Dressed-state analysis of three-level quantum-jump experiments

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Recent experiments with single atomic ions isolated in electromagnetic traps have demonstrated abrupt changes in observed fluorescence intensities, which have been interpreted as evidence for quantum jumps as the atom changes energy levels. This presents the seeming paradox of discontinuously changing observables from a system whose dynamical evolution is governed by the continuous optical Bloch equations. In this paper I present an analysis of the dynamics of a model three-level atom interacting with two laser fields in terms of the dressed states of the atom-laser system. These dressed states are eigenstates of a Hamiltonian which includes laser-atom interactions, and thus they simplify the discussion of the time evolution of the combined laser-atom system. This dressed-state description is particularly useful in the regime in which both laser intensities are strong enough to saturate their respective transitions. I calculate the radiative lifetimes of the different species of dressed states and the branching ratios for decay to other dressed states, and demonstrate the consequences of these quantities for the observed fluorescence pattern. The calculated duration of bright and dark periods shows a pronounced dependence on the detunings of the lasers.

I. INTRODUCTION

The recently developed experimental capability to isolate single cold ions in electromagnetic traps provides an opportunity for novel investigations of the details of the interaction of light and matter. Such experiments can probe subtle properties which are often masked by the large numbers of atoms or ions in conventional experiments. In particular, the potential for the direct observation of so-called quantum jumps of an atom as it changes state has attracted a great deal of experimental[1–5] and theoretical[6–13] interest. Such experiments present the seeming paradox of discontinuous changes in experimentally observed quantities in a system whose dynamics are described by continuous differential equations. In this paper I analyze the dynamics of such quantum-jump experiments in terms of the dressed states of the atom-laser system. The dressed states, being eigenstates of a Hamiltonian including laser-atom interactions, simplify the analysis of the time evolution of the system, and also offer a simple means of calculating many experimentally accessible quantities. I will find the dressed states relevant to a model quantum-jump experiment and I will calculate the fluorescence pattern that would be detected as a function of laser intensities and laser detunings.

The system I will consider in this paper consists of three atomic levels which are coupled by two single-mode radiation fields as indicated in Fig. 1. The eigenstates of the atomic Hamiltonian with no radiation present will be labeled $|0\rangle$, $|1\rangle$, and $|2\rangle$. For convenience I will refer to the energy difference between $|0\rangle$ and $|1\rangle$ as corresponding to the emission or absorption of a blue photon and the difference between $|0\rangle$ and $|2\rangle$ as corresponding to emission or absorption of a red photon. Ω_B and Ω_R represent the blue and red Rabi frequencies, respectively, and Δ_B and Δ_R the detuning of the blue and red lasers from the zero-field atomic resonances. The spontaneous decay rates of the states are given by Γ_B and Γ_R. For ease of calculation of the dressed states I will assume that both of the transitions are strongly driven, i.e., $\Omega_B \gg \Gamma_B$, $\Omega_R \gg \Gamma_R$, and I will assume that the blue transition is much more strongly driven than the red transition, i.e., $\Omega_B \gg \Omega_R$.

According to the original proposal of Dehmelt,[14] a weak red transition could be detected by monitoring the resonance fluorescence of the strong blue transition. The blue fluorescence would always be "on" and detectable unless the atom happens to have made one of its infrequent transitions to $|2\rangle$ by the absorption of a red pho-

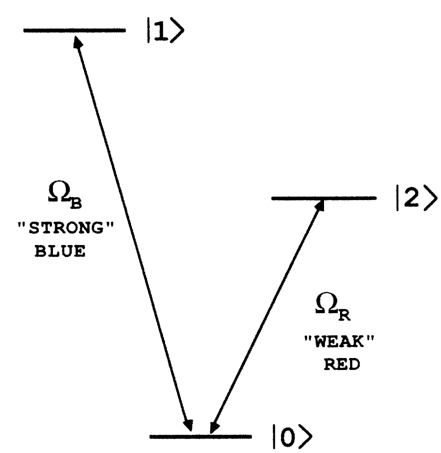

FIG. 1. "V" configuration for a single-atom quantum-jump experiment. The labeled states are eigenstates of the atomic Hamiltonian alone. The 0-1 transition is "strong" and the 0-2 transition is "weak." The 0-1 transition will be referred to as blue and the 0-2 transition as red.

ton. This transition abruptly terminates the bright period of blue fluorescence and the blue fluorescence remains off as long as the atom is shelved in the long-lived state $|2\rangle$ because the atom cannot absorb and subsequently reemit blue photons while it is in $|2\rangle$.

The validity of such a simple picture of the three-state problem is not evident when at least one of the transitions is strongly driven because the zero-field atomic states $|0\rangle$, $|1\rangle$, and $|2\rangle$ are no longer eigenstates of the problem when the atom-radiation field coupling is not small. In this case the probability amplitudes of these states evolve in time yielding a three-state generalization of Rabi flopping. When the pure atomic states $|0\rangle$, $|1\rangle$, and $|2\rangle$ are employed as basis states, the time evolution of the system is given by the optical Bloch equations[15] which completely specify the density-matrix elements $\rho_{ij}(t)$. The recovery of the discontinuous fluorescence signal predicted by the simple-minded analysis mentioned above is not immediately apparent from such continuous equations.

