more conventional spin dynamics is that Langevin dynamics seems more generic to other statistical mechanical systems (e.g., liquids and their glass transition). Having decided upon Langevin dynamics, it is simpler to consider soft-spin field theories with Potts symmetry rather than "hard" lattice models. The appropriate field theory with p-state Potts symmetry[2,15] is $(-\infty<\phi_i^a<\infty)$ in an external magnetic field is

$$\beta H=-\beta\sum_{a=1}^{p-1}\sum_{i<j}J_{ij}\phi_i^a\phi_j^a+\frac{r_0}{2}\sum_{a=1}^{p-1}\sum_{i=1}^{N}\phi_i^a\phi_i^a+\frac{g_3}{3}\sum_{a,b,c=1}^{p-1}\sum_{i=1}^{N}Q_{abc}\phi_i^a\phi_i^b\phi_i^c+\frac{1}{4}\sum_{a,b,c,d=1}^{p-1}\sum_{i=1}^{N}T_{abcd}\phi_i^a\phi_i^b\phi_i^c\phi_i^d-\sum_{a=1}^{p-1}\sum_{i=1}^{N}h_i^a\phi_i^a\;, \tag{2.1a}$$

where $\beta=T^{-1}$ (with Boltzmann's constant k_B taken to be unity), $a,b,\ldots$ denote spin components in a $(p-1)$-dimensional vector space, and (i,j) denote lattice sites. The first term in Eq. (2.1a) couples sites together and J_{ij} denotes the interaction between sites i and j. The MF ferromagnetic Potts model corresponds to

$$J_{ij}=\frac{J}{N}\;, \tag{2.1b}$$

and for the MF PG model the $\{J_{ij}\}$ are random and frustrated and we assume that they are Gaussian distributed,

$$P(J_{ij})=\left[\frac{N}{2\pi J^2}\right]^{1/2}\exp\left[-\frac{(J_{ij})^2}{2J^2}N\right]. \tag{2.1c}$$

The N dependences in Eqs. (2.1b) and (2.1c) are chosen such that well-defined thermodynamic limits exist. The last four terms in Eq. (2.1a) are single site terms. Q and T are Potts coupling constants given by

$$Q_{abc}=\sum_{l=1}^{p}e_a^le_b^le_c^l \tag{2.2a}$$

and

$$T_{abcd}=u_0S_{abcd}+f_0F_{abcd}\;, \tag{2.2b}$$

with

$$S_{abcd}=\tfrac{1}{3}(\delta_{ab}\delta_{cd}+\delta_{ac}\delta_{bd}+\delta_{ad}\delta_{bc}) \tag{2.2c}$$

and

$$F_{abcd}=\sum_{l=1}^{p}e_a^le_b^le_c^le_d^l\;. \tag{2.2d}$$

The $\{\mathbf{e}^l\}$ $(l=1,2,\ldots,p)$ determine the Potts symmetry and they are given elsewhere.[2,15] Some useful identities are

$$\sum_{l=1}^{p} e_a^l e_b^l = p\delta_{ab} , \tag{2.3a}$$

$$\sum_{a=1}^{p-1} e_a^l e_l^{l'} = p\delta_{ll'} - 1 , \tag{2.3b}$$

$$\sum_{l=1}^{p} e_a^l = 0 . \tag{2.3c}$$

B. Dynamical equations

As mentioned above, we assume Langevin dynamics for $\phi_i^a(t)$,

$$\Gamma_0^{-1}\partial_t \phi_i^a(t) = -\frac{\partial(\beta H)}{\partial \phi_i^a(t)} + \xi_i^a(t) . \tag{2.4a}$$

Here Γ_0 is a bare kinetic coefficient which sets the microscopic time scale and $\xi_i^a(t)$ is a Gaussian random noise with zero mean and variance,

$$\langle \xi_i^a(t)\xi_j^b(t')\rangle = \frac{2}{\Gamma_0}\delta_{ij}\delta_{ab}\delta(t-t') . \tag{2.4b}$$

In usual systems Eq. (2.4b) ensures an approach to equilibrium. In the dynamical calculation two physical quantities of interest are the equilibrium spin-spin correlation function

$$C_{ab}(t-t') = \delta_{ab}C(t-t') = \frac{1}{N}\sum_{i=1}^{N}[C_{ii}^{ab}(t,t')] \tag{2.5a}$$

and the linear response function

$$G_{ab}(t-t') = \delta_{ab}G(t-t') = \frac{1}{N}\sum_{i=1}^{N}[\chi_{ii}^{ab}(t,t')], \quad t>t' \tag{2.5b}$$

where

$$C_{ij}^{ab}(t,t') \equiv \langle \phi_i^a(t)\phi_j^b(t')\rangle_\xi \tag{2.5c}$$

and

$$\chi_{ij}^{ab}(t,t') \equiv \frac{\partial\langle \phi_i^a(t)\rangle_\xi}{\partial h_j^b(t')} . \tag{2.5d}$$

Here the square brackets denote (for the PG case) an average over the random interactions and h_i^a denotes an external magnetic field. We have also used that in the paramagnetic phase both C and G are diagonal in the vector indices a,b. In general, causality yields the relation

$$G(t) = -\Theta(t)\partial_t C(t) \tag{2.5e}$$

with $\Theta(t>0)=1$ and $\Theta(t<0)=0$.

