Field theory of pattern identification

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Based on the psychological experimental fact that images in mental space are transformed into other images for pattern identification, a field theory of pattern identification of geometrical patterns is developed with the use of gauge field theory in Euclidean space. Here, the "image" or state function $\psi[X]$ of the brain reacting to a geometrical pattern X is made to correspond to the electron's wave function in Minkowski space. The pattern identification of the pattern X with the modified pattern $X+\Delta X$ is assumed to be such that their images $\psi[X]$ and $\psi[X+\Delta X]$ in the brain are transformable with each other through suitable transformation groups such as parallel transformation, dilatation, or rotation. The transformation group is called the "image potential" which corresponds to the vector potential of the gauge field. An "image field" derived from the image potential is found to be induced in the brain when the two images $\psi[X]$ and $\psi[X+\Delta X]$ are not transformable through suitable transformation groups or gauge transformations. It is also shown that, when the image field exists, the final state of the image $\psi[X]$ is expected to be different, depending on the paths of modifications of the pattern X leading to a final pattern. The above fact is interpreted as a version of the Aharonov and Bohm effect of the electron's wave function [A. Aharonov and D. Bohm, Phys. Rev. **115**, 485 (1959)]. An excitation equation of the image field is also derived by postulating that patterns are identified maximally for the purpose of minimizing the number of memorized standard patterns.

I. INTRODUCTION

Pattern identification is considered as one of the most fundamental types of information processing in the brain for recognizing the physical world.[1] The objects to be identified are not limited to geometrical patterns, but extend to more general physical objects, such as a classical or quantum particle. For example, the wave function of a moving electron at a point in Minkowski space is identified with that at another point through an electromagnetic field (or group transformation). In the process of the pattern identification of geometrical patterns, the various features are extracted from the object, analyzed, and finally reorganized to give rise to its image in the brain. The constructed image is compared with the ones stored in memory through suitable transformation groups such as rotation, parallel translation, dilatation, or mirror inversion. The group property of the transformations is important from the point of view of "information contraction." In other words, many patterns are reduced to standard ones through the group transformations. The mechanism of some basic group transformations is considered to be inherently present in the brain. Discovery of new group transformations enables us to find the identity between physical quantities which were considered to be totally different. In order to make pattern-recognition instruments similar to human visual perception, much theoretical effort has been made on how to extract invariant features from geometrical patterns under given transformation groups.[2] Recently, a remarkable psychological experiment has been done to show that the time required to identify the same patterns with different spatial orientations increases linearly in proportion to their angular differences.[3] This fact suggests that the image produced in the brain is rotated for identification with the original image.

In the present paper, based on the above-mentioned experimental facts, we introduce a postulate that geometrical patterns are identified when their images constructed in the brain can be transformed into each other under suitable transformation groups. Recognizing the image as a physical quantity with invariant features under given group transformations, and using the mathematical structure of gauge field theory[4] in Euclidean space, we develop an identification theory of geometrical patterns. Here the image $\psi[X]$ of a geometrical pattern X is made to correspond to the quantum wave function of an electron, the pattern X to the Minkowski space coordinates, and the "image potential" introduced through transformation groups to the vector potential of the electromagnetic (or gauge) field in Minkowski space. An "image field" corresponding to the electromagnetic field is also introduced through the image potential. The mathematical relation between the image potential and the image field is the same as that between the connection coefficient and the curvature tensor in differential geometry. If the difference between two images cannot be canceled out by suitable coordinate transformations in the image space, the image field is induced in the brain and the two corresponding patterns are interpreted as having different information contents. In other words, the difference in information content is represented by the excitation of the image field in the brain. The concept of the image field will give a new method to explain optical illusions or the hysteresis effect in visual perception, which is a version of the Aharonov and Bohm effect of the electron's wave function.[5] Postulating that patterns are identified maximally for the purpose of minimizing the number of memorized standard patterns, we derive an excitation equation of the image field.

