

Interrupted fluorescence, quantum jumps, and wave-function collapse

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The phenomenon of antibunching provides important support for the physical principle of the collapse of the wave function following a detection of a photon. In this paper we show that interrupted fluorescence in a three-level, V-configuration atom, with one transition strongly driven and one weakly driven, can be regarded as evidence supporting the complementary principle of "collapse by nondetection." Use of this principle leads to the immediate deduction of remarkably simple general expressions for the average bright and dark periods, which allows these quantities to be calculated for both coherent and incoherent excitation without the need to solve the density-matrix equations, or even to find their long-term steady-state solutions. Thus the concept of collapse by nondetection appears to be not only valid but is also useful for predicting physically observable phenomena.

I. INTRODUCTION

The phenomenon of antibunching of light¹⁻³ from a single two-level atom has a very simple and striking physical interpretation in terms of the detection of a photon "reducing the wave packet."² Immediately after (in retarded time) a photon detection, the atom has a probability of unity of being in the lower state. The observations of antibunching can indeed be taken as evidence of the sudden projection of the atom into the lower state, associated with the sudden conversion of *a priori* probability to a different *a posteriori* probability by the measurement process.⁴

Recently another single-atom effect has received attention.⁴⁻¹² When a three-level atom with states $|1\rangle$, $|0\rangle$, $|2\rangle$ in a V configuration has the $|1\rangle$ - $|0\rangle$ transition strongly driven and the $|0\rangle$ - $|2\rangle$ transition weakly driven as shown in Fig. 1, the fluorescent light from the $|1\rangle$ - $|0\rangle$ transition may sometimes be emitted

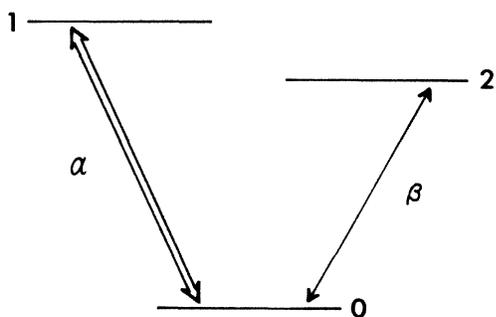


FIG. 1. Schematic outline of energy levels involved in the quantum jumps. States $|0\rangle$ and $|1\rangle$ are strongly coupled by the interaction α , the metastable state $|2\rangle$ is weakly coupled to $|0\rangle$ by the interaction β .

as a series of bright periods of rapid photon emission^{11,12} interrupted by dark intervals of no photon emission. Cook and Kimble⁵ have given a simple description of this process for incoherent irradiation but, in doing so, they assumed from the outset that sudden jumps between bright and completely dark periods do indeed occur.⁸ The assumption leads to difficulties when the incoherent excitation is replaced by coherent excitation with the weak transition driven at the unperturbed atomic resonance frequency, for which the same picture does not apply.^{4,10} Other authors^{4,8} have been able to show that in some circumstances dark periods occur and in others they do not, but unfortunately these treatments lack the simplicity of the Cook-Kimble arguments.

In this paper we develop a simple theory of interrupted fluorescence based on an extension to the concept of the projection into the lower state by the detection of a photon. Recently Dicke¹³ has discussed the apparently paradoxical situation where the failure to detect something can also produce a change in the state of a system, that is, a collapse of the wave packet. Such a failure can, in some circumstances, provide sufficient information to convert *a priori* probabilities to *a posteriori* probabilities in as drastic a manner as a successful detection event. For our present purposes, in order to ensure that the absence of a detection event in the measurement process will convey sufficient information to alter the state vector significantly, we must consider the measurement process to extend over a finite sampling period Δt , which is sufficiently long that if a detection event can occur during Δt , then there is a probability approaching unity that at least one such event will occur during this period.

The approach of this paper, which is in a similar spirit to that of the original work of Cook and Kimble,² shows that interrupted fluorescence is indeed an experimentally accessible example of wave-packet reduction by nondetection. We find that use of this measurement model

enables us to obtain a general formula for the average length of dark and bright periods for both incoherent and coherent excitation and to calculate these lengths in a very simple manner, which involves neither solving the density-matrix equations nor even finding their long-time steady-state solutions. The results obtained are in accord with those obtained by more complicated methods and agree where appropriate with relevant experiments performed so far. The results thus lend support to the validity of the measurement model, and the simplicity with which these results have been obtained indicates its usefulness.