For the PG case we need to carry out the average over the quenched random interactions. In the mean-field (or saddle-point) approximation this is easily accomplished[2,16] with the dynamical functional integral formalism of De Dominicis[17] and Janssen *et al.*[18] In the $N\to\infty$ limit, the mean-field equation of motion for $\phi_i^a(\omega)$, the Fourier transform of $\phi_i^a(t)$, averaged over the random interactions, is[2]

$$\begin{aligned}\phi_i^a(\omega) = {} & G_0(\omega)[f_i^a(\omega)+h_i^a(\omega)] - G_0(\omega)g_3\sum_{b,c=1}^{p-1} Q_{abc}\int\frac{d\omega_1}{2\pi}\phi_i^b(\omega_1)\phi_i^c(\omega-\omega_1) \\ & - G_0(\omega)\sum_{b,c,d=1}^{p-1} T_{abcd}\int\frac{d\omega_1}{2\pi}\int\frac{d\omega_2}{2\pi}\phi_i^b(\omega_1)\phi_i^c(\omega_2)\phi_i^d(\omega-\omega_1-\omega_2) ,\end{aligned} \tag{2.6a}$$

with $G_0(\omega)$ a renormalized bare propagator

$$G_0^{-1}(\omega) = r_0 - i\omega\Gamma_0^{-1} - \mu\int_0^\infty dt\, e^{i\omega t}G(t) \tag{2.6b}$$

and

$$\mu \equiv \beta^2 J^2 , \tag{2.6c}$$

and $f_i^a(\omega)$ a renormalized noise term

$$\langle f_i^a(\omega)f_j^b(\omega')\rangle = 2\pi\delta(\omega+\omega')\delta_{ij}\delta_{ab} \times \left[\frac{2}{\Gamma_0} + \mu\int_{-\infty}^{+\infty} dt\, e^{i\omega t}C(t)\right] . \tag{2.6d}$$

Note that Eqs. (2.6) is an effective single site equation of motion for the PG problem. For the mean-field Potts ferromagnetic model we will work directly with Eqs. (2.4).

C. Dynamical stability matrix for the PG model

The equation of motion for the Potts variable $\phi_i^a(\omega)$ [cf. Eqs. (2.6)], is derived by solving the self-consistent equations for the Q fields which are introduced to decouple the multisite interactions. The multisite interactions are obtained when the quenched average over the random bonds is performed. The equations for the Q fields are obtained by a saddle-point solution of a dynamical field theory.[2,16] Around the chosen saddle-point (SP) solution there are Gaussian fluctuations which determine the stability of the SP solution.[19] Thus one expands the field theory for the Q fields around the mean-field solution, i.e., let $\delta Q^{\alpha\beta} = Q^{\alpha\beta} - \bar{Q}^{\alpha\beta}$. The resulting generating functional (or partition function) for the Gaussian fluctuations in the dynamical Q fields is

$$Z = \int D[\delta Q]\exp[-NA_2(\delta Q)] , \tag{2.7a}$$

with A_2 the quadratic form

$$A_2(\delta Q)=\frac{\mu}{2}\sum_{a,b,c,d}\sum_{\alpha,\beta,\gamma,\nu}\int dt_1\int dt_2\int dt_3\int dt_4\, \delta Q_a^{\alpha\beta}(t_1,t_2)[A^{\alpha\beta\gamma\nu}\delta_{ca}\delta_{db}\delta(t_3-t_1)\delta(t_4-t_2)$$
$$-\mu C_{abcd}^{\alpha\beta\gamma\nu}(t_1,t_2,t_3,t_4)]$$
$$\times\delta Q_{cd}^{\gamma\nu}(t_3,t_4)\Theta(t_4-t_3)\Theta(t_2-t_1)\ . \tag{2.7b}$$

Here the notation is identical to that used in Ref. 19 except we also have vector labels. The Greek superscripts take on values 1 or 2 and

$$\begin{aligned}&A^{1122}=A^{2211}=A^{1221}=A^{2112}=1\\ &A^{\alpha\beta\gamma\nu}=0 \quad \text{otherwise}\ ,\end{aligned} \tag{2.8a}$$

and

$$\begin{aligned}C_{abcd}^{\alpha\beta\gamma\nu}(t_1,t_2,t_3,t_4)=&\langle\psi_a^\alpha(t_1)\psi_b^\beta(t_2)\psi_c^\gamma(t_3)\psi_d^\nu(t_4)\rangle_{\rm MF}\\ &-\langle\psi_a^\alpha(t_1)\psi_b^\beta(t_2)\rangle_{\rm MF}\\ &\times\langle\psi_c^\gamma(t_3)\psi_d^\nu(t_4)\rangle_{\rm MF}\ ,\end{aligned} \tag{2.8b}$$

where $\langle\ \rangle_{\rm MF}$ indicates that this correlation function is to be calculated in the MF approximation, i.e., the probability measure is determined by the mean-field dynamical Lagrangian. The fields ψ_a^α are

$$\begin{aligned}&\psi_a^1(t)=\phi_i^a(t)\ ,\\ &\psi_a^2(t)=i\hat{\phi}_i^a(t)\ ,\end{aligned} \tag{2.8c}$$

where $i\hat{\phi}$ is the auxiliary response field which is introduced in the dynamical functional integral approach to enforce the equation of motion.[16–18]

The correlation functions defined by Eqs. (2.7) can be related to the dynamic spin-glass susceptibilities by adding a source term to the field theory from which the mean-field equations of motion for $\phi_i^a(\omega)$ are obtained. One finds ($t_1<t_2$ and $t_3<t_4$)

$$\langle\delta Q_{ab}^{\alpha\beta}(t_1,t_2)\delta Q_{cd}^{\gamma\nu}(t_3,t_4)\rangle=\frac{1}{\mu^2N^2}\sum_{i,j=1}^{N}[\langle\Phi_{ia}^\alpha(t_1)\Phi_{ib}^\beta(t_2)\Phi_{jc}^\gamma(t_3)\Phi_{jd}^\nu(t_4)\rangle_\xi]_{\rm MF}$$
$$-\frac{1}{\mu^2}\bar{Q}_{ab}^{\alpha\beta}(t_1,t_2)\bar{Q}_{cd}^{\gamma\nu}(t_3,t_4)+\frac{1}{\mu N}\delta_{ac}\delta_{bd}\delta(t_1-t_3)\delta(t_2-t_4)A^{\alpha\beta\gamma\nu}\ . \tag{2.9a}$$