By now it is well known that such abrupt changes in the observed fluorescence do occur in real single-ion experiments[1–4] similar to the model under investigation in this study, and the origin of such discontinuities in terms of the optical Bloch equations has been investigated by several authors.[6–12] In this paper I propose a simple alternative view of the problem of fluorescence from a three-level atom. To avoid the seeming paradox of discontinuous fluorescence levels from a system with continuously evolving probability amplitudes it is fruitful to consider the dressed-state[16] picture of the three-level atom interacting with two single-mode radiation fields. In the limit where spontaneous emission can be treated as a small perturbation, the eigenstates of the complete Hamiltonian (including atom-field interactions) can be found exactly. The composition of the eigenstates of this complete Hamiltonian is time independent and the probability amplitudes of the states are constant. Using such a "dressed-state" basis of states greatly simplifies the physical picture of the three-level atom in two radiation fields. Furthermore, the occurrence of dark periods in the fluorescence from the single atom can be analyzed simply in terms of the lifetimes of the dressed states against radiative decay and branching ratios of the dressed states as they decay to lower-energy dressed states.

(Cohen-Tannoudji and Dalibard have applied what they term a dressed-atom approach to this problem in Ref. 13. In the bulk of their work, however, the states are "dressed" only to the extent that the numbers of photons in the relevant laser modes are included in the characterization of the states. The basis states they employ are eigenstates of the free-atomic and free-field Hamiltonians, but are not eigenstates of a Hamiltonian including laser-atom interactions. These states are not stationary states, and the existence of dark and bright periods must be inferred from analysis of the time-varying coefficients of these basis states. The approach of Ref. 13 is particularly useful at low laser intensities, whereas the present paper is applicable to the regime where both transitions are saturated, and the completely dressed states give a clear physical picture of the dynamics.)

I will begin this paper with a discussion of the dressed states appropriate to the problem at hand. I will find the dressed states for a model atom in the limit in which spontaneous emission can be ignored. I will then derive the lifetimes of the dressed states by the addition of spontaneous emission to the problem and demonstrate the existence of long-lived dressed states and discuss the observable consequences of these long-lived states. The properties of the dressed states will be demonstrated to depend critically on the detunings of the lasers in ways that should be readily discernible in the fluorescence pattern detected in a single-ion experiment.

II. DRESSED STATES OF THE THREE-LEVEL ATOM

The complete Hamiltonian for the problem includes the sum of the internal atomic energy, the energy in the two modes of the radiation field, and the atom-field interaction energy. If we include the interaction with the fields of the two laser modes, but ignore the interaction with the empty modes of the radiation field which are responsible for spontaneous decay, the rotating-wave-approximation Hamiltonian is

$$\begin{aligned} H &= H_{\text{atom}} + H_{\text{field}} + H_{\text{atom-field}} \\ &= \hbar(\omega_B - \Delta_B)|1\rangle\langle 1| + \hbar(\omega_R - \Delta_R)|2\rangle\langle 2| \\ &\quad + \hbar\omega_B a_B^{+} a_B + \hbar\omega_R a_R^{+} a_R \\ &\quad + (a_B V_{01}|0\rangle\langle 1| + a_R V_{02}|0\rangle\langle 2| + \text{H.c.})\,, \end{aligned} \tag{1}$$

where ω_B and ω_R are the frequencies of the blue and red photons which have detunings $\Delta_B = \omega_B - \omega_{01}$, $\Delta_R = \omega_R - \omega_{02}$ from the zero-field atomic resonance frequencies, a_B, $a_B^{\dagger}$, a_R, and $a_R^{\dagger}$ are the annihilation and creation operators for the blue and red photons, respectively, and V_{01} and V_{02} are the matrix elements (not necessarily dipole) connecting the zero-field atomic states.

If the coupling between the atom and the laser fields is ignored, the eigenstates of the combined atom-laser system are simply specified by $|S, N_B, N_R\rangle$, where S indicates the state of the atom ($S=0$, 1, or 2) and N_B and N_R represent the number of photons in the blue and red radiation fields, respectively. These eigenstates are grouped in nearly degenerate three-state manifolds as indicated in Fig. 2. The states within a given manifold are separated by the detunings of the lasers, and the manifolds are separated in energy from nearby manifolds by the energy of the blue and red photons. This is in contrast to the more simple energy ladder of the two-state atom dressed with a single radiation field.

The atom-field coupling, which is responsible for stimulated absorption and emission processes, connects the states within a given manifold. This coupling is related to the conventional Rabi rates in the following manner:

$$\frac{2}{\hbar}\langle 0,N_B,N_R \mid H_{A-F} \mid 1,N_B-1,N_R\rangle = \frac{2}{\hbar}V_{01}\sqrt{N_B}$$
$$= \Omega_B \exp(i\phi_B)\ , \tag{2a}$$

$$\frac{2}{\hbar}\langle 0,N_B,N_R \mid H_{A-F} \mid 2,N_B,N_R-1\rangle = \frac{2}{\hbar}V_{02}\sqrt{N_R}$$
$$= \Omega_R \exp(i\phi_R)\ . \tag{2b}$$

The states such as $|0,N_B,N_R\rangle$ are not eigenstates of the Hamiltonian which includes the atom-field coupling. The procedure for obtaining the new eigenstates is standard and has been previously applied to several problems in optical physics,[17,18] and will be reviewed here. The "dressing" of the states simply corresponds to the diagonalization of a 3×3 matrix representation of the Hamiltonian because the atom-laser coupling only connects states within a given three-state manifold. This diagonalization mixes the states within each manifold resulting in three new eigenstates which are linear combinations of the uncoupled atom-laser states $|0,N_B,N_R\rangle$, $|1,N_B-1,N_R\rangle$, and $|2,N_B,N_R-1\rangle$. These linear combinations are the dressed states of the combined atom-laser system. The exact composition of the states depends, of course, on the detunings and intensities of the two laser fields and will in general be given by a unitary transformation $\tilde{U}$,

$$|I\rangle = \sum_i u_{Ii}\,|i\rangle\ , \tag{3}$$

where upper case letters refer to states in the coupled basis and lower case letters to states in the uncoupled basis, and u_{Ii} are the elements of the transformation $\tilde{U}$. The dressed states are still grouped in three-state manifolds as in Fig. 2, but with new energy splittings within the manifolds.