II. SAMPLING PERIOD

The photons which are to be detected are those from the strong transition $|1\rangle \rightarrow |0\rangle$. We consider the measurement process to be that of observing these photons over a finite sampling-time period Δt which is sufficiently large that, if the atom is in one of the strong transition states $|1\rangle$ or $|0\rangle$ during that time, then there is a probability approaching unity that at least one photon will be detected before the end of this period. This means that we need Δt to be much greater than α^{-1} and γ_1^{-1} , where α is a characteristic frequency denoting the strong coupling strength and γ_1 is the spontaneous decay rate for the strong transition. We assume that the coupling strength β for the weak transition and its decay rate γ_2 is very much smaller than α and γ_1 , which allows us to have Δt simultaneously (a) much greater than α^{-1} and γ_1^{-1} and (b) much smaller than β^{-1} and γ_2^{-1} .

The detection of at least one strong-transition photon at some time during Δt immediately projects the atom into state $|0\rangle$, so that the density-matrix element ρ_{00} becomes 1 and the density-matrix element ρ_{22} becomes zero. The condition (b) in the preceding ensures that the continuous density-matrix evolution equations will predict that there is an insignificant change in ρ_{22} during the remainder of Δt , so that $\rho_{22} \simeq 0$ at the end of Δt , but condition (a) allows a rapid evolution of the atomic state in the $|0\rangle$ - $|1\rangle$ plane of Hilbert space during Δt , so that ρ_{11} and ρ_{00} can differ significantly from 0 and 1. Condition (a) ensures that the nondetection of a strong-transition photon during Δt projects the atom into a state orthogonal to the $|0\rangle$ - $|1\rangle$ plane, that is, into $|2\rangle$, so that $\rho_{22} \simeq 1$ at the end of Δt from condition (b).

Thus the reduction or collapse of the wave packet, by either the detection of at least one photon during Δt or by the nondetection of a photon in this time, gives a definite value to ρ_{22} of either 0 or 1 at the end of Δt . The collapse associated with the nondetection event also gives the associated definite values $\rho_{11} = \rho_{00} = 0$ at the end of Δt , but the detection event does not give a value of unity to ρ_{00} at the end of Δt because of condition (a). In fact, following a detection event there will in general be sufficient time for the occupation probability of unity to be shared between $|0\rangle$ and $|1\rangle$ and ρ_{00} will reach a quasiequilibrium value $\bar{\rho}_{00}$ at the end of the Δt period. For example, $\bar{\rho}_{00}$ will be $\frac{1}{2}$ if $\alpha \gg \gamma_1$.¹⁴ Although it introduces this slight complication in the state of the atom following a detection event, compared with $\rho_{00} = 1$ had we chosen Δt to be

much smaller, condition (a) is necessary for a nondetection event to provide enough information to collapse the wave function sufficiently for the atom to be found in a definite state, state $|2\rangle$, with near certainty.

III. PERIODS OF LIGHT AND DARKNESS

A. Bright periods

We let the time $t=0$ be such that it immediately follows a sampling period Δt in which at least one strong-transition photon has been detected. Thus at $t=0$ the atom is in the $|0\rangle$ - $|1\rangle$ plane of Hilbert space and $\rho_{22}(0)=0$. To find the average length of a bright period we wish to find the "life expectancy" T_B , i.e., the expectation value of the time in which the atom will remain *continuously* in the $|0\rangle$ - $|1\rangle$ plane. If the atom is still in this plane at a later time $t=t_1$, e.g., as found by the observation of at least another photon in the sampling period just prior to t_1 , then the life expectancy at t_1 will also be T_B . Thus if we consider the complementary possibilities of remaining or not remaining in the plane until t_1 , with the associated outcomes that the life expectancy is increased to T_B+t_1 or is reduced to an amount less than t_1 , we can write the relation