The average on the left-hand side of Eq. (2.9a) is with respect to the theory defined by Eqs. (2.7). The fields Φ_{ia}^α are

$$\begin{aligned}&\Phi_{ia}^1(t)=i\hat{\phi}_i^a(t)\ ,\\ &\Phi_{ia}^2(t)=\phi_i^a(t)\ ,\end{aligned} \tag{2.9b}$$

and

$$\bar{Q}_{ab}^{\alpha\beta}(t_1,t_2)=\frac{1}{N}\sum_{i=1}^{N}[\langle\Phi_{ia}^\alpha(t_1)\Phi_{ib}^\beta(t_2)\rangle_\xi]_{\rm MF}\ . \tag{2.9c}$$

For example, the PG susceptibility is given by[19]

$$\chi_{\rm PG}(\Omega,\Omega')=\int d(t_1-t_3)d(t_4-t_2)\exp[i\Omega(t_1-t_3)+i\Omega'(t_2-t_4)]\chi_{\rm PG}(t_1-t_3,t_4-t_2)\ , \tag{2.10a}$$

with

$$\chi_{\rm PG}(t_1-t_3,t_4-t_2)=N\mu^2\lim_{\substack{t_2,t_4\to\infty\\ t_1,t_3\to-\infty}}\sum_{a,b}\left[\langle\delta Q_{aa}^{21}(t_1,t_2)\delta Q_{bb}^{12}(t_3,t_4)\rangle-\frac{\delta_{ab}}{\mu N}\delta(t_1-t_3)\delta(t_2-t_4)\right]$$
$$=\frac{1}{N}\sum_{i,j=1}^{N}\sum_{a,b=1}^{p-1}[\chi_{ij}^{ab}(t_1-t_3)\chi_{ji}^{ba}(t_4-t_2)]\ . \tag{2.10b}$$

In principle, the static PG susceptibility calculated with equilibrium statistical mechanics should be given by

$$\chi_{\rm PG}=\lim_{\Omega,\Omega'\to 0}\chi_{\rm PG}(\Omega,\Omega')\ . \tag{2.10c}$$

For nonergodic system the $\chi_{\rm PG}$ defined by Eq. (2.10c) is not necessarily equal to the susceptibility calculated with equilibrium statistical mechanics. Finally, we note that the propagators defined by Eq. (2.7a) can be written as the solution of the integral equation[19]

$$\sum_{\mu_1,\mu_2}\sum_{e,f}\int dt_5\int dt_6[\delta(t_1-t_5)\delta(t_2-t_6)\delta_{ae}\delta_{bf}A^{\alpha\beta\mu_1\mu_2}-\mu C_{abef}^{\alpha\beta\mu_1\mu_2}(t_1,t_2,t_5,t_6)]$$
$$\times\langle\delta Q_{ef}^{\mu_1\mu_2}(t_5,t_6)\delta Q_{cd}^{\gamma\nu}(t_3,t_4)\rangle\Theta(t_6-t_5)=\frac{\delta_{\alpha\gamma}\delta_{\beta\nu}}{\mu N}\delta_{ac}\delta_{bd}\delta(t_1-t_3)\delta(t_2-t_4)\ . \tag{2.11}$$

It is shown in Sec. IV that the above integral equation for χ_{PG} can be decoupled from the rest and can be solved when the limits indicated in Eq. (2.10b) are satisfied.

III. REVIEW OF DYNAMICS IN THE REGULAR POTTS FERROMAGNET

In this section we first show how external perturbations can probe metastable states in conventional spin models with first-order phase transitions. Following this we show that equilibrium time correlation functions are well behaved as the temperature is approached where metastable states first exist. We conclude by interpreting our results.

A. Macroscopic equation for external perturbations

The starting equations are given by Eqs. (2.4), (2.1a), and (2.1b). Defining

$$\phi_a(t)\equiv\frac{1}{N}\sum_{i=1}^{N}\phi_i^a(t) \tag{3.1a}$$

and

$$\bar{\phi}_a(t)=\langle\phi_a(t)\rangle_\xi\ , \tag{3.1b}$$

these equations give (as $N\to\infty$)

$$\begin{aligned}(\Gamma_0^{-1}\partial_t+r_0-\beta J)\bar{\phi}_a(t)&=-g_3\sum_{b,c=1}^{p-1}Q_{abc}\frac{1}{N}\sum_{i=1}^{N}\langle\phi_i^b(t)\phi_i^c(t)\rangle_\xi\\&\quad-\sum_{b,c,d=1}^{p-1}T_{abcd}\frac{1}{N}\sum_{i=1}^{N}\langle\phi_i^b(t)\phi_i^c(t)\phi_i^d(t)\rangle_\xi\ .\end{aligned} \tag{3.1c}$$

In usual (hard) MF lattice models the averages decouple and one is left with a closed equation for $\bar{\phi}$.[8] In soft-spin models, however, even in the MF limit there are fluctuations in the length or size of the spin variables.[20] However, in Sec. III B we show that these length fluctuations are always nonsingular in the regular case and that we can replace Eq. (3.1c) by

$$\begin{aligned}(\Gamma_0^{-1}\partial_t+r_0-\beta J)\bar{\phi}_a(t)=&-g_3\sum_{b,c=1}^{p-1}Q_{abc}\bar{\phi}_b(t)\bar{\phi}_c(t)\\&-\sum_{b,c,d=1}^{p-1}T_{abcd}\bar{\phi}_b(t)\bar{\phi}_c(t)\bar{\phi}_d(t)\ .\end{aligned} \tag{3.1d}$$

Equation (3.1) is a reliable equation as long as the underlying first-order transition is only weakly discontinuous. For the Potts model this can always be adjusted to be true by choosing either a small g_3 or by choosing p to be close to 2 [cf. Eq. (3.3c)].

