X-ray absorption in atoms under intense laser fields

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(Received 27 August 1987)

The absorption of x rays in atoms assisted by an intense laser field is considered. The general expression for the x-ray absorption cross section (σ_x) as a function of the laser field is obtained in the intense-field regime using the Seely and Harris approximation [Phys. Rev. A 7, 1064 (1973)]. We find that in this regime, and when the two photon fields are polarized in the same direction, the cross section σ_x decreases as ε^{-1}, ε being a factor proportional to the intense-field amplitude. When the field is further increased to the limit of superintense fields, σ_x varies as ε^{-3}. A physical explanation for this behavior is provided.

I. INTRODUCTION

The advent of the laser has stimulated the study of multiphoton processes in atoms,[1-4] plasmas,[5,6] and solids.[7-9] In particular, multiphoton absorption has been investigated (in Ref. 10) in connection with the problem of x-ray absorption in atoms. In Ref. 10 the authors considered the changes in the cross section of deep-level absorption of x rays in an atom due to the simultaneous irradiation of the system by an intense optical laser field. Considering the electromagnetic interaction between the x-ray photon and the electron as the perturbation, transition probabilities were calculated from which the x-ray cross section σ_x was obtained. Formally, the formula for the cross section obtained in Ref. 10 solves the problem of the influence of the intense field on the processes of electron scattering. But the result (containing an infinite series of Bessel functions) is not satisfactory from the physical point of view. So it is necessary to learn to analyze such expressions when the argument of the Bessel function is not small and, consequently, it is impossible to restrain the infinite series in n (n is an integer) to the first few terms.

In this paper we shall consider the x-ray absorption in atoms in the presence of an intense laser field by using a similar approach to the one used by Seely and Harris[6] in the study of multiphoton inverse bremsstrahlung in plasmas. They noticed that when the argument of the Bessel function is large (intense-field regime), in the whole series only those terms for which the order is equal to the argument are essential and they approximated a typical series of Bessel functions by the sum of two δ functions, as shown below. This approximation is simple and very convenient in treating the problem in the intense-field regime and it has successfully been employed for the study of multiphoton processes in solids.[8,9,11,12]

We have therefore considered an x-ray beam interacting with a deep-level atomic electron in the simultaneous presence of an intense laser field. The laser field is considered to be small relative to atomic fields but strongly interacting with the outgoing electron represented by a plane wave. Considering the electromagnetic interaction between the x-ray photon and the electron to be the perturbation, transition probabilities are calculated between the unperturbed states. The electrons are treated quantum mechanically, while the laser field is treated classically, since there are a large number of photons in the same state. Having obtained the transition probability the x-ray absorption cross section is calculated using the Seely and Harris approximation. One finds a simple expression for the cross section with a similar behavior to that obtained, for instance, in processes such as atomic ionization in ultrastrong laser fields.[3]

II. CALCULATIONS

The S matrix which describes the electronic transition between states i and f due to the interaction of the electron, x-ray, and photon field in the presence of a superintense laser field is taken as

$$S_{fi}=-\frac{i}{\hbar}\int_{-\infty}^{+\infty}dt\,d^3\mathbf{r}\,\psi_f^*(\mathbf{r},t)H'(t)\psi_i(\mathbf{r},t)\ , \tag{1}$$

where $\psi_f(\mathbf{r},t)$ is the intense-field-dependent wave function for the ejected electron, $\psi_i(\mathbf{r},t)$ is the intense-field-free wave function for the bound electron (considered to be, for the sake of simplicity, the hydrogenlike $1s$ state), and

$$\begin{aligned}H'(t)&=-(e/mc)\,\mathbf{A}_x\cdot\mathbf{P}\\&=-(e/mc)A_{0x}(\hat{\mathbf{z}}\cdot\mathbf{P})\exp(i\omega_x t)\end{aligned} \tag{2}$$

is the interaction Hamiltonian arising from a linearly polarized x-ray radiation, ω_x being the x-ray frequency.

The initial electronic state ψ_i is in this case given by

$$\psi_i(\mathbf{r},t)=(\pi a^3)^{-1/2}\exp\left[-\frac{r}{a}-\frac{iE_b t}{\hbar}\right]\ , \tag{3}$$

where $E_b=Z^2e^2/2a_0$ represents the binding energy of the electron to the nucleus and $a=a_0/Z$, with a_0 the Bohr radius and Z the nuclear charge of the atom.