$$T_B = P_{10}(t_1)(t_1 + T_B) + [1 - P_{10}(t_1)]ft_1, \quad (1)$$

where the survival probability $P_{10}(t_1)$ is the probability of remaining *continuously* in the $|0\rangle$ - $|1\rangle$ plane during t_1 , i.e., the probability that *no* transitions to $|2\rangle$ have occurred. The second term in Eq. (1) is the probability of not surviving t_1 multiplied by the resulting time gained in the event of not surviving the whole of t_1 . The result of not surviving t_1 is zero life expectancy after t_1 , and the second term incorporates the amount of time survived after the beginning of the t_1 interval and before its end. This is somewhere between 0 and t_1 , i.e., ft_1 , where $0 \leq f \leq 1$. From (1),

$$T_B = \frac{t_1}{1 - P_{10}(t_1)} - t_1(1 - f). \quad (2)$$

This is true for all values of t_1 , but to eliminate the unknown f we note that $P_{10}(t_1) \rightarrow 1$ as $t_1 \rightarrow 0$ and so we take the limit as $t_1 \rightarrow 0$, in which case the first term dominates, giving

$$T_B = \lim_{t_1 \rightarrow 0} \frac{t_1}{1 - P_{10}(t_1)}. \quad (3)$$

Now $1 - P_{10}(t_1)$ is the probability that at least one transition to $|2\rangle$ occurs in the time t_1 , which includes, for example, the probability of a transition to $|2\rangle$ and then back to $|0\rangle$. Thus for general times t_1 , $1 - P_{10}(t_1)$ will *not* be the same as $\rho_{22}(t_1)$, which is simply the probability of being in $|2\rangle$ at t_1 . However, for times t_1 sufficiently short, $1 - P_{10}(t_1)$ approaches $\rho_{22}(t_1)$ because the dominant term in each will arise from the single-transition probability $|0\rangle \rightarrow |2\rangle$, which greatly outweighs any multiple-transition probability as $t_1 \rightarrow 0$. We thus have simply

$$T_B^{-1} = \lim_{t_1 \rightarrow 0} \frac{\rho_{22}(t_1)}{t_1} = \left[\frac{d\rho_{22}}{dt} \right]_{t=0}, \quad (4)$$

because $\rho_{22}(t_1) = \rho_{22}(t_1) - \rho_{22}(0)$ as $\rho_{22}(0) = 0$.

B. Dark periods

To find the average length of a dark period, if any indeed exists, we wish to solve the following problem: If no photons are detected during Δt , and therefore the atom state has been projected into $|2\rangle$ at $t=0$, what is the expectation value T_D of the time in which the atom remains continuously in $|2\rangle$? An argument corresponding to that above for the bright periods gives

$$T_D^{-1} = \lim_{t_1 \rightarrow 0} \frac{1 - P_2(t_1)}{t_1}, \quad (5)$$

where the survival probability $P_2(t_1)$ is the probability of remaining continuously in $|2\rangle$ during the time t_1 and so is not equal to $\rho_{22}(t_1)$ for general values of t_1 . Again, however, when t_1 is very small, $1 - P_2(t_1)$, the probability of there being at least one transition out of $|2\rangle$ in t_1 , approaches its dominant term, the probability of one transition out of $|2\rangle$, which is also the limit of $\rho_{22}(0) - \rho_{22}(t_1)$, which here is $1 - \rho_{22}(t_1)$, for small t_1 . Thus we have

$$T_D^{-1} = - \left[\frac{d\rho_{22}}{dt} \right]_{t=0}, \quad (6)$$

to be used in conjunction with $\rho_{22}(0) = 1$.

In order to use (4) or (6) with $\rho_{22}(0) = 0$ or 1, to find T_B or T_D , the values of the other matrix elements $\rho_{ij}(0)$ will also need to be determined. As explained earlier, because the sampling period Δt just prior to $t=0$ is sufficiently long for many transitions between $|0\rangle$ and $|1\rangle$ to occur, the appropriate values to use for $\rho_{ij}(0)$ are $\bar{\rho}_{ij}$, the quasiequilibrium, or short-time steady-state, values which correspond to the particular (effectively constant) value of ρ_{22} . To find $\bar{\rho}_{ij}$ we keep ρ_{22} as a constant in the appropriate density-matrix equations, set $\dot{\rho}_{ik}$ to zero for $\rho_{ik} \neq \rho_{22}$, and solve algebraically to find $\bar{\rho}_{ij}$ as a function of ρ_{22} . This process is similar to adiabatic elimination, as is illustrated in the examples in Sec. IV.

