PHYSICAL REVIEW A VOLUME 37, NUMBER 8 APRIL 15, 1988

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# Strong squeezing in the Jaynes-Cummings model

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(Received 14 December 1987)

Using the well-known solution for a two-level atom interacting with a single mode of radiation (Jaynes-Cummings model), we reanalyze the possibility of squeezing. We found that if only the initial coherent state has a mean photon number exceeding 10 (for a resonant case) significant squeezing can be achieved. Its strength can be arbitrarily large for increasing average photon number. Finally, we solve our model in the situation when the em field is damped and analyze the impact of damping on both squeezing and revival phenomena.

Squeezing phenomena continue to attract much attention. Especially squeezed states of light (e.g., states with lower variance of an electric-field operator than the vacuum state) were investigated both from theoretical and experimental points of view (for a review, see Ref. 1). Recently squeezing of light was observed in experiments.[2]

The possibility of squeezing phenomenon for a single mode of radiation interacting with a two-level atom under the rotating-wave approximation (RWA) was analyzed by several authors.[3-6] A paper of Meystre and Zubairy[3] discusses squeezing in the Jaynes-Cummings model for the resonant case when the electromagnetic (em) field is resonant with the atom. They analyze the situation when the initial state of radiation is a coherent state. However their discussion is restricted to the case when the mean photon number is smaller than 10. There they found that squeezing does not exceed 20%.

Analyzing the time evolution of the system in the resonant case when the mean photon number is larger ($n_0 > 10$), we have found that the squeezing effects get stronger for large $n_0$. It means that strongly squeezed states may be obtained if only the initial mean photon number is large enough. We present an analytic formula describing squeezing in the asymptotic limit $n_0 \rightarrow \infty$. Considering a more general situation when the em field is not exactly resonant to the two-level atom, we found that squeezing effect can be even stronger out of resonance than for the resonant case.

The Jaynes-Cummings model is one of the simplest quantum-optical systems. On the other hand, to get a more realistic physical model, several corrections have to be taken into account. To get a more realistic picture we solve the equations of motion when the em field is damped. Analyzing those solutions we could find a range of the damping constant for which revivals of atomic inversion already disappear but squeezing still persists.

The Jaynes-Cummings model consists of a two-level atom interacting with a single mode of radiation which is nearly resonant with the atomic gap. The system's time evolution is governed by a Hamiltonian (RWA):

$$H_{JC} = \omega_0 S_3 + (\omega_0 + \Delta) a^\dagger a + \lambda (a^\dagger S_- + a S_+) \qquad (1)$$

where $a$ is the annihilation of the radiation mode and $S_-$ and $S_+$ are the standard atomic transition operators for a two-level system ($S_3 = S_+ S_- - S_- S_+$).

The solution of the Schrödinger equation can be easily evaluated. For $\Delta = 0$ we obtain (time has been scaled in units of $1/\lambda$):

$$|\psi(t)\rangle = \sum_{n=0}^{\infty} [\cos(\sqrt{n+1}t)\alpha_n + \sin(\sqrt{n+1}t)\beta_{n+1}]\,|1,n\rangle + [\cos(\sqrt{n}t)\beta_n - \sin(\sqrt{n}t)\alpha_{n-1}]\,|-1,n\rangle \ , \qquad (2)$$

where

$$\alpha_n = \langle 1,n|\psi_0\rangle,\ \beta_n = \langle -1,n|\psi_0\rangle \ [|\psi_0\rangle = |\psi(t=0)\rangle] \ .$$

Using this solution of the Schrödinger equation, we are going to calculate mean value and dispersion of a generalized electric field operator:

$$E_\phi = a e^{i\phi} + a^\dagger e^{-i\phi} \ . \qquad (3)$$

[The dispersion of an operator $A$ is equal to $\sigma^2(A) = \langle\psi|A^2|\psi\rangle - (\langle\psi|A|\psi\rangle)^2$.] For $\phi = 0$ and

$\phi=\pi/2$, $E_\phi$ is proportional to standard in and out of phase operators $E_{in}=c(a+a^\dagger)$ and $E_{out}=ic(a-a^\dagger)$, respectively.