A Volkov-type wave function is used to represent the outgoing electron modulated by the intense field and it is given by

$$\psi_f(\mathbf{r},t)=\exp\frac{i}{\hbar}\left[\mathbf{p}\cdot\mathbf{r}+\frac{1}{2m}\int_0^t\left|\mathbf{p}-\frac{e}{c}\mathbf{A}_L(t')\right|^2 dt'\right] . \quad (4)$$

If we assume the laser field to be a linearly polarized wave of frequency ω, the vector potential can be written as

$$\mathbf{A}_L(t)=A_{0L}\hat{\mathbf{z}}\cos(\omega t) . \quad (5)$$

Substituting Eqs. (2)–(5) into (1), the matrix element S_{fi} is found to be

$$S_{fi}=-\frac{ie}{mc\hbar}p_z A_{0x}\left[\frac{1}{\pi a^3}\right]^{1/2}\frac{8\pi a^3}{(1+a^2p^2/\hbar^2)^2}\times\int_{-T/2}^{+T/2}dt\,\exp\frac{i}{\hbar}\left[\epsilon t+\frac{\lambda}{\omega}\sin(\omega t)\right] , \quad (6)$$

where

$$\epsilon=-(p^2/2m)+\hbar\omega_x-E_b ,$$

$$\lambda=(eE_{0L}/m\omega)p_Z,\quad p_Z\equiv p\cos(\mathbf{p},\hat{\mathbf{z}}) .$$

Here E_{0L} is the intense-laser-field amplitude. Using a Bessel function expansion for $\exp[(i\lambda/\hbar\omega)\sin(\omega t)]$, the time integration for the S matrix can be performed and the transition probability per unit time can be obtained,

$$\frac{|S_{fi}|^2}{T}=\frac{128\pi^2A_{0x}^2p_Z^2a^3}{m^2c^2(1+a^2p^2/\hbar^2)^4\hbar}\sum_{n=-\infty}^{+\infty}J_n^2(\lambda/\hbar\omega)\times\delta(\epsilon-n\hbar\omega) , \quad (7)$$

where J_n is the Bessel function of order n.

In obtaining Eq. (6), we have omitted the term $e^2A_L^2(t)/2mc^2$ from the argument of the exponential function. This term represents the kinetic energy shift for an electron having a momentum $\mathbf{p}$, in the presence of the laser field $\mathbf{E}_L(t)$. This may easily be understood by noticing that, similarly, the second-order energy shift of the ground state appearing in the expression for $\psi_i(\mathbf{r},t)$, Eq. (3), is given by

$$\Delta E_b=\sum_I\left[\frac{\langle 0|H_1|I\rangle\langle I|H_1|0\rangle}{E_I-E_b}\right] ,$$

where $(E_I-E_b)\gg\hbar\omega$. Here $|I\rangle$ and E_I are the excited-state eigenfunctions and eigenvalues, respectively, and

$$H_1=-(ie\hbar/mc)\nabla\cdot\mathbf{A}_L(t) .$$

Using the fact that $[\mathbf{r},H_1]=i\hbar\mathbf{p}/m$, we find that

$$\Delta E_b=\frac{e^2A_L^2(t)}{2mc^2\hbar^2}\langle 0|[z,p_z]|0\rangle=\frac{e^2A_L^2(t)}{2mc^2} ,$$

which equals the uniform energy shift of the final electron state. We thus obtain the result that ΔE_b exactly cancels the $A_L^2(t)$ term in the argument of the exponential function in Eq. (6). It thus follows that the energy E_b is truly the unperturbed ground-state energy of the bound electron.

The presence of the multiphoton processes is evident from the energy-conservation requirement in the argument of the δ function in (7). The interpretation is that positive values of n correspond to the absorption of n photons, negative values correspond to the emission of $|n|$ photons, and summation over all n's must be carried out to add up the contributions of all such processes to the absorption cross section.

To obtain the x-ray scattering cross section we divide the transition probability per unit time [Eq. (7)] by the incident flux of x rays and sum over all possible values of p, the electron momentum. One gets

$$\sigma_x=\frac{32\pi e^2a^3}{m^2c\omega_x\hbar^3}\times\sum_{n=-\infty}^{+\infty}\int_0^\infty dp\int_0^\pi d\theta\frac{p^4\cos^2\theta\sin\theta}{(1+a^2p^2/\hbar^2)^4}J_n^2\left[\frac{\lambda}{\hbar\omega}\right]\times\delta(\epsilon-n\hbar\omega) . \quad (8)$$

In the regime of intense fields, $\lambda\gg\hbar\omega$ and the argument of the Bessel function in Eq. (8) is large. Of course, $\lambda\gg\hbar\omega$ depends upon the direction of $\mathbf{p}$. However, since we shall be mainly interested in $\mathbf{p}$ parallel to the field amplitude $\mathbf{E}_0$, which is the direction of acceleration of the electrons in the field, the condition $\lambda\gg\hbar\omega$ is essentially $|\mathbf{E}_0|$ large. For large values of argument, the Bessel function is small, except when the order is equal to the argument. The sum over n in Eq. (8) may then be written approximately as[6]

$$\sum_{\substack{n=-\infty\\ n\neq 0}}J_n^2(\lambda/\hbar\omega)\delta(\epsilon-n\hbar\omega)\simeq\tfrac{1}{2}[\delta(\epsilon-\lambda)+\delta(\epsilon+\lambda)] . \quad (9)$$