IV. EXCITATION METHODS

A. Incoherent excitation

The rate equations are⁴

$$\dot{\rho}_{11} = -p\rho_{11} + q\rho_{00}, \quad (7)$$

$$\dot{\rho}_{22} = s\rho_{00} - r\rho_{22}, \quad (8)$$

where $q = B_1 W_1$, $p = q + \gamma_1$, $s = B_2 W_2$, and $r = s + \gamma_2$, with γ , B , and W being the Einstein coefficients and spectral radiation density. To use (8) to find T_B and T_D we must use the quasiequilibrium value $\bar{\rho}_{00}$ in place of ρ_{00} . This is easily found by setting $\dot{\rho}_{11}$ equal to zero in (7) and using $\bar{\rho}_{11} + \bar{\rho}_{00} = 1 - \rho_{22}$ as the second equation in the two unknowns $\bar{\rho}_{11}$ and $\bar{\rho}_{00}$. Substituting the resulting value

for $\bar{\rho}_{00}$ into (8) gives

$$\dot{\rho}_{22} = \frac{sp(1 - \rho_{22})}{p + q} - r\rho_{22}. \quad (9)$$

To find T_B we use (9) in conjunction with $\rho_{22}(0) = 0$, and it then follows immediately from (4) that

$$T_B = (p + q)/sp. \quad (10)$$

We note that this, the average length of a bright period, is finite and thus there must also be dark periods. To find T_D we put $\rho_{22} = 1$ in (9) and use (6), giving simply

$$T_D = 1/r. \quad (11)$$

In the limit of a saturated strong transition, the expression for T_B reduces to $2/(B_2 W_2)$: the saturation reduces the ground-state occupation probability to $\frac{1}{2}$ during the bright periods, which periods are terminated by stimulated transitions to the shelf state at a rate $B_2 W_2$. In the opposite limit of a strong γ_1 , T_B is given by $1/(B_2 W_2)$ reflecting the fact that the ground-state probability remains nearly unity during a bright period. T_D is $1/(B_2 W_2 + \gamma_2)$, reflecting the fact that the dark period is terminated by either a spontaneous or a stimulated emission from the shelf state. We note that when both transitions are saturated, $T_B/T_D = 2$, in accord with the equal long-time occupation probabilities for each state, as discussed in Sec. V.

B. Coherent excitation

We consider the case where the strong transition $|1\rangle - |0\rangle$ is driven on resonance with a strong Rabi frequency 2α and the weak transition is driven at weak Rabi frequency 2β by an amount Δ off resonance. The appropriate density-matrix equations, in the usual rotating reference frame, are

$$\dot{\rho}_{20} = i\rho_{20}(\Delta + i\gamma_2/2) + i\beta(\rho_{00} - \rho_{22}) - i\alpha\rho_{21}, \quad (12)$$

$$\dot{\rho}_{21} = i\rho_{21}[\Delta + i(\gamma_1 + \gamma_2)/2] - i\alpha\rho_{20} + i\beta\rho_{01}, \quad (13)$$

$$\dot{\rho}_{22} = i\beta(\rho_{02} - \rho_{20}) - \gamma_2\rho_{22}. \quad (14)$$

To use (14) we need to know the quasiequilibrium value $\bar{\rho}_{02}$. To find this we treat ρ_{22} as a constant in (12), set $\dot{\rho}_{20}$ to zero, and replace ρ_{20} , ρ_{21} , and ρ_{00} by $\bar{\rho}_{20}$, $\bar{\rho}_{21}$, and $\bar{\rho}_{00}$. To find $\bar{\rho}_{21}$ in terms of $\bar{\rho}_{20}$ we set $\dot{\rho}_{21}$ to zero in (13) and also ignore the terms $\beta\bar{\rho}_{01}$ because of the assumed smallness of β . Substituting the resulting expression for $\bar{\rho}_{21}$ into the equation obtained from (12) allows us to find $\bar{\rho}_{02}$ in terms of $\bar{\rho}_{00}$ and ρ_{22} . Combining this with its complex conjugate gives, with a trivial simplification $\gamma_1 \gg \gamma_2$,

$$i\beta(\bar{\rho}_{02} - \bar{\rho}_{20}) = A(\Delta)(\bar{\rho}_{00} - \rho_{22}), \quad (15)$$

where

$$A(\Delta) = \frac{\gamma_1 \alpha^2 \beta^2}{(\Delta^2 - \alpha^2)^2 + \gamma_1^2 \Delta^2 / 4}. \quad (16)$$