II. FIELD THEORY OF PATTERN IDENTIFICATION

Let us consider first a stationary geometrical pattern in three-dimensional space. A finite volume of three-dimensional space within a field of vision is supposed to be digitized into n spatial elements v_i, $i=1,2,\ldots,n$ of equal shape. The spatial pattern is assumed to be represented by the light intensity x_i, $i=1,2,\ldots,n$ or brightness of each spatial element. An n-dimensional pattern space equipped with an orthogonal coordinate system is introduced, and the light intensity of each spatial element is scaled on each coordinate. Accordingly, a point vector

$$\chi=(x_1,x_2,\ldots,x_n)$$

in the pattern space corresponds to a geometrical pattern within our visual field. Similarly, let us introduce an M-dimensional image space in the brain, in which the image produced corresponding to pattern χ is represented by a point vector $\psi[\chi]$. The image $\psi[\chi]$ is also interpreted as the state vector representing the distribution of fired neurons in the brain reacting to the stationary pattern χ, so that the image is regarded as a physical reality to be observed in a theoretical sense.

For a temporally varying pattern, its image $\psi[\chi]$ is considered to be constructed in a way that depends not only on its light-intensity distribution, but also on its temporal change. For example, let us consider a round trip of a temporal pattern modification such that a horizontally elongated elliptic circle is gradually changed into a vertically elongated one via a true circle, and vice versa. Then the first turning point at which the true circle is recognized in the forward path is, in general, perceived as different from the second one which is recognized in its return path. The discrepancy in the turning points shows an optical illusion and is interpreted in such a way that the image corresponding to a temporally varying pattern depends not only upon its light-intensity distribution, but also upon its temporal change. Taking this into account, we introduce a simple assumption that the image corresponding to a temporally varying pattern is constructed in a way that depends not only on its light-intensity distribution $(x_1,x_2,\ldots,x_n)$, but also on the velocity $(\dot{x}_1,\dot{x}_2,\ldots,\dot{x}_n)$ of its temporal change. In other words, the pattern vector χ for a temporally varying pattern is assumed to represent a $2n$-dimensional vector, $(x_1,x_2,\ldots,x_n,\dot{x}_1,\dot{x}_2,\ldots,\dot{x}_n)$.

Now, the image vector $\psi[\chi]$ corresponding to a stationary or temporally varying pattern is assumed to be expanded with the use of the basis vectors ζ_a, $a=1,2,\ldots,M$ as

$$\psi[\chi]=\sum_{a=1}^{M}\psi^a[\chi]\zeta_a[\chi]\ , \tag{1}$$

where ψ^a is the component of ψ along the ζ_a axis, and hereafter, repeated indices are assumed to be summed, so that Eq. (1) is simply written as

$$\Psi[\chi]=\psi^a[\chi]\zeta_a[\chi]\ . \tag{1$'$}$$

Now we introduce transformation matrices $R_a^{\,b}$ of the basis vectors,

$$\zeta_a'[\chi]=R_a^{\,b}[\chi,\alpha_1,\alpha_2,\ldots,\alpha_p]\zeta_b[\chi]\ , \tag{2}$$

which is assumed to compose a continuous Lie group with p parameters $\alpha_1,\alpha_2,\ldots,\alpha_p$. Especially, for infinitesimal transformation, we have

$$R[\chi;\epsilon^1,\epsilon^2,\ldots,\epsilon^p]=1+\mathcal{E}^r[\chi]U_r\ , \tag{3}$$

where $\mathcal{E}^r[\chi], r=1,2,\ldots,p$ are infinitesimal parameters and U^r, $r=1,2,\ldots,p$ are constant square matrices of rank M, independent of χ, which satisfy the group condition

$$[U_r,U_s]=C_{rs}^{t}U_t\ . \tag{4}$$

Here $C_{r,s}^t$, $r,s,t=1,2,\ldots,p$ are the structure constants of the continuous group. The group property of the transformation U^r is essential to the "information contraction" in the sense that various modified images are reduced to standard ones.