In the special case where $\Delta_B=\Delta_R=0$ it is easy to find an analytic form of the transformation and the explicit compositions of the dressed states are

$$|\alpha\rangle = \frac{1}{\sqrt{2}}\left[\,|0,N_B,N_R\rangle + \frac{\Omega_B e^{-i\phi_B}}{(\Omega_B^2+\Omega_R^2)^{1/2}}\,|1,N_B-1,N_R\rangle + \frac{\Omega_R e^{-i\phi_R}}{(\Omega_B^2+\Omega_R^2)^{1/2}}\,|2,N_B,N_R-1\rangle\right]\ , \tag{4a}$$

$$|\beta\rangle = \frac{1}{\sqrt{2}}\left[-|0,N_B,N_R\rangle + \frac{\Omega_B e^{-i\phi_B}}{(\Omega_B^2+\Omega_R^2)^{1/2}}\,|1,N_B-1,N_R\rangle + \frac{\Omega_R e^{-i\phi_R}}{(\Omega_B^2+\Omega_R^2)^{1/2}}\,|2,N_B,N_R-1\rangle\right]\ , \tag{4b}$$

$$|\gamma\rangle = -\frac{\Omega_R e^{i\phi_R}}{(\Omega_B^2+\Omega_R^2)^{1/2}}\,|1,N_B-1,N_R\rangle + \frac{\Omega_B e^{i\phi_B}}{(\Omega_B^2+\Omega_R^2)^{1/2}}\,|2,N_B,N_R-1\rangle\ . \tag{4c}$$

The energies of the zero-detuning dressed states which were given explicitly in Eq. (4) are

$$E_\alpha = N_B\hbar\omega_B + N_R\hbar\omega_R + \tfrac{1}{2}\hbar(\Omega_B^2+\Omega_R^2)^{1/2}\ , \tag{5a}$$

$$E_\beta = N_B\hbar\omega_B + N_R\hbar\omega_R - \tfrac{1}{2}\hbar(\Omega_B^2+\Omega_R^2)^{1/2}\ , \tag{5b}$$

$$E_\gamma = N_B\hbar\omega_B + N_R\hbar\omega_R\ . \tag{5c}$$

It is important to remember that these states are not eigenstates of either photon number or of the atomic Hamiltonian, but are rather the linear combinations of such states which are stationary eigenstates whose time evolution is trivial.

The energies of the dressed states of a model system of a three-state atom interacting with two modes of a radiation field are plotted in Fig. 3. Figure 3 shows the three levels of a given manifold. There are, of course, many similar manifolds of higher and lower energies. For the calculations displayed in Fig. 3, and for all subsequent

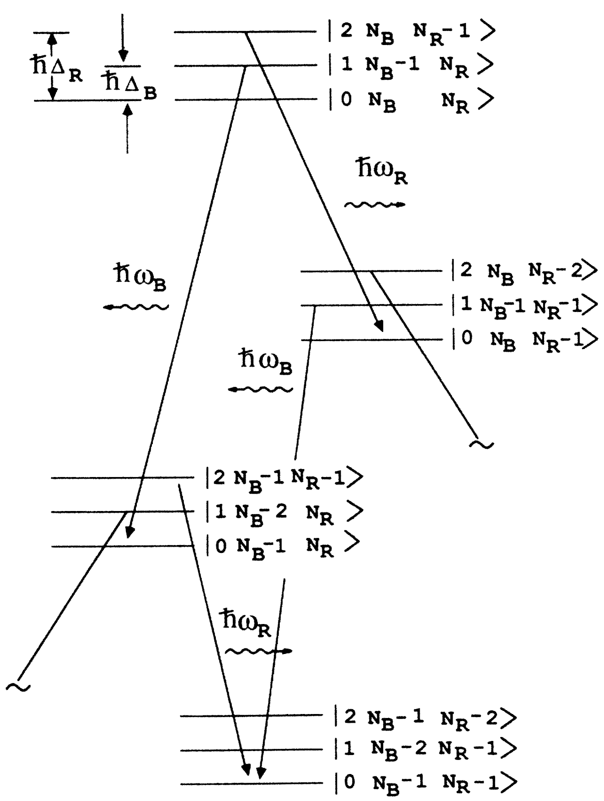

FIG. 2. Schematic energy levels of the eigenstates of the combined atom-field Hamiltonian which does not include laser-atom interactions. If the laser-atom interaction is included the states will still be grouped in three-state manifolds, but the energy splitting between states will be altered, and spontaneous decays will be allowed from all states.