Substituting for $i\beta(\rho_{02} - \rho_{20})$ in (14) yields

$$\dot{\rho}_{22} = A(\Delta)\bar{\rho}_{00} - [A(\Delta) + \gamma_2]\rho_{22}. \quad (17)$$

To find T_B we put $\rho_{22} = 0$ in (17) and use (4) to give

$$T_B = A^{-1}(\Delta)\bar{\rho}_{00}^{-1}. \quad (18)$$

Now $\bar{\rho}_{00}$ takes the value $\frac{1}{2}$ for $\alpha \gg \gamma_1$, i.e., for saturation of the strong transition, and tends to 1 for $\gamma_1 \gg \alpha$. In general, it has the value, obtained simply for the well-known on-resonance two-level atom, of $(\alpha^2 + \gamma_1^2/4)/(2\alpha^2 + \gamma_1^2/4)$ (see, e.g., Ref. 14). Thus because $A^{-1}(\Delta)$ is also finite for $\Delta \neq 0$, T_B will be finite and dark periods will exist. For $\Delta = 0$, $A(\Delta)$ in (16) approaches zero for $\alpha \gg \beta, \gamma_1$, and T_B will tend to infinity, making any dark periods exceedingly rare, in agreement with previous predictions.^{4,7,8}

To obtain T_D we set $\rho_{22} = 1$, and thus $\bar{\rho}_{00} = 0$, in (17) and use (6) to yield

$$T_D = \frac{1}{A(\Delta) + \gamma_2}. \quad (19)$$

This is in agreement with the expression obtained by Cohen-Tannoudji and Dalibard⁷ by a dressed-atom approach.

The ratio T_D/T_B is easily found from (19) and (18),

$$\frac{T_D}{T_B} = \frac{A(\Delta)\bar{\rho}_{00}}{A(\Delta) + \gamma_2}. \quad (20)$$

The two extreme values of $\bar{\rho}_{00}$ are 1 if $\gamma_1 \gg \alpha$ and $\frac{1}{2}$ if $\alpha \gg \gamma_1$, and the behavior of T_D/T_B as a function of Δ is determined by (20). In order to compare the former case with the results of Cohen-Tannoudji and Dalibard, we consider the special values of α and β such that $\alpha^2\gamma_2 = \beta^2\gamma_1$. Substitution of this relation into (20) with $\bar{\rho}_{00} = 1$ produces a curve with a maximum of $\frac{1}{2}$ at $\Delta = 0$ and a width at half-height of $4\sqrt{2}\alpha^2/\gamma_1$, in agreement with their results. Similarly, taking $\bar{\rho}_{00} = \frac{1}{2}$ for $\alpha \gg \gamma_1$ and the special relation $4\beta^2 = \gamma_1\gamma_2$, we find a curve for T_D/T_B which is double peaked with maxima of $\frac{1}{4}$ at $\Delta = \pm\alpha$ and with widths at half-height of $2^{-1/2}\gamma_1$, also in accord with the results of Cohen-Tannoudji and Dalibard.⁷

V. PHOTON EMISSION RATES

The period T_B is the time during which the atom is continuously in, or very near to, the $|0\rangle$ - $|1\rangle$ plane of Hilbert space. One might thus expect the behavior of the atom during this time, and thus the statistics of the strong-transition photons which are emitted in the period, to be very similar to that associated with a two-level atom. For example, during this period the average photon emission rate would be simply proportional to $\bar{\rho}_{11}$, the quasiequilibrium value of ρ_{11} obtainable from two-level dynamics.