Now let us examine the identification condition of the infinitesimal variation $\Delta\chi=(\delta\chi^1,\ldots,\delta\chi^n)$ of pattern χ. We have, from Eq. (1),

$$\psi[\chi+\Delta\chi]=\psi^a[\chi+\Delta\chi]\zeta_a[\chi+\Delta\chi]\ . \tag{5}$$

Now for the purpose of the identification of $\psi[\chi+\Delta\chi]$ with $\psi[\chi]$, we introduce an assumption based on the experimental facts[3] that the basis vectors $\zeta_a[\chi+\Delta\chi]$ are transformed into the new ones $\zeta_a[\chi]$ through the infinitesimal transformation

$$\zeta_a[\chi+\Delta\chi]=(1+\epsilon_\mu^r\delta\chi^\mu U_r)_a^b\zeta_b[\chi]\ . \tag{6}$$

ϵ^r in Eq. (3) is chosen as $\epsilon_\mu^r[\chi]\delta\chi^\mu$, $\mu=1,2,\ldots,n$, taking account of the fact that the infinitesimal parameter $\epsilon^r[\chi]$ should be proportional to the small variation $\delta\chi$ of the pattern χ. Substituting Eq. (6) into Eq. (5) and using Eq. (1), we have

$$\psi[\chi+\Delta\chi]=\psi[\chi]+(\partial_\mu\psi^a+\epsilon_\mu^r[\chi]U_{rb}^a\psi^b)\delta\chi^\mu\zeta_a[\chi]\ , \tag{7}$$

where the second term on the right-hand side means the deviation of the image $\psi[\chi+\Delta\chi]$ from $\psi[\chi]$ left after the coordinate transformation (6) for identification. In other words, a new information quantity representing the difference of $\psi[\chi+\Delta\chi]$ from $\psi[\chi]$ is included in the second term. Further, a noteworthy point is that the parameters $\mathcal{E}_\mu^r[\chi]$ of the transformation group depend, in general, upon the pattern itself. This represents the phenomenon of optical illusion, such that if two different patterns are varied following the same geometrical transformation rule, their corresponding images change, in general, in a different manner. Figure 1, sketched with the use of a well-known psychological phenomenon, shows that the dilatation rate in image space is different, depending on the pattern itself.

Introducing a covariant derivative of Ψ through

$$\nabla_\mu \Psi^a = \partial_\mu \Psi^a + \epsilon^r_\mu U^a_{rb} \psi^b = \partial_\mu \psi^a + A^{\,a}_{\mu b} \psi^b \ , \tag{8}$$

we have, from Eq. (7),

$$\Psi[X + \Delta X] = \Psi[X] + \nabla_\mu \Psi^a \zeta_a \delta X^\mu \ , \tag{9}$$

where ∂_μ means $\partial / \partial X^\mu$, and $A^{\,a}_{\mu b}$ denotes $\epsilon_{\mu r} U^{ra}_b$ and is regarded as the connection coefficient in differential geometry of image space. The identification is assumed to be possible when the norm of the difference vector $\psi[X + \Delta X] - \psi[X]$ is smaller than a critical distance d_c,

$$\| \psi[X + \Delta X] - \psi[X] \| < d_c \ . \tag{10}$$

A sufficient condition in order for the inequality (10) to be fulfilled is found, from Eq. (9), to be given by the condition of the parallel displacement of the image vector $\psi[X]$ in the image space with the connection coefficient $A^{\,a}_{\mu b}$,