The factor $\frac{1}{2}$ may be verified by the integrating both sides of Eq. (9) with respect to ϵ. The first δ function corresponds to the absorption and the second to the emission of $\lambda/\hbar\omega$ photons of the intense laser field. Since $\lambda\gg\hbar\omega$, only multiphoton processes are significant. Substituting Eq. (9) into Eq. (8), the x-ray cross section becomes

$$\sigma_x=\frac{16\pi e^2a^3}{m^2c\omega_x\hbar^3}\int_0^\infty\frac{dp\,p^4}{(1+a^2p^2/\hbar^2)^4}\times\int_0^\pi\cos^2\theta\sin\theta\,d\theta[\delta(\epsilon-\lambda)+\delta(\epsilon+\lambda)] . \quad (10)$$

On carrying out the θ integration using the δ functions, Eq. (10) becomes

$$\sigma_x = \frac{16\pi e^2 a}{m^2 c\hbar\omega_x \alpha_0^3} \int_0^\infty d\tilde{p}\, \tilde{p} \frac{(E_x - E_b - b\tilde{p})^2}{(1+\tilde{p})^4} \quad (\alpha_0 \equiv eE_0/m\omega) \ . \tag{11}$$

In (11) $\tilde{p}$ is a dimensionless quantity and $b \equiv \hbar^2/2ma^2$. The integration over $\tilde{p}$ is immediate and we finally obtain for the x-ray cross section (normalized to the zero laser field x-ray cross section σ_0) the expression

$$\frac{\sigma_x}{\sigma_0} = \frac{1}{8} \frac{(1+1/\varepsilon)^2}{\eta^{7/2}\varepsilon(1+1/\eta+\varepsilon/\eta)^2} \ , \tag{12}$$

with

$$\varepsilon \equiv 2eE_0 a/\hbar\omega \ ,$$

$$\eta \equiv 2ma^2(\hbar\omega_x - E_b)/\hbar^2 \ .$$

III. DISCUSSION

In Sec. II we obtained the absorption cross section for the x rays in atoms in the presence of an intense laser field in the limit of intense fields by making use of the Seely and Harris approximation of plasma physics.[6] This intense-field result is of use only when processes involving many photons ($n \gg 1$), as is the case here, are being considered. With the present-day pulsed lasers available, this intense-field result, Eq. (12), is more likely to be of great interest than the low-field result treated in Ref. 10.

Equation (12) immediately tells us that there will be x-ray absorption only if $\hbar\omega_x > E_b$, as expected. The intense-field regime of an optical laser field means that $E_0 \sim 10^8$ V/cm, and since the available x-ray sources emit quanta $\hbar\omega_x$ in the range 1–100 keV, $\eta \sim 10^2 - 10^4$, so that σ_x/σ_0 is seen from (12) to decrease rapidly as ε^{-1}. This is a behavior which is reminiscent of other phenomena involving multiphoton processes, such as multiphoton ionization in atomic systems.[3] If we further increase the field to the limit of superintense fields corresponding to intensities of the order of 10^{18} W cm^{-2} (which is the critical intensity for our dipole approximation[13]), we find from (12) that in this case σ_x/σ_0 drops very rapidly as ε^{-3}. Physically this may be understood as follows: Consider the problem of one electron in the laser field $\mathbf{A}_L(t)$ and moving in the Coulomb potential V of the nucleus; we have

$$H = \frac{1}{2m} |\mathbf{p} - (e/c)\mathbf{A}_L(t)|^2 + V = H_0 + V \ .$$

In the case of superintense fields, such that $|e\mathbf{A}_L| \gg |V|$, the above Hamiltonian may be approximated by the first term H_0, which is the free-particle Hamiltonian. In order words, for ε very large the electron has large kinetic energy and is no longer affected by the nucleus; the electron-nucleus interaction becomes "frozen." Alternatively, we may say that the effective coupling between the final electronic state and the initial Coulomb wave function is very much reduced. This results in the reduction of σ_x/σ_0 in superintense fields.

In conclusion, we have made use in this paper of the Seely and Harris approximation[6] of plasma physics to calculate the x-ray absorption cross section in atoms in the presence of an intense laser field. This intense-field approximation, Eq. (9), leads to a straightforward determination of the cross section without need of power-series expansions or graphical solutions such as the ones used in Ref. 10. Moreover, this approximation is able to reproduce very well universal features reminiscent of other phenomena involving multiphoton processes and could be of great utility for investigating high-field effects in other problems involving electron-photon scattering.

ACKNOWLEDGMENTS

The authors wish to thank the (CNPq) Conselho Nacional de Pesquisas for financial support. The authors also want to thank the University of Brasília for financial support.

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