Note, that for the pair of operators $E_\phi$ and $E_{\phi+\pi/2}$ we have ($[E_\phi,E_{\phi+\pi/2}]=2i$),

$$\sigma(E_\phi)\sigma(E_{\phi+\pi/2})\geq 1 \ . \qquad (4)$$

During time evolution our system exhibits squeezing e.g., for some $\phi$ and $t$ we have $\sigma(E_\phi(t))<1$. Note that because of a symmetry of $H_{JC}$ under transformation $S_-\to S_-e^{i\xi}$ $a\to ae^{i\xi}$ we can restrict ourselves only to the case of real $\alpha$. Discussing the general squeezing ellipse for $\alpha=\alpha_0e^{i\xi}$ ($\alpha_0$ is real) we found that $f(\phi)=\sigma(E_\phi(t))$ has minima for $\phi=\xi$ and $\xi+\pi$ (and maxima for $\phi=\xi+\pi/2$ and $\xi+3\pi/2$). It means that for real $\alpha$ the in-phase electric field operator is the one with maximal squeezing. So let us define $\eta(t)=\sigma(E_{\phi=0}(t))$ as a squeezing parameter. We stress that our definition of the generalized electric field ensures our dimensionless squeezing parameter $\eta(t)$ to be normalized to 1 on vacuum.

First we analyze the time evolution of our system in the resonant case ($\Delta=0$), when the initial state is a coherent state (more precisely $|\psi_0\rangle=|\alpha\rangle[\cos(\theta)|1\rangle+\sin(\theta)\times|-1\rangle]$). The time dependence of the squeezing parameter $\eta(t)$ for different initial mean photon numbers $n_0$ ($n_0=\langle\psi_0|a^\dagger a|\psi_0\rangle$) is presented in Fig. 1(a). The evolution of $\eta(t)$ has the same properties for all $n_0>10$. First the dispersion increases, then for a period of time squeezing appears and attains its maximum value [e.g., minimum of $\eta(t)$] for a certain time $t_{sq}$. Finally, for $t\gg t_{sq}$ the dispersion $\eta(t)$ increases. The effect of maximal squeezing is increasing with $n_0$. In Fig. 1(b), the dependence of $\eta(t_{sq})$ on $n$ is plotted. We clearly see that strong squeezing is possible if only the photon number of the initial coherent state is large enough. On the other hand, the Heisenberg inequality is far from being saturated, so these states of em field are not minimal uncertainty packets.

In the asymptotic limit $n_0\to\infty$ the dispersion $\eta(\tau)$ is described by a simple formula ($\tau=t/(n_0)^{1/2}$; for derivation see the Appendix)

$$\eta(\tau)=1+\tfrac{1}{2}\tau\sin(\tau)+[\tfrac{1}{2}\tau\sin(\tau/2)]^2 \ . \qquad (5)$$

Analyzing this expression we found that squeezing appears in the asymptotic limit periodically and its strength tends to 100%. In Fig. 2 the evolution $\eta(\tau)$ is plotted for $n_0=1600$ in comparison to the asymptotic behavior.

The results stated above describe the case when $\Delta=0$, $\theta=0$ and do not depend significantly on the change of $\theta$ ($\theta=0$ corresponds to an initially fully excited atom, $\theta=\pi$ to atom in its ground state, etc.). Considering the nonresonant case ($\Delta\neq0$), the situation is slightly different. In Fig. 3 the maximal squeezing parameter $\eta(t_{sq})$ is plotted versus $\Delta$. W see that in the latter case the squeezing effect can be stronger.

An interesting situation occurs when the em field in the cavity is damped. Using a standard model of damping of the em field, we obtain the following equation of motion for the density matrix (Ref. 7; limit of zero temperature):

$$\frac{d}{dt}\rho=i[\rho,H_{JC}]+\Gamma(2a\rho a^\dagger-\rho a^\dagger a-a^\dagger a\rho) \ . \qquad (6)$$

Although this equation is linear, it is not easy to solve it for large photon number because of numerical restric-