$$\nabla_\mu \psi^a = 0 \ . \tag{11}$$

The integrability condition of Eq. (11) is given with the use of Eq. (8) as

$$F^{\,a}_{\mu\nu b} = 0 \ , \tag{12}$$

where $F^{\,a}_{\mu\nu b}$ is the curvature tensor defined by

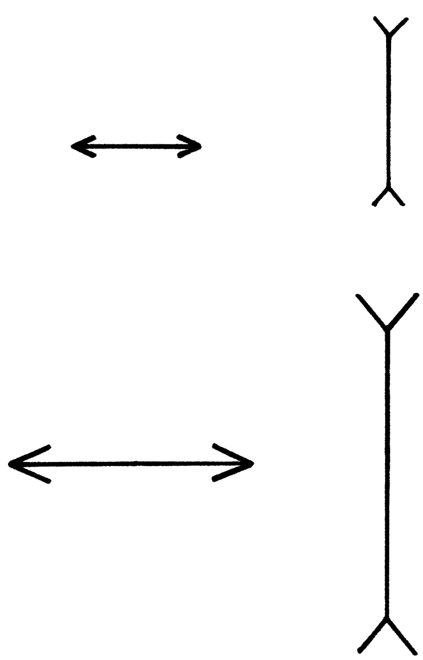

FIG. 1. The above figure is an example of the optical illusion that the dilatation rate in image space is, in general, different from that in pattern space. Here two pairs of similar patterns with the same size ratio 2:1 are shown. However, the size ratio of the pair of the patterns on the right side is recognized as smaller than that of the similar patterns on the left side. This example shows that the size ratio of the images of two similar patterns in the image space differs, in general, from that in the pattern space. The above consideration suggests the fact that the parameter $\mathscr{E}^r_\mu$ of the coordinate transformations (6) performed in image space for pattern identification depends, in general, upon the pattern itself.

$$F^{\,a}_{\mu\nu b} = \partial_\mu A^{\,a}_{\nu b} - \partial_\nu A^{\,a}_{\mu b} + [A_\mu, A_\nu]^a_b \ . \tag{13}$$

Hereafter, $F^{\,a}_{\mu\nu b}$ is referred to as the "image field" derived from the "image potential" $A_{\mu\nu}$. Here $[A_\mu, A_\nu]$ represents the commutation relation, $A_\mu A_\nu - A_\nu A_\mu$. Especially if the transformation groups in the image space are commutative $[A_\mu, A_\nu] = 0$, the image field reduces to the electromagnetic field in the n-dimensional Euclidean space,

$$F^{\,a}_{\mu\nu b} = \partial_\mu A^{\,a}_{\nu b} - \partial_\nu A^{\,a}_{\mu b} \ , \tag{14}$$

where $\mu, \nu = 1, 2, \ldots, n$. The identification condition (12) in the above case is satisfied when the potential $A^{\,a}_{\mu b}$ is the gradient field.

In the gauge field theory of the electromagnetic field, $\psi[X]$, X, and $A^{\,a}_{\mu b}$ correspond, respectively, to the complex wave function of the electron, coordinates in four-dimensional Minkowski space, and the vector potential of the electromagnetic field. The group transformations in image space correspond also to the rotation group in the two-dimensional charge space of the complex wave function. Some of the transformations, such as parallel translation or mirror inversion, are considered to be inherently present in the brain, whereas others are newly acquired by external learning or creation in the brain itself. The creation is considered to be closely related to the spontaneous generation of images due to the off-equilibrium property of the brain. It is interesting to study the creation of groups from the viewpoint of pattern formation.[6] If it is possible to find a new transformation group, we can identify the two patterns, which were considered to be different, with each other. In other words, if the integrability condition (11) is satisfied, the pattern identification is established through the suitable coordinate transformations forming the parameter-Lie group. Conversely, if the difference between two images cannot be cancelled out by the suitable transformation groups that are present in the brain, then the image fields are induced and their corresponding patterns are interpreted as having different information contents. The difference between two patterns is represented by the induction of the image field in the brain. The image field in the image space is shown to satisfy the commutation relation of the covariant derivative (8),

$$[\nabla_\mu, \nabla_\nu] \psi^a = F^{\,a}_{\mu\nu b} \psi^b \ . \tag{15}$$

Let us suppose the situation that a pattern is transformed into a final one via two different routes of geometrical transformations. Then, Eq. (15) shows that the final images that are constructed after passing through each route are, in general, different from each other. The optical illusion stated above is considered as a version of the Aharonov and Bohm (AB) effect of the quantum wave function in electromagnetic potentials, and is expected to be observed in psychological experiments of pattern identification. Here the two-dimensional space of the complex wave function corresponds to the M-dimensional image space that was introduced in this section. Namely, the complex wave function of the material wave has the two components, real and imaginary parts, whereas the image vector $\psi[X]$ has M components. The phase of the two-dimensional wave function in the AB effect corresponds to that of the M-dimensional image space. The phase discrepancy between the two different routes of the pattern transformations results in the difference between two final images. The optical illusion seems to have a close relation with the hysteresis effects[6] that are observed in visual perception.