To discuss Eq. (3.1d) it is convenient to use the $\{e^l\}$ as basis vectors and write

$$\bar{\phi}_a(t)=\sum_{l=1}^{p}e_a^l\bar{\phi}_l(t)\ . \tag{3.2a}$$

Assuming a magnetization in the $l=l_0$ direction gives

$$\begin{aligned}(\Gamma_0^{-1}\partial_t+r_0-\beta J)\bar{\phi}_{l_0}(t)&=-g_3p(p-2)\bar{\phi}_{l_0}^2(t)\\&\quad-[u_0(p-1)+f_0p(p^2-3p+3)]\bar{\phi}_{l_0}^3(t)\ .\end{aligned} \tag{3.2b}$$

Note that for $p=2$ the Potts model reduces to an Ising model and Eq. (3.2b) predicts a continuous mean-field transition at $r_0=\beta_cJ=J/T_c$. For $p>2$ the Potts model has a discontinuous transition at $T_c>J/r_0$. Above the true thermodynamic transition the Potts model has well-defined metastable states and the temperature where these states first appear can be obtained from the nontrivial long-time solutions to Eq. (3.2b). Let

$$m\equiv\lim_{t\to\infty}\bar{\phi}_{l_0}(t)\ . \tag{3.3a}$$

Equation (3.2b) gives

$$m=0 \tag{3.3b}$$

or

$$m^2+g_3\frac{p(p-2)m}{\Delta}+\frac{r_0}{\Delta}\left[1-\frac{\beta J}{r_0}\right]=0\ , \tag{3.3c}$$

with

$$\Delta\equiv u_0(p-1)+f_0p(p^2-3p+3)\ . \tag{3.3d}$$

Equation (3.3c) yields

$$m_\pm=-g_3\frac{p(p-2)}{2\Delta}\pm\frac{1}{2}\left[\frac{g_3^2p^2(p-2)^2}{\Delta^2}-\frac{4r_0}{\Delta}\left[1-\frac{\beta J}{r_0}\right]\right]^{1/2}\ . \tag{3.3e}$$

Since m must be real, Eq. (3.3e) is a physical solution only if

$$T<T_A=\frac{J}{r_0}[1-g_3^2p^2(p-2)^2/4\Delta r_0]^{-1}\ , \tag{3.4}$$

where T_A is the temperature where well-defined metastable states first exist. This identification can also be justified by using statistical mechanical methods to study the regular Potts model.

Equation (3.2b) can be easily solved analytically. The complete solution is of no real interest to us here. For a comparison to the PG case, we sketch in Fig. 1 the fixed points (FP) of Eq. (3.2b) and indicate their stability properties. For $T>T_A$ the FP $m_\pm$ are complex and unstable and the only stable FP is the $m=0$ solution. For $T<T_A$ both the $m=m_-$ and $m=0$ FP are stable while the $m=m_+$ is an unstable FP. The physical conclusion is that if $\bar{\phi}_{l_0}(t=0)<m_+$ then for long times there will be a continuous slowing down and eventually there will be a freezing to a finite magnetization for $T<T_A$ given by $m=m_-$. Otherwise the long-time net magnetization is zero. We interpret this result further below. Here we note that for $\bar{\phi}_{l_0}(t=0)<m_+$ the external perturbation has to be macroscopic.

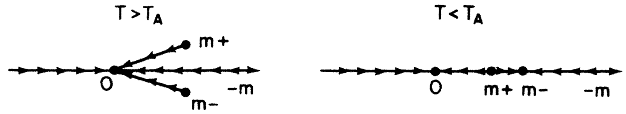

FIG. 1. Fixed points of Eq. (3.2b) and their stability properties for $T \gtrless T_A$.

B. Equilibrium time correlation functions as $T \to T_A$

To calculate equilibrium time correlation functions in the regular Potts model we treat the nonlinearities in Eqs. (2.4) as small and construct a self-consistent one-loop theory. Such an approximation is sufficient to illustrate the general structure of the theory and draw some general conclusions. This approximation is also identical to the approximation used in our previous work on the PG model. We calculate two distinct correlation functions, $\mathrm{Im}\omega > 0$,

$$\hat{C}_{ii}^{ab}(\omega) = \int_0^\infty dt\, e^{i\omega t} \langle \phi_i^a(t) \phi_i^b(0) \rangle_\xi \tag{3.5a}$$

and

$$\hat{C}^{ab}(\omega) = \frac{1}{N} \sum_{i,j} \hat{C}_{ij}^{ab}(\omega) . \tag{3.5b}$$