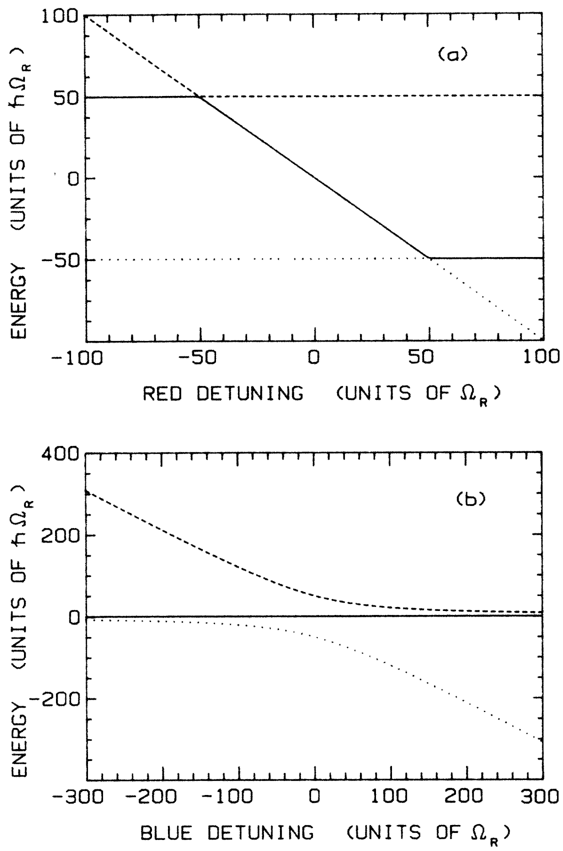

FIG. 3. Energy level of the dressed states in a given manifold as a function of detuning. All quantities are measured in terms of the red Rabi frequency Ω_R. (a) $\Delta_B=0$ as Δ_R is varied. (b) $\Delta_R=0$ as Δ_B is varied.

calculations in this paper, I have used the following relationship between atomic parameters:

$$\Omega_B=100\Omega_R \ . \tag{6}$$

The avoided curve crossings at the Autler-Townes-like resonances that are evident in Fig. 3 have important consequences for the observable properties of the states that will be discussed in the following sections of this paper.

III. SPONTANEOUS EMISSION AND DARK PERIODS

The dressed states present the advantage of simplifying any discussion of the time evolution of the atom-field system. If the combined atom-field system is initially in one of the dressed states, it can, to first approximation, always be considered to be in one of the three species of dressed states. The only change of state occurs upon the irreversible spontaneous emission of a photon, which simply projects the wave function onto dressed states in lower-energy manifolds. The system will then remain in this eigenstate until another spontaneous emission event occurs. Thus, the weak coupling to the nonlaser modes of the radiation field results in a cascade down the energy-level diagram of the dressed atom-laser system in Fig. 2.

In order to predict the occurrence of dark periods in the fluorescence of the single three-state atom, it is only necessary to examine the lifetimes of the stationary states of the atom-field system. The existence of a long-lived state implies the existence of periods during which no fluorescent photons are emitted, and the lifetime of the state gives the mean length of such dark periods. Such a long-lived state interrupts the otherwise relatively rapid cascade down the energy level diagram. Viewed in this light, the fluorescence from the atom is analogous to the decay cascade of a radioactive nucleus. The so-called quantum jumps occur whenever the combined laser-atom system irreversibly decays to a lower-energy eigenstate.

Spontaneous emission is the result of the coupling of the atom to the empty, nonlaser modes of the radiation field. To account for this, the Hamiltonian must be augmented to include a nonzero coupling between states in different manifolds. In the uncoupled basis representation of Fig. 2 the allowed spontaneous transitions from the topmost manifold of states are from $|1,N_B-1,N_R\rangle$ to $|0,N_B-1,N_R\rangle$ and from $|2,N_B,N_R-1\rangle$ to $|0,N_B,N_R-1\rangle$ corresponding to the emission of a blue and red photon, respectively, into nonlaser modes of the field. In the coupled-state representation, transitions are potentially allowed from all of the dressed states in a given manifold. This is because all of the states, except for special detunings, contain some of $|1,N_B-1,2\rangle$ and $|2,N_B,N_R-1\rangle$ in the linear combination comprising the states. In the dressed states the matrix elements (not necessarily dipole) between states in neighboring manifolds are given by

$$\begin{aligned} d_{IJ} &= \langle I|(d_B|0\rangle\langle 1|+d_R|0\rangle\langle 2|)+\text{H.c.})|J\rangle \\ &= \sum_{i,j}\langle i|u_{iI}^*[(d_B|0\rangle\langle 1|+d_R|0\rangle\langle 2|) \\ &\qquad\qquad +\text{H.c.}]u_{Jj}|j\rangle \\ &= \begin{cases} u_{1I}^*u_{J0}d_B & \text{for emission of blue photon,} \\ u_{2I}^*u_{J0}d_R & \text{for emission of red photon,} \end{cases} \end{aligned} \tag{7}$$

where d_B and d_R are the matrix elements in the uncoupled basis. The rate of transfer Γ_{IJ} from a dressed state of species I to a dressed state of species J is proportional to $|d_{IJ}|^2$. The total rate of spontaneous emission out of a dressed state $|I\rangle$ is given by the sum of the rates to the three states in the manifold which is one red photon lower in energy and the three states in the manifold which is one blue photon lower,