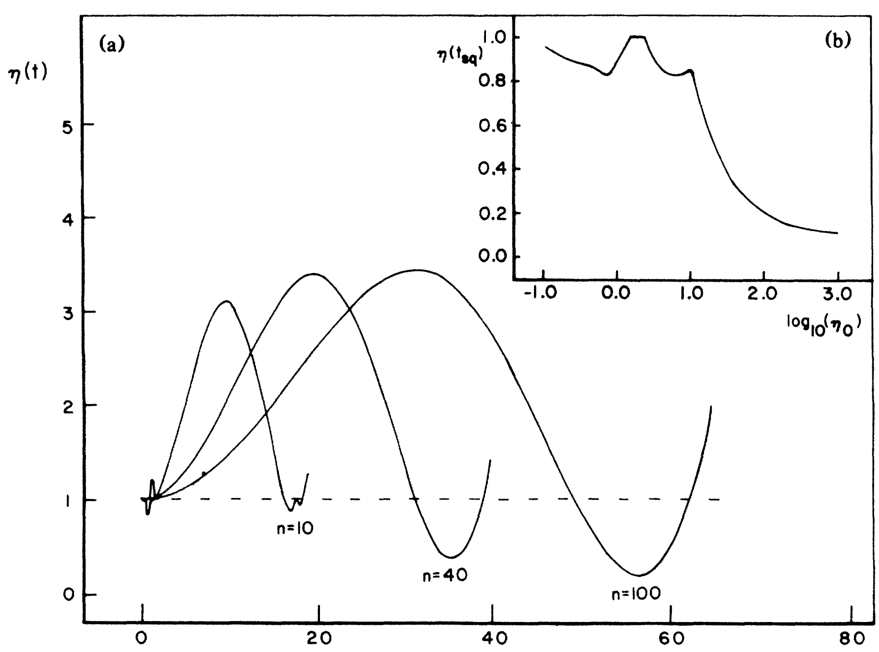


FIG. 1. (a) The squeezing parameter $\eta(t)$ is plotted vs $t$ for different average photon numbers ($n_0=10,40,100$) ($t$ is scaled in units of $1/\lambda$). (b) The maximal squeezing parameter $\eta(t_{sq})$ is plotted vs $\log_{10}(n_0)$ ($n$ is the initial average photon number and $t$ is scaled in units of $1/\lambda$).

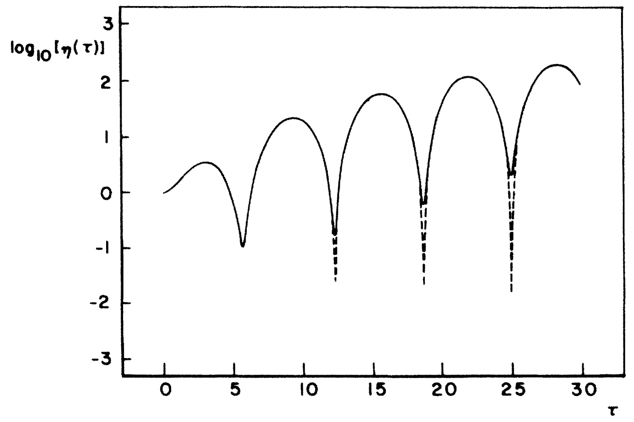


FIG. 2. $\log_{10}[\eta(\tau)]$ is plotted for $n_0=1600$ (full curve) in comparison to the asymptotic evolution $\log_n[\eta(\tau)]$ (dashed line).

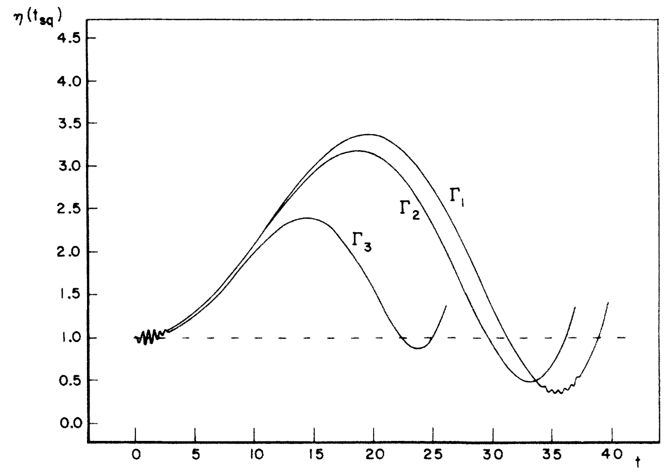


FIG. 4. Time dependence of the squeezing parameter $\eta(t_{sq})$ for different damping constants $\Gamma_1=0$, $\Gamma_2=1/8t_{sq}$, $\Gamma_3=1/t_{sq}$ ($t$ is scaled in units of $1/\lambda$). The initial mean photon number is $n_0=40$.

tions. We solve the evolution using an approximation for the "step evolution" operator $U(\Delta t)=\exp[(L_\gamma+L_0)\Delta t]$

$$\exp[(L_\gamma+L_0)\Delta t]\cong\exp(L_\gamma\Delta t)\exp(L_0\Delta t)\ , \tag{7}$$

where $L_0\rho=i[\rho,H_{JC}]$ and $L_\gamma\rho=\Gamma(2a\rho a^\dagger-a^\dagger a\rho-\rho a^\dagger a)$. Such the approximation (the Trotter product formula[8]) originates from the identity

$$e^{(A\Delta t+B\Delta t)}=e^{A\Delta t}e^{B\Delta t}e^{([A,B]/2(\Delta t)^2+\cdots}$$

in the limit $\Delta t\rightarrow 0$ (we may omit the commutator part). Finally, we get

$$\rho_{n+1}=\exp(L_\gamma\Delta t)\exp(-iH_{JC}\Delta t)\rho_n\exp(iH_{JC}\Delta t)\ , \tag{8}$$

where $\rho_n=\rho(t=n\Delta t)$. We check carefully the numerical stability for $\Delta t\rightarrow 0$.