To one-loop order and as $N \to \infty$, these correlation functions can be calculated by standard methods.[21] The final results are $\hat{C}^{ab} = \delta_{ab} \hat{C}$,

$$\hat{C}_{ii}(\omega) = \frac{C_{ii}(t=0)}{[-i\omega + \bar{r}_0 \Gamma(\omega)]} , \tag{3.6a}$$

with the equal time spin correlation function given by

$$C_{ii}(t=0) = \frac{1}{\bar{r}_0} = \{ r_0 - 2p^2(p-2) g_3^2 C_{ii}^2(t=0) + C_{ii}(t=0)[u_0(p+1) + 3f_0^2] \}^{-1} . \tag{3.6b}$$

$\Gamma(\omega)$ is a renormalized kinetic coefficient

$$\frac{1}{\Gamma(\omega)} = \frac{1}{\Gamma_0} + 2p^2(p-2) g_3^2 \int_0^\infty dt\, e^{i\omega t} C_{ii}^2(t) . \tag{3.6c}$$

The equation for $\hat{C}(\omega)$ is given by Eqs. (3.6) with the *only* change being that r_0 is replaced by $r_0 - \beta J$.

Some conclusions can now be drawn. Examining Eqs. (3.6) and Eqs. (3.2)–(3.4), it is clear that the equilibrium time correlation functions will *not* exhibit any anomalies as $T \to T_A^+$. As already mentioned this is because the equilibrium time correlation functions do *not* probe the part of phase space where metastable states exist. We represent this as follows. If we define a one-dimensional phase space by (essentially the magnetic susceptibility) the collective coordinate

$$\chi_{ab} = \sum_{i,j=1}^N \phi_i^a \phi_j^b \tag{3.7}$$

then it is clear that in a paramagnetic state $\chi \sim O(N)$, while in a metastable ferromagnetic state $\chi \sim O(N^2)$. Since all fluctuations are finite it is clear that the paramagnetic and metastable ferromagnetic states are in disjoint parts of phase space. The important physical conclusion, when compared with the PG case discussed in Sec. IV is that as $T \to T_A^+$ equilibrium time correlation functions decay in the usual way and there is no anomalous linear transport.

IV. DYNAMICS IN THE POTTS GLASS MODEL

In this section we first review the results of I and compare them to the results in Sec. III. Following this we consider the dynamical stability of the SP solution and fluctuations about the SP solution as $T \to T_A^+$. We conclude by relating the transition at T_A to an ergodic to nonergodic transition.

A. Mean-field solution

In papers I we approximately solved Eq. (2.6) for $\mathrm{Im}\omega > 0$,

$$\hat{C}(\omega) = \int_0^\infty dt\, e^{i\omega t} C(t) . \tag{4.1}$$

The approximation was a self-consistent one-loop result and it is technically very similar to the theory given in Section III B. The PG undergoes either a continuous or discontinuous transition depending on how large the cubic term in the field theory is compared to the quadratic term. Here we are interested in the discontinuous case and in papers I it was argued that our results are reliable only if the transition is weakly discontinuous. The result for $\hat{C}(\omega)$ is

$$\hat{C}(\omega) = \frac{C(t=0)}{[-i\omega + \bar{r}_0 \Gamma(\omega)]} , \tag{4.2a}$$

with the equal time spin correlation function given by

$$C(t=0) \equiv \frac{1}{\bar{r}_0} = \{ r_0 - \mu C(t=0) - 2p^2(p-2) g_3^2 C^2(t=0) - 2p^2(p-2) g_3^2 C^2(t=0) + C(t=0)[u_0(p+1) + 3f_0 p^2] \}^{-1} . \tag{4.2b}$$

$\Gamma(\omega)$ is a renormalized kinetic coefficient

$$\Gamma^{-1}(\omega) = \Gamma_0^{-1} + \mu \int_0^\infty dt\, e^{i\omega t} C(t) + 2p^2(p-2) g_3^2 \int_0^\infty dt\, e^{i\omega t} C^2(t) . \tag{4.2c}$$

For use below we note that in the time domain, $\phi(t) = C(t)/C(0)$ satisfies the equation

$$\nu_0^{-1} \dot{\phi}(t) + \phi(t) + \lambda_1 \int_0^t dt_1 \phi(t-t_1) \dot{\phi}(t_1) + \lambda_2 \int_0^t dt_1 \phi^2(t-t_1) \dot{\phi}(t_1) = 0 , \tag{4.3a}$$

with $\phi(t=0) = 1$, $\nu_0 \equiv \bar{r}_0 \Gamma_0$, and the nonlinear coupling constants are given by

$$\lambda_1=\frac{\mu}{\bar{r}_0^2},\quad \lambda_2=\frac{2p^2(p-2)}{\bar{r}_0^3}g_3^2\ . \tag{4.3b}$$

Comparing Eqs. (4.2) and (3.6) we note that although structurally they are very similar there is a crucial difference: In the mean-field PG model the off-site-diagonal correlation functions are given zero weight so that

$$\hat{C}_{ab}(\omega)=\frac{1}{N}\sum_{i=1}^{N}[\hat{C}_{ii}^{ab}(\omega)]=\frac{1}{N}\sum_{i,j=1}^{N}[\hat{C}_{ij}^{ab}(\omega)]\ . \tag{4.4}$$

The technical consequence of this is that all the correlation functions that appear in Eqs. (4.2) are identical.