$$\begin{aligned} \Gamma_I &= \sum_J (|u_{1I}^2|u_{J0}|^2\Gamma_B+|u_{2I}|^2|u_{J0}|^2\Gamma_R) \\ &= |u_{1I}|^2\Gamma_B+|u_{2I}|^2\Gamma_R \ , \end{aligned} \tag{8}$$

where Γ_B and Γ_R are the spontaneous decay rates of the uncoupled states which are proportional to $|d_B|^2$ and $|d_R|^2$, respectively. For the special case of the zero-detuning states of Eq. (3), the emission rates are

$$\Gamma_\alpha=\frac{1}{2}\left[\frac{\Gamma_B\Omega_B^2}{\Omega_B^2+\Omega_R^2}+\frac{\Gamma_R\Omega_R^2}{\Omega_B^2+\Omega_R^2}\right]$$

$$\approx\tfrac{1}{2}\Gamma_B\ , \tag{9a}$$

$$\Gamma_\beta=\Gamma_\alpha\ , \tag{9b}$$

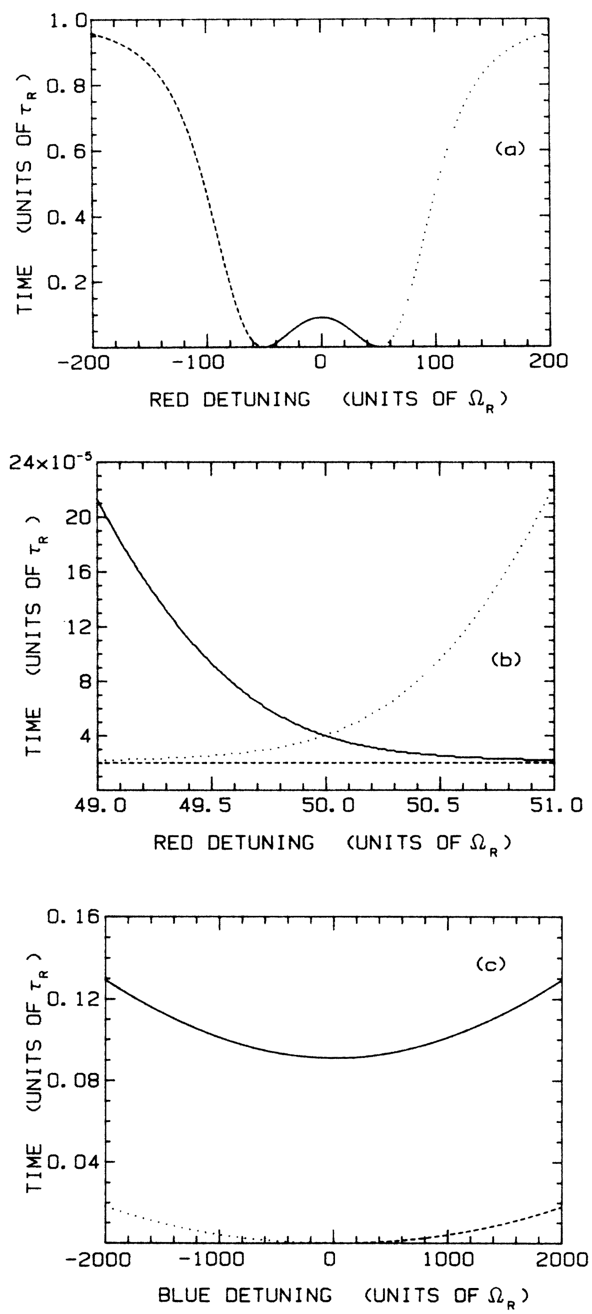

FIG 4. Lifetimes of the dressed states in a given manifold as a function of detuning in terms of $\tau_R=1/\Gamma_R$. Note that all graphs contain the lifetimes for three different states. (a) $\Delta_B=0$ as Δ_R is varied. (b) Detail of (a) near $\Delta_R=50\Omega_R$ resonance. (c) $\Delta_R=0$ as Δ_B is varied.

$$\Gamma_\gamma=\frac{\Gamma_B\Omega_R^2}{\Omega_B^2+\Omega_R^2}+\frac{\Gamma_R\Omega_B^2}{\Omega_B^2+\Omega_R^2}$$

$$\approx\Gamma_R+\frac{\Omega_R^2}{\Omega_B^2}\Gamma_B\ . \tag{9c}$$

The lifetimes of the dressed states are simply the inverses of the total emission rates, and the longest lifetime corresponds to the length of the dark periods in the observed fluorescence. The variation in the lifetimes as a function of detuning is displayed in Fig. 4. For this model calculation I have used the same relationship between matrix elements as that used to calculate the dressed-state energies of Fig. 3 as well as the assumption that $\Gamma_B=10^5\Gamma_R$, which is reasonable if we assume that $\omega_B\sim2\omega_R$. The lifetimes of the dressed states exhibit a strong dependence on detuning of the lasers, and especially so for the red laser. In general, there is one relatively long-lived state and two states with much shorter lifetimes. In Fig. 3(a) the long-lived state at a given value of the detuning is the state which corresponds to the diagonally decreasing energy at that detuning. In Fig. 3(b) the long-lived state is that which corresponds to the level that changes only imperceptibly in energy as the blue laser is tuned. These states correspond most closely with the long-lived uncoupled state $|2,N_B,N_R-1\rangle$. At the detunings of the laser which correspond to the positions of the avoided crossings in the energy level diagrams of Fig. 3 the states can mix strongly, thus altering the lifetimes of the states. It should be noted that even a small mixing of the long-lived state with a rapidly decaying state will have pronounced effects due to the great disparity in lifetimes of the unmixed states.