This is verified by explicit calculations of ρ_{11} from the appropriate rate or Bloch equations.^{4,8} Such calculations show two-level dynamics on short-time scales with associated quasisteady states being achieved. At longer times the shelving transition comes into play and reduces the

quasi-steady-state value. The photon statistics similarly reflect this disparity in time scales: at short times the degree of second-order coherence is that for a two-state system.^{4,8}

During the period T_D following a bright period there are no photons emitted, so that the average transition rate over the total time $T_B + T_D$ is proportional to $\bar{\rho}_{11}T_B(T_B + T_D)^{-1}$. This rate, however, must be proportional to $\rho_{11}(\infty)$, the long-time three-level equilibrium value, and equating the two gives

$$\frac{T_D}{T_B} = \frac{\bar{\rho}_{11}}{\rho_{11}(\infty)} - 1. \quad (21)$$

Thus, for example, in cases where $\bar{\rho}_{11} \approx \rho_{11}(\infty)$, which will be so when setting $\beta = 0$ in the formula for $\rho_{11}(\infty)$ to obtain $\bar{\rho}_{11}$ makes very little difference to $\rho_{11}(\infty)$, the ratio approaches zero. This is essentially the argument used by Knight *et al.*⁴ to demonstrate the absence of dark periods. In general, $\bar{\rho}_{11} > \rho_{11}(\infty)$ as shown in earlier work^{4,8} and indeed the existence of periods of darkness can be inferred directly from the existence of a significant hump in the graph of ρ_{11} against time, which shows the difference between the quasisteady state and the real steady-state values.

As a check on Eq. (21) we consider the incoherent excitation case for which $\bar{\rho}_{11}$ and $\rho_{11}(\infty)$ are easily found.⁴ We find from (21)

$$\frac{T_D}{T_B} = \frac{sp}{r(p+q)}, \quad (22)$$

which agrees precisely with the ratio found from (10) and (11), which confirms, at least for this case, that during a bright period the photon statistics are very similar to two-level atom photon statistics.

VI. DISCUSSION

It is interesting to compare the approach in this paper with that of Cohen-Tannoudji and Dalibard,⁷ who introduced a new function $\omega_2(\tau)$ which generates the probability of detecting the next photon at time τ after one at time zero. This represents a distribution function for there to be a delay of τ between emissions. $\omega_2(\tau)$ is related to the probability $P(\tau)$ that *no* emissions occur between 0 and τ by

$$P(\tau) = 1 - \int_0^\tau \omega_2(\tau') d\tau', \quad (23)$$

i.e.,

$$\omega_2(\tau) = -\frac{dP(\tau)}{dt}. \quad (24)$$

We can derive $P(\tau)$ as the survival probability within a dressed-atom manifold of states.

The hypothesis that there are two very different time constants allows the separation of terms,

$$P(\tau) = P_{\text{short}}(\tau) + P_{\text{long}}(\tau), \quad (25)$$

where

$$P_{\text{long}}(\tau) = P_L \exp(-\tau/\tau_L) \quad (26)$$

and $P_{\text{short}}(\tau)$ tends to zero rapidly with a short-time scale $\tau_S \ll \tau_L$. Time is now subdivided into bins of length θ such that $\tau_S \ll \theta \ll \tau_L$ and an interval between light pulses is defined as short or long depending on whether it is less than or greater than θ . Then $P_{\text{short}}(\theta) \simeq 0$ because this component of $P(\tau)$ decays very rapidly, and so $P(\theta) \simeq P_L$, which will be the probability of a long interval because $P(\theta)$, is just the probability of no photon in the interval θ .

This leads eventually to the calculation of average long and short intervals. It is only for the average long interval, or dark period, that we can make a direct connection with our approach in this paper. This is given by

$$T_L = \frac{1}{P_L} \int_{\theta}^{\infty} \omega_2(\tau) \tau d\tau. \quad (27)$$

Using (24) to integrate this by parts and neglecting the terms which are much smaller than the main term gives

$$T_L = \frac{1}{P_L} \int_{\theta}^{\infty} P_{\text{long}}(\tau) d\tau. \quad (28)$$