To discuss the PG transition we first define the Edwards-Anderson order parameter

$$\lim_{t\to\infty} C(t)\equiv q_{\rm EA}=q\equiv\frac{\bar{q}}{\bar{r}_0}\ . \tag{4.5a}$$

Assuming $C(t)$ has a time persistent part with a nonzero q and a decaying part, Eqs. (4.2), (4.3), and (4.5) yield the equation of state

$$\lambda_2\bar{q}^{\,3}+\bar{q}^{\,2}(\lambda_1-\lambda_2)=\bar{q}(\lambda_1-1)\ . \tag{4.5b}$$

Examining Eq. (4.5b) one finds that there is a weakly discontinuous phase transition with a discontinuity in q of $O(\varepsilon)$ at T_A if

$$\lambda_2=1+\varepsilon \tag{4.6a}$$

with $\varepsilon>0$ but small. For small ε the solution to Eq. (4.5b) is $\bar{q}=0$, or

$$\bar{q}_\pm\simeq\varepsilon_2\pm\frac{1}{2}\left[\varepsilon^2-4\left[1-\frac{\mu}{\bar{r}_0^2}\right]\right]^{1/2}\ . \tag{4.6b}$$

Since $\bar{q}$ must be real and greater than zero, Eq. (4.6b) is a physical solution only if

$$T<T_A=\frac{J}{\bar{r}_0}(1-\varepsilon^2/4)^{-1/2}\ . \tag{4.6c}$$

In papers I and paper II T_A was shown to be identical to the temperature where well-defined metastable states first exist according to equilibrium or Thouless-Anderson-Palmer approaches to spin glasses.

We stress that the above conclusion is very distinct from the regular case discussed in Sec. III. Here we see that fluctuation effects probe the part of the phase space where metastable states exist. Further insight can be gained by examining the FP's of Eq. (4.3a) and their stability. As we indicate in Fig. 2 for $T>T_A$, the FP $\bar{q}_\pm$ are complex and unstable and the only stable FP is the $\bar{q}=0$ solution. For $T<T_A$ both the $\bar{q}=q_+$ and $\bar{q}=0$ FP are stable while the $\bar{q}=\bar{q}_-$ FP is unstable. The physical conclusion is that since $\phi(t=0)=1>\bar{q}_+$ for long times there will always be a continuous slowing down and an eventual freezing to a finite $\bar{q}_{\rm EA}$ for $T<T_A$ given by $\bar{q}=\bar{q}_+$. We also note that Fig. 2 for the fluctuations in the PG case is very similar to Fig. 1 for the regular model with external macroscopic perturbations.

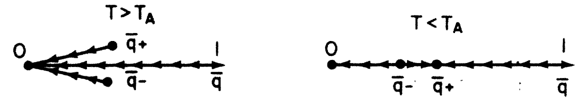

FIG. 2. Fixed points of Eq. (4.3a) and their stability properties for $T\gtrless T_A$.

B. Dynamical stability of the SP solution

To understand how the transition at T_A takes place we proceed in several steps. We first note that the argument at the end of Sec. III B is not directly applicable to the PG case. The ensemble and disorder average of χ_{ab} given by Eq. (3.7) is of $O(N)$ in both the paramagnetic phase and PG phase. Superficially this already indicates that in terms of two-point functions the phase spaces for the paramagnetic and PG phase are close to one another. This is relevant since the free energy of the metastable PG phase is in terms of $q_{\rm EA}$, the long time limit of a disorder averaged two-point time correlation function.

Although the above argument is correct, it might be argued that Eq. (3.7) is not the correct analogous collective coordinate for the PG model. Collective coordinates should be defined in terms of fluctuation in the appropriate order parameter, and for the spin systems this leads to the choice of the susceptibility matrix. This notion leads to Eq. (3.7) as the appropriate collective coordinate for the paramagnetic to ferromagnetic transition. However, for the random system the susceptibility matrix is itself random with zero trace. Thus the collective coordinate should involve a square of the random matrix. The appropriate collective coordinate should be something related to a PG susceptibility.[3,5] For example,

$$X\equiv\sum_{a,b}\sum_{i,j}\langle\phi_i^a\phi_j^a\rangle\langle\phi_i^b\phi_j^a\rangle\ , \tag{4.7}$$

where $\langle\ \rangle$ denotes an equilibrium ensemble average. In the paramagnetic phase $[X]\sim O(N)$, while the metastable PG phase $[X]\sim O(q_{\rm EA}^2N^2)$. In terms of this collective coordinate the phase space for the paramagnetic state and the metastable PG phase are very far apart. It then follows that there can be a spontaneous phase transition only if fluctuations are divergent as $T\to T_A^+$. To investigate this point we have studied the dynamic PG susceptibility. First note that in the strict limit given in Eq. (2.10b), the Eq. (2.11) can be solved exactly.[19] One finds

$$\chi_{\rm PG}(\Omega,\Omega')=\frac{(p-1)\mu^2G(\Omega)G(\Omega')}{[1-\mu G(\Omega)G(\Omega')]}\ , \tag{4.8a}$$

and for the static PG susceptibility

$$\chi_{\rm PG}=\frac{(p-1)\mu^2G^2(0)}{[1-\mu G^2(0)]}\ . \tag{4.8b}$$

In the limits indicated in Eq. (2.10b), Eq. (2.11) is exactly solvable because the mean-field correlation function $C^{\alpha\beta\mu_1\mu_2}$ factorizes and because in this limit $\langle\delta Q^{21}\delta Q^{12}\rangle$ is not coupled to any other propagator. Two things should be noted about Eqs. (4.8). First, Eq. (4.8b) is identical to the result which follows from equilibrium statistical mechanics. Second, the Eqs. (4.8) are nonsingular at T_A given by Eqs. (4.6). Thus if there are divergent fluctuations at T_A they must be of dynamical origin.