IV. DURATION OF BRIGHT PERIODS

In the preceding sections I have demonstrated a simple interpretation of the occurrence of dark periods in the fluorescence emitted from the three-level atom. In order to analyze how often such dark periods occur, or equivalently, the duration of the bright periods, it is only necessary to consider the branching ratios of each of the dressed states to the dressed states in lower-energy manifolds under the influence of spontaneous emission. For convenience in the following discussion I will label the longest-lived species of dressed state $|I\rangle$. The bright periods will be those in which the decay cascade includes only the two short-lived species of states which I will label $|J\rangle$ and $|K\rangle$. The probability of having N cascades entirely within the subsystem comprised of the short-lived states $|J\rangle$ and $|K\rangle$ followed by a cascade to the long-lived state $|I\rangle$,

$$P_N=(1-b)^N b\ , \tag{10}$$

where b is the branching ratio to the long-lived state $|I\rangle$ from either $|J\rangle$ or $|K\rangle$. Expansion of the dressed states in terms of the uncoupled basis states gives a simple expression for this branching ratio which is independent of the species of the short-lived state, i.e.,

$$b=\frac{\Gamma_{JI}}{\Gamma_{JI}+\Gamma_{JJ}+\Gamma_{JK}}$$
$$=\frac{\Gamma_{KI}}{\Gamma_{JI}+\Gamma_{JJ}+\Gamma_{JK}}$$
$$=|a_{I0}|^2 . \tag{11}$$

(Note that it is possible with some nonzero probability to enter a dark period immediately after the emission of either a blue or a red photon, and that it is possible to end a dark period with the emission of either color also. This is because each of the dressed states can contain some mixture of all three of the bare atomic states and so can, in principle, decay to any of the other species of dressed state.)

The mean duration of such an N cascade process is given by

$$\tau_N=(N_J\tau_J+N_K\tau_K) , \tag{12}$$

where N_J and N_K are the number of cascades from states J and K, respectively ($N_J+N_K=N$), and τ_J and τ_K are the mean lifetimes of these states. If p_J and p_K are the probabilities that a given cascade is to states J and K, respectively, during this N event cascade, then N_J and N_K are given by p_JN and p_KN, and the mean duration of a bright period is

$$\tau_B=\sum_N \tau_N p_N$$
$$=(p_J\tau_J+p_K\tau_K)\sum_N N(1-b)^N b$$
$$=(p_J\tau_J+p_K\tau_K)\frac{1-b}{b} . \tag{13}$$

The only remaining difficulty is the calculation of the probabilities p_J and p_K. These probabilities are evaluated in the Appendix with the result that

$$p_J=\frac{|a_{J0}|^2}{|a_{J0}|^2+|a_{K0}|^2} , \tag{14a}$$

$$p_K=\frac{|a_{K0}|^2}{|a_{J0}|^2+|a_{K0}|^2} . \tag{14b}$$

Figure 5 displays the results for the mean duration of the bright periods as a function of detuning calculated using Eq. (13), and Fig. 6 shows the ratio of the length of the dark periods to the bright periods. The duration of the bright periods near the zero-detuning condition becomes unrealistically large, and actually approaches infinity in the limit of the approximations used in this paper. Examination of the zero-detuning states of Eq. (4) shows that the branching ratio to the long-lived state $|\gamma\rangle$ from either $|\alpha\rangle$ or $|\beta\rangle$ is zero and the population is coherently trapped[18] in the two short-lived states in this special

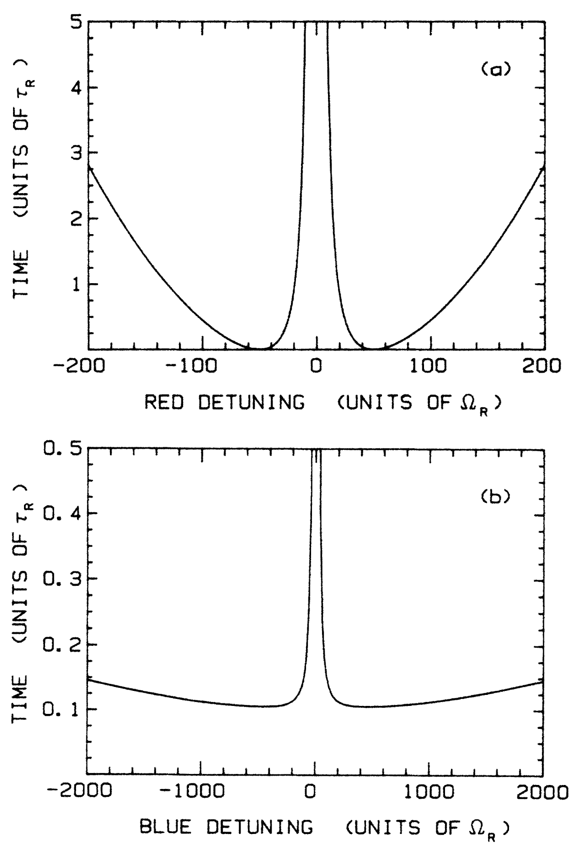

FIG. 5. Duration of the bright periods as a function of detuning in terms of $\tau_R=1/\Gamma_R$. (a) $\Delta_B=0$ as Δ_R is varied. (b) $\Delta_R=0$ as Δ_B is varied.