We recognize $P_{\text{long}}(\tau)/P_L$ as the survival probability that, if the atom is initially in the manifold at time θ , it will still be surviving against weak-transition photon emission at τ . Writing this as $[P_2(\delta\tau)]^n$, where $P_2(\delta\tau)$ is the survival probability for a period $\delta\tau$ and $\tau = n \delta\tau$, we can write (28), upon setting $\theta \simeq 0$, as

$$T_L = \lim_{\delta t \rightarrow 0} \sum_{n=0}^{\infty} [P_2(\delta\tau)]^n \delta\tau = \lim_{\delta t \rightarrow 0} \frac{\delta t}{1 - P_2(\delta\tau)}, \quad (29)$$

where we have summed the geometric series. Comparison of (29) with (5) shows the connection between our approach and that of Cohen-Tannoudji and Dalibard.⁷ Such a comparison between the bright periods derived by the two approaches is not as straightforward, but the dark-period comparison is the more important because (5) is based on our collapse by the nondetection measurement model. This provides direct evidence that this model incorporates, in a different form, the essential physics input of the approach of Cohen-Tannoudji and Dalibard. Indeed, a closer analysis shows a clear correspondence between θ and our Δt .

VII. CONCLUSION

The concept of the collapse of the wave packet, or reduction of the wave function, upon detection of a photon has proved to be a useful means of understanding the phenomenon of antibunching both qualitatively and quantitatively. Indeed, antibunching can be regarded² as strong evidence supporting this fundamental physical principle of measurement. Zoller *et al.*⁸ have incorporated the principle of wave-packet collapse upon detection into their study of quantum jumps. In this paper we have made use of an extension of this concept, collapse by nondetection, to analyze the phenomenon of interrupted fluorescence. We find that the concept leads directly to remarkably simple general expressions for T_B and T_D , the average length of the bright and dark periods from a

single V -configuration three-level atom with the $|1\rangle$ - $|0\rangle$ transition strongly driven and the $|0\rangle$ - $|2\rangle$ transition weakly driven. The measurement process involves observation of strong-transition photons over a finite sampling time, and the result of the measurement, either the detection or nondetection of at least one photon in this time, projects the atomic state either onto the $|0\rangle$ - $|1\rangle$ plane of Hilbert space or onto the orthogonal state $|2\rangle$. The general expressions for T_B and T_D are simple to apply for both coherent and incoherent excitation without the need to solve the density-matrix equations or even to find the long-time steady-state solutions. The results obtained appear to be in agreement with those found by more complicated theoretical methods and agree with experimental results^{11,12} where appropriate. Thus our results show that interrupted fluorescence can be regarded as strong evidence supporting the physical principle of wave-function collapse by nondetection, a concept discussed by Dicke in a different context. Our results also suggest that wave-function collapse, at least of this type, is associated with the gaining of sufficient information to cause a significant difference between the *a priori* and *a posteriori* probabilities, rather than being caused, for example, by some direct physical interaction involved in the measurement process.

Our results also support the picture of the atomic-state "jumping" between the $|0\rangle$ - $|1\rangle$ plane and the state $|2\rangle$ with the associated switching on and off of the strong-transition fluorescence. The time taken for the jump is very much less than T_B and T_D and so will appear as a sudden, discontinuous change on a long-time scale. However, we cannot verify that this apparent discontinuity persists on a short-time scale. We cannot say that the time occupied by the jump is much shorter than the minimum measurement sampling period Δt which is necessary to project, *with reasonable certainty*, the atomic state onto $|2\rangle$. Taking the sampling time to shorter-time scales than a few times the average time between strong-transition photon emissions during a bright period seriously reduces the information produced by a nondetection event. This reduces the difference between *a priori* and *a posteriori* probabilities, so that atomic state is not completely projected onto $|2\rangle$. For sampling times much smaller than the average time between strong-transition photons, a nondetection event provides virtually no information and barely alters the wave function at all. Thus we cannot say that the time taken for a jump to the shelving state $|2\rangle$ is less than approximately γ_1^{-1} for $\alpha \gg \gamma_1$.

As an added note, since this paper was first submitted, a paper by Porrati and Putterman¹⁵ has been published which, while taking a very different approach from that of this paper, also associates interrupted fluorescence with wave-function collapse following a failure to observe a photon.

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