To study the possible dynamical fluctuations we have calculated how the limiting process in Eq. (2.10b), used in solving Eq. (2.11), is approached. It is not hard to show that for $\tau \equiv t_4 - t_1$ large, but finite, that $\langle \delta Q^{21} \delta Q^{12} \rangle$ still does not effectively couple to any other propagator. From Eq. (2.11) we conclude that the behavior of $\langle \delta Q^{21} \delta Q^{12} \rangle$ will be determined by the inverse of the function

$$F_{abcd}(t_1-t_3, t_4-t_2, \tau = t_4 - t_1) = \delta_{ac}\delta_{bd}\delta(t_1-t_3)\delta(t_2-t_4) - \mu C^{1221}_{abcd}(t_1,t_2,t_3,t_4) \ . \tag{4.9a}$$

If the function

$$F_{abcd}(\Omega,\Omega',\tau) = \int d(t_1-t_3)d(t_4-t_2) \times \exp[i\Omega(t_1-t_3)+i\Omega'(t_4-t_2)] \times F_{abcd}(t_1-t_3,t_4-t_2,\tau) \tag{4.9b}$$

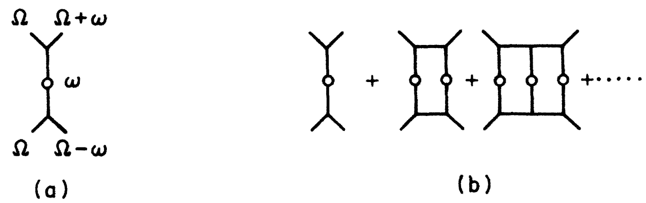

FIG. 3. (a) Diagram leading to the last term in Eq. (4.10a). (b) Diagrams leading to Eq. (4.11c). The notation used is defined in Ref. 21.

is almost zero on a time scale τ for $T \gtrsim T_A$, then we will conclude that dynamical fluctuations are divergent although we know that for $T = T_A + 0^+$ and $\tau \to \infty$, the fluctuations are finite.

We have calculated F by the perturbative method used in IV A. The first relevant diagram is shown in Fig. 3(a). Setting $\Omega = \Omega'$ the result for $F(\equiv F^{(1)})$ at this order is

$$F^{(1)}_{abcd}(\Omega,\Omega,\tau) = \delta_{ac}\delta_{bd}[1-\mu G^2(\Omega)] - 4g_3^2\mu G^2(\Omega)\sum_{b'} Q_{acb'}Q_{bdb'}\int \frac{d\omega}{2\pi} e^{i\omega\tau} G(\Omega+\omega)G(\Omega-\omega)C(\omega) \ . \tag{4.10a}$$

Because we are interested in the dynamical metastable states, into which the system freezes at $T = T_A$, we utilize an appropriate definition of dynamical metastability. In this context it can be shown that for T sufficiently close to T_A the correlation function acquires a very near plateau value for times greater than the initial microscopic decay time of $C(t)$ but less than the decay time of $C(t)$. Under these conditions, for $T \to T_A^+$, $C(\omega)$ continuously crosses over to

$$C(\omega)\,|_{T=T_A} = q2\pi\delta(\omega) + \text{a nonsingular part} \ . \tag{4.10b}$$

For $T \gtrsim T_A$ and for τ less than the decay time of $C(t)$, Eq. (4.10a) can be approximated by

$$F^{(1)}_{abcd}(\Omega,\Omega,\tau<\tau_c) \simeq \delta_{ac}\delta_{bd}[1-\mu G^2(\Omega)] - 4g_3^2\mu G^4(\Omega) q \sum_{b'} Q_{acb'}Q_{bdb'} \ , \tag{4.11a}$$

where τ_c is the decay time of $C(t)$ and q is the intermediate time plateau value of $C(t)$ for $v_0^{-1} \ll t \ll \tau_c$. From Eqs. (2.10) we see that only certain vector labels are relevant for the PG susceptibility. We consider

$$F^{(1)}(\Omega,\Omega,\tau<\tau_c) = \sum_{a,c} F^{(1)}_{aacc}(\Omega,\Omega,\tau<\tau_c) = (p-1)[1-\mu G^2(\Omega) - 4g_3^2 p^2(p-2)\mu \times G^4(\Omega) q] \ . \tag{4.11b}$$

In the same time regime the diagrams shown in Fig. 3(b) also contribute and need to be taken into account.[22] They form a geometric series and one obtains

$$F(\Omega,\Omega,\ \tau<\tau_c) \simeq \frac{(p-1)}{[1-4g_3^2p^2(p-2)qG^2(\Omega)]} \times [1-4g_3^2p^2(p-2)qG^2(\Omega) - \mu G^2(\Omega)] \ . \tag{4.11c}$$

For $\Omega\tau_c > 1$ the causality relation[23]

$$G(\Omega) = C(t=0) + i\Omega\hat{C}(\Omega) \simeq \frac{1}{\bar{r}_0} - q \tag{4.12a}$$

can be used to rewrite Eq. (4.11c) as ($\bar{q} = \bar{r}_0 q$)

$$F(\Omega\tau_c > 1,\ \tau<\tau_c) \simeq \frac{(p-1)}{[1-2\lambda_2\bar{q}(1-\bar{q})^2]} \times [3\lambda_2\bar{q}^{\,2} + 2\bar{q}(\lambda_1-\lambda_2) - (\lambda_1 - 1)] + O(q^3) \ , \tag{4.12b}$$

where we have used that near $T_A, \lambda_1 \simeq 1$, $\lambda_2 \simeq 1+\varepsilon$ and $\varepsilon \ll 1$. Examining Eq. (4.12b), one sees that it is proportional to the $\bar{q}$ derivative of the equation of state given by Eq. (4.5b). This equation defines a PG spinodal point at T_A.[5]

We can now draw some conclusions. For $T \to T_A^+$, $\tau_c \to \infty$ and the dynamic PG susceptibility is proportional to

$$\chi_{\rm PG} \sim [3\lambda_2\bar{q}^{\,2} + 2\bar{q}(\lambda_1-\lambda_2) - (\lambda_1-1)]^{-1} \ . \tag{4.13}$$

For $T \to T_A^+$, $\chi_{\rm PG}$ is singular and for τ fixed the singular behavior is approached continuously. The PG susceptibility effectively has the same value for $T = T_A^-$. Physically our results imply that correlations grow for times $\tau < \tau_c$, but for very long times, $\tau \gg \tau_c$, they decay to microscopic correlations. Technically our results predict a divergent *time-dependent* correlation length. This concept is probably generic to many glassy systems.