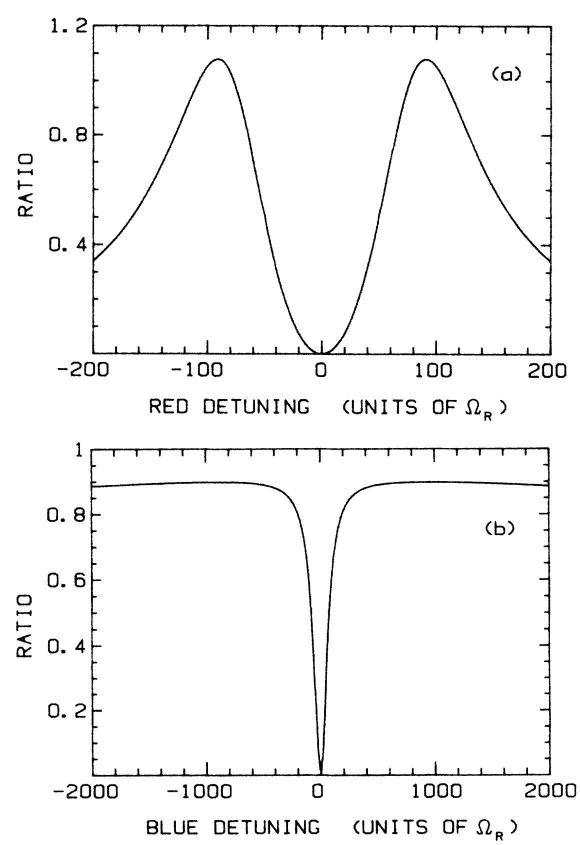

FIG. 6. Ratio of the duration of dark periods to bright periods as a function of detuning. (a) $\Delta_B=0$ as Δ_R is varied. (b) $\Delta_R=0$ as a Δ_B is varied.

case. This complete trapping exists because I ignored spontaneous emission in the calculation of the explicit forms of the dressed states.

V. ANALYTIC APPROXIMATION

In order to develop an analytic approximation for the lifetimes of the states as a function of detuning, I will first consider approximations for the dressed states. Except near the avoided curve crossings of Fig. 3, good approximations to the states are simply the eigenstates of the Hamiltonian when $\Omega_R = V_{02} = 0$. In Fig. 3(a), where $\Delta_B = 0$, the two states whose energies are flat as a function of detuning in a given region are approximately the dressed states of a two-level atom interacting with the blue laser

$$|I\rangle = \frac{1}{\sqrt{2}}\left[\exp\left[\frac{i\phi_B}{2}\right]|0,N_B,N_R\rangle + \exp\left[-\frac{i\phi_B}{2}\right]|1,N_B-1,N_R\rangle\right], \quad (15a)$$

$$|J\rangle = \frac{1}{\sqrt{2}}\left[\exp\left[\frac{i\phi_B}{2}\right]|0,N_B,N_R\rangle - \exp\left[-\frac{i\phi_B}{2}\right]|1,N_B-1,N_R\rangle\right], \quad (15b)$$

and the state whose energy decreases with increasing red detuning is approximately the uncoupled state

$$|K\rangle = |2,N_B,N_R-1\rangle . \quad (15c)$$

Near the level crossings two of the states mix appreciably because of their near degeneracy. If we neglect the coupling to the nondegenerate state we can analytically diagonalize the resulting 2×2 Hamiltonian. For $\Delta_R > 0$ it is the states $|J\rangle$ and $|K\rangle$ that mix near the crossing, and close enough to the crossing so that $\Omega_B - 2\Delta_R \ll \Omega_B$, the new eigenstates are

$$|I'\rangle = |I\rangle = \frac{1}{\sqrt{2}}\left[\exp\left[\frac{i\phi_B}{2}\right]|0,N_B,N_R\rangle + \exp\left[-\frac{i\phi_B}{2}\right]|1,N_B-1,N_R\rangle\right], \quad (16a)$$

$$\begin{aligned}|J'\rangle &= \exp\left[\frac{i\phi^*}{2}\right]\cos\theta|J\rangle + \exp\left[-\frac{i\phi^*}{2}\right]\sin\theta|K\rangle \\ &= \frac{1}{\sqrt{2}}\exp\left[\frac{i\phi^*}{2}\right]\left[\exp\left[\frac{i\phi_B}{2}\right]\cos\theta|0,N_B,N_R\rangle - \exp\left[-\frac{i\phi_B}{2}\right]\cos\theta|1,N_B-1,N_R\rangle\right] \\ &\quad + \exp\left[-\frac{i\phi^*}{2}\right]\sin\theta|2,N_B,N_R-1\rangle ,\end{aligned} \quad (16b)$$

$$\begin{aligned}|K'\rangle &= \exp\left[\frac{i\phi^*}{2}\right]\sin\theta|J\rangle - \exp\left[-\frac{i\phi^*}{2}\right]\cos\theta|K\rangle \\ &= \frac{1}{\sqrt{2}}\exp\left[\frac{i\phi^*}{2}\right]\left[\exp\left[\frac{i\phi_B}{2}\right]\sin\theta|0,N_B,N_R\rangle - \exp\left[-\frac{i\phi_B}{2}\right]\sin\theta|1,N_B-1,N_R\rangle\right] \\ &\quad - \exp\left[-\frac{i\phi^*}{2}\right]\cos\theta|2,N_B,N_R-1\rangle ,\end{aligned} \quad (16c)$$