III. FIELD EQUATION OF IMAGE FIELD

First we note the Jacobi identity,

$$\{[\nabla_\lambda,[\nabla_\mu,\nabla_\nu]]+[\nabla_\mu,[\nabla_\nu,\nabla_\lambda]]+[\nabla_\nu,[\nabla_\lambda,\nabla_\mu]]\}\psi^a=0 \ . \tag{16}$$

Substituting Eq. (12) into Eq. (13), we have a field identity equation,

$$\nabla_\lambda F^a_{\mu\nu b}+\nabla_\mu F^a_{\nu\lambda b}+\nabla_\nu F^a_{\lambda\mu b}=0 \ , \tag{17}$$

where the covariant derivative of $F_{\mu\nu}$ is given by

$$\nabla_\lambda F^a_{\mu\nu b}=\partial_\lambda F^a_{\mu\nu b}+[A^a_{\lambda c},F^c_{\mu\nu b}] \ . \tag{18}$$

Following the analogy with the electromagnetic field, we consider the excitation of the image field. First, the Lagrangian $\mathcal{L}$ of the field is defined as

$$\mathcal{L}=F^a_{\mu\nu b}F^b_{\mu\nu a}=\mathrm{Tr}(F_{\mu\nu}F_{\mu\nu}) \ , \tag{19}$$

where Tr means taking the trace of. The Lagrangian (19) has the covariant property under the coordinate transformation (2) in the image space. This is shown by noting the expression of $F_{\mu\nu}$ in the new coordinate frame, ζ'_a, $a=1,2,\ldots,M$,

$$F'_{\mu\nu}=RF_{\mu\nu}R^{-1} \ . \tag{20}$$

Using Eq. (20) and the cyclic property of the trace operator, we have the covariant property,

$$\mathcal{L}'=\mathrm{Tr}(F'_{\mu\nu}F'_{\mu\nu})=\mathrm{Tr}(F_{\mu\nu}F_{\mu\nu})=\mathcal{L} \ . \tag{21}$$

Now let us introduce an important postulate, that the image field is excited following the least-action principle,

$$\delta\mathcal{L}=0 \ , \tag{22}$$

which is necessary in order for the information contraction that is brought about by pattern identification to be maximized by minimizing image-field excitations. Here the terminology of "information contraction" is used in the sense that various modified patterns are reduced to memorized standard patterns. Using Eqs. (13) and (16) and the variational principle (22), we have the field equation,

$$\partial_\nu F^b_{\lambda\nu a}+[A_\nu,F_{\lambda\nu}]^b_a=0 \ , \tag{23}$$

where λ runs over $1,2,\ldots,n$. As in the case with electromagnetic field equations, we can introduce the source $j_\lambda[X]$ of the image field on the right-hand side of Eq. (23),

$$\partial_\nu F^b_{\lambda\nu a}+[A_\nu,F_{\lambda\nu}]^b_a=j^b_{\lambda a}[X] \ . \tag{24}$$

Taking into account the fact that the image field is excited when the images of two different patterns are not transformable through suitable group transformations, we can define the relative information quantity between the two patterns with the use of the Lagrangian $\mathcal{L}$. Here we note that the definition of the relative information quantity has the covariant property (21).

IV. CONCLUSIONS

Based on the psychological experimental fact that images in mental space are transformed into each other for pattern identification, a field theory of images was developed with the use of the methods of gauge theory. It was shown that the image field is excited in the brain when the two images $\psi[X]$ and $\psi[X+\Delta X]$ corresponding, respectively, to the geometrical patterns X and $X+\Delta X$, are not transformable through suitable group transformations. The image field is expected to be observed physiologically as an excited state of the neuronal network in the brain. It is also a forthcoming important problem to find how the group transformations are realized in the neurobiological network. On introducing the postulate that the pattern is identified maximally for information contraction, we derived the excitation equation of the image field, and the covariant expression of relative information quantity between two patterns was defined as excited field energy.

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