We conclude that in the PG there is an ergodic to nonergodic phase transition at T_A due to the divergent time scale τ_C. Most important for understanding the mechanism of this transition is the fact that the equilibrium PG susceptibility is not equal to the zero frequency dynamic PG susceptibility. Effectively there are divergent dynamic fluctuations as $T \to T_A^+$. The paramagnetic state is interpreted as dynamically unstable for $T < T_A$, and in the mean field Potts model there will be a transition at T_A to a "metastable" glassy phase.

Finally we mention that the ergodic to nonergodic phase transition can be characterized by an order parameter which measures the difference between a quantity calculated via a complete ensemble average and that same quantity calculated by an infinite time average. Such an order parameter is not unique and we will not pursue this point. Using restricted phase-space averages the statistical mechanics of the metastable states with finite $q_{\rm EA}$ for $T < T_A$ have been discussed elsewhere.[5]

It should be emphasized that the inequality of the zero frequency limit of the dynamic PG susceptibility and the equilibrium statistical mechanical PG susceptibility $\chi_{\rm PG}^{\rm SM}[\Lambda = \lim_{\Omega,\Omega' \to 0} \chi_{\rm PG}(\Omega,\Omega') - \chi_{\rm PG}^{\rm SM} \neq 0]$ is not due to violations of the fluctuation-dissipation theorem (FDT). This particularly important since a similar inequality of the SG susceptibility calculated using time average and purely equilibrium methods for a system[24] exhibiting a continuous transition led Sompolinsky and Zippelius to conclude that the FDT is violated. In our analysis, which is valid for $T = T_A^+$, the FDT is certainly obeyed and yet it has been established that $\Lambda \neq 0$. This is a clear manifestation of the breakdown of the ergodic theorem, which implies that phase-space averages are no longer equal to time averages. Thus at $T = T_A$ the system undergoes a transition from an ergodic phase to a nonergodic phase. Although these conclusions have been reached using Langevin dynamics for the spin motion, the results should be valid for any dynamics as long as the correct equilibrium distribution is obtained in the infinite time limit

At T_A^-, the system is frozen[5] into one of an exponentially large number of statistically similar glass states. Since these states all have the same correlation functions, it is not necessary to specify which state the system freezes into at T_A. A detailed discussion on the behavior for $T < T_A$ is given in Ref. 5.

V. DISCUSSION

We conclude with a few additional remarks.

(1) The dynamics in non-mean-field Potts models is not clear. We expect that for $T < T_A$ there will be finite domains with properties similar to the global metastable states predicted by mean-field theory. The long time dynamics would then be governed by activated transport, as the domains change from one metastable state to another. Experimentally, we would expect a break in the slope of the temperature dependence of the relaxation times as one passes through T_A.

There is a remarkable similarity between the dynamics in the regular model subjected to a macroscopic perturbation and the subsequent decay to a metastable ferromagnetic state and the decay of the fluctuation-induced transition in the glassy case for $T < T_A$ (compare Figs. 1 and 2). It is likely that the methods developed to describe nucleation out of a metastable state for regular systems[24] will prove useful for describing the long time dynamics in non-mean-field glassy models for $T < T_A$. More detailed arguments along these lines are given in paper II.

(2) Our results are probably generic for SG models with discontinuous Edwards-Anderson order parameter which have well-defined metastable states in the mean-field limit above their true equilibrium transition temperature. We can speculate that in more general glassy problems such as structural glasses where random interactions are not put in, that similar behavior will occur if the order parameter (as opposed to the interactions) is a stochastic or random variable. This conjecture is motivated by the observation that the most crucial aspect of our calculations appear to be that the magnetization at each lattice site has a distribution with zero mean and square average equal to $q_{\rm EA}$.[5,25] It is easy to imagine that liquid-state theories for solidification admit solutions with these properties.[26] If such solutions exist we would expect that they could be located by dynamical theories for fluctuations.

Along these lines it is also relevant to point out that if a mean-field spin (or liquid) model is considered with a regular transition at T_1 and a SG spinodal at T_A with $T_A < T_1$. Then our arguments would suggest that the maximum amount of supercooling would be to T_A, where the system would spontaneously form a spin-glass state. This is similar to the situation which probably occurs in the recent dynamic theories[27] for the structural glass transition with T_1 replaced by the crystallization temperature or density.

(3) The most important reason why fluctuation effects can locate the metastable PG phase is that the symmetry of the paramagnetic state and the isotropic spin glass state is identical. The reason for this is that the Edwards-Anderson[28] order parameter distinguishing the paramagnetic phase and the spin-glass phase is in a sense a microscopic order parameter associated with fluctuations in the magnetization at each site.[29] On the other hand, the fully ordered ferromagnetic phase is distinguished by the presence of a non-zero macroscopic variable, namely, the magnetization and hence the symmetry of the phase is perceptibly different. Because the above arguments only rely on broken symmetry considerations, we believe they should be applicable to any generic random system and in particular to the structural glass transition.

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