where the parameters θ and ϕ^* are determined by

$$\cos 2\theta = -\frac{2\Delta_R - \Omega_B}{[2\Omega_R^2 + (2\Delta_R - \Omega_B)^2]^{1/2}} , \quad (17a)$$

$$\sin 2\theta = \frac{\sqrt{2}\Omega_B}{[2\Omega_R^2 + (2\Delta_R - \Omega_B)^2]^{1/2}} , \quad (17b)$$

$$\phi^* = \phi_R - \frac{\phi_B}{2} , \quad (17c)$$

and the energies of the approximate eigenstates are given by

$$\frac{E_{I'}}{\hbar} = \frac{\Omega_B}{2} , \quad (18a)$$

$$\frac{E_{J'}}{\hbar}=-\tfrac{1}{4}\{\Omega_B+2\Delta_R-[(\Omega_B-2\Delta_R)^2+2\Omega_R^2]^{1/2}\}\ , \tag{18b}$$

$$\frac{E_{K'}}{\hbar}=-\tfrac{1}{4}\{\Omega_B+2\Delta_R+[(\Omega_B-2\Delta_R)^2+2\Omega_R^2]^{1/2}\}\ . \tag{18c}$$

Examination of the coefficients of the uncoupled states which comprise these approximate eigenstates yields the following decay rates:

$$\Gamma_{I'}=\frac{\Gamma_B}{2}\ , \tag{19a}$$

$$\Gamma_{J'}=\frac{\Gamma_B\cos^2\theta}{2}+\Gamma_R\sin^2\theta\ , \tag{19b}$$

$$\Gamma_{K'}=\frac{\Gamma_B\sin^2\theta}{2}+\Gamma_R\cos^2\theta\ . \tag{19c}$$

The energy levels as a function of blue detuning displayed in Fig. 3(b) can be understood in a similar manner. The state whose energy is flat as a function of detuning corresponds to the long-lived state $|K\rangle$ which is almost entirely $|2,N_B,N_R-1\rangle$. The other two levels are the dressed states of the two-level atom, blue laser system. Except for very large blue detunings both of these states will be relatively short lived. Because there is no localized region in which a single short-lived state interacts with the long-lived state, a simple analytic approximation like that developed for the case of red detunings is not possible.

VI. CONCLUSIONS

I have introduced a simple interpretation of the dynamics of three-level quantum-jump experiments in terms of the dressed states, which are eigenstates of a Hamiltonian that include laser-atom interactions. In the limit in which both atomic transitions are strongly driven I have calculated the lifetimes of the dressed states and the mean duration of dark and bright periods in the observed fluorescence from the atom. While these calculations will not hold strictly when the transitions are not strongly driven, the dynamic picture offered by the dressed-state basis still gives a clear qualitative way in which to interpret the abrupt changes in the observed fluorescence levels.

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APPENDIX

In the view I have presented of the spontaneous decay cascade, each decay is a Markov process whose result depends only on the decaying state and not on the previous history of the laser-atom system. Such a process can be described with a matrix containing the branching ratios between the two states acting on a real vector characterizing which state the system is in. If I represent the states $|J\rangle$ and $|K\rangle$ with the two-dimensional vectors

$$|J\rangle=\begin{bmatrix}1\\0\end{bmatrix},\quad |K\rangle=\begin{bmatrix}0\\1\end{bmatrix}\ , \tag{A1}$$

and the transition matrix $\tilde{M}$ as

$$\tilde{M}=\begin{bmatrix}\dfrac{\Gamma_{JJ}}{\Gamma_{JJ}+\Gamma_{JK}} & \dfrac{\Gamma_{KJ}}{\Gamma_{KJ}+\Gamma_{KK}}\\ \dfrac{\Gamma_{JK}}{\Gamma_{JJ}+\Gamma_{JK}} & \dfrac{\Gamma_{KK}}{\Gamma_{kj}+\Gamma_{kk}}\end{bmatrix}\ , \tag{A2}$$

then the probabilities p_J and p_K of being in states J and K after N cascades are given by

$$\begin{bmatrix}p_J\\p_K\end{bmatrix}=\tilde{M}^N|A\rangle\ , \tag{A3}$$

where $|A\rangle$ is the initial state of the system. when the branching ratio to the long-lived state is small, i.e., $b \ll 1$, then the number of cascades N in any bright period will be very large. Therefore, values for p_J and p_K can always be taken from the large N limit of Eq. (A3),

$$\lim_{N\to\infty}\tilde{M}^N=\begin{bmatrix}\dfrac{|a_{J0}|^2}{|a_{J0}|^2+|a_{K0}|^2} & \dfrac{|a_{J0}|^2}{|a_{J0}|^2+|a_{K0}|^2}\\ \dfrac{|a_{K0}|^2}{|a_{J0}|^2+|a_{K0}|^2} & \dfrac{|a_{K0}|^2}{|a_{J0}|^2+|a_{K0}|^2}\end{bmatrix}\ , \tag{A4}$$

which gives

$$\begin{bmatrix}p_J\\p_K\end{bmatrix}=\begin{bmatrix}\dfrac{|a_{J0}|^2}{|a_{J0}|^2+|a_{K0}|^2}\\ \dfrac{|a_{K0}|^2}{|a_{J0}|^2+|a_{K0}|^2}\end{bmatrix}\ , \tag{A5}$$

which is independent of the initial state of the system.

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