The evolution $\eta(t)$ for different damping constants is presented in Fig. 4. When damping is increasing the effect of squeezing is getting weaker and occurs for shorter times. Note that when damping increases the mean photon number in the cavity decreases. The result plotted in Fig. 4 is similar to the situation presented in Fig. 1(a) e.g., to different dependencies $\eta(t)$ for various initial photon numbers $n_0$. Comparing these two results we conclude, that decrease of squeezing in a damped cavity may be viewed as a consequence of reduction of photon number due to damping.

One of the characteristic features of the Jaynes-Cummings model is that the atomic inversion (e.g., $\langle S_3\rangle$) undergoes damped oscillations with successive revivals.[9] In Figs. 5(a) and 5(b) we compare the atomic inversion for $\Gamma=0$ and $\Gamma=1/8t_{sq}$, respectively. We see that damping losses are destroying revivals in a cavity[7] [see Figs. 5(a) and 5(b)]. We stress, that for this damping the squeezing effect is still present (see Fig. 4). So we may conclude that squeezing effect in the Jaynes-Cummings model is more robust than revivals.

Summarizing our communication, we found that the Jaynes-Cummings model, which is one of the simplest quantum-optical systems, initiated by coherent em fields gives strong squeezing of light. This squeezing phenomenon seems not to be extremely sensitive to damping caused by imperfection of a cavity. Our results also confirm that the long time-scale interaction of even arbitrary strong coherent em wave with matter cannot be correctly described in the semiclassical approach.

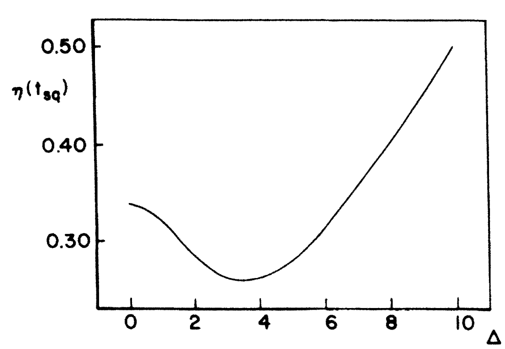


FIG. 3. The maximal squeezing parameter $\eta(t_{sq})$ is plotted for different detunings ($\Delta$ is scaled in units of $\lambda$).

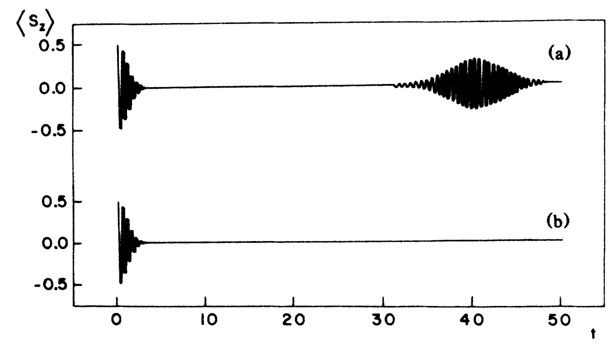


FIG. 5. Atomic inversion plotted vs $t$ for (a) $\Gamma=0$ and (b) $\Gamma=1/8t_{sq}$ ($t$ is scaled in units of $l/\lambda$).

We are very grateful to J. Haus who encouraged us to analyze the case of damped em field. We wish to thank I. Białynicki-Birula, J. Mostowski, and K. Rzążewski for essential remarks and fruitful discussions.

## APPENDIX

The expression for the mean value of operators $E_0$ and $E_0^2$ have the form $(p_n = e^{-n_0}(n_0)^n/n!)$:

$$\langle E_0\rangle = \sqrt{n_0}\,\mathrm{Re}\sum_{n=0}^{\infty} p_n\left\{\left[1-\left(\frac{n+2}{n+1}\right)^{1/2}\right]\exp[i(\sqrt{n+1}+\sqrt{n+2})t]+\left[1+\left(\frac{n+2}{n+1}\right)^{1/2}\right]\exp[i(\sqrt{n+2}-\sqrt{n+1})t]\right\},$$

$$\langle E_0^2\rangle = n_0\,\mathrm{Re}\sum_{n=0}^{\infty} p_n\left\{\left[1-\left(\frac{n+3}{n+1}\right)^{1/2}\right]\exp[i(\sqrt{n+1}+\sqrt{n+3})t]+\left[1+\left(\frac{n+3}{n+1}\right)^{1/2}\right]\exp[i(\sqrt{n+3}-\sqrt{n+1})t]\right\}$$
$$-\mathrm{Re}\sum_{n=0}^{\infty} p_n \exp[2i\sqrt{n+1}\,t]+2(n_0+1)\ . \qquad \text{(A1)}$$

Now we will calculate the dispersion of $E_0$ in the asymptotic limit $n_0 \rightarrow \infty$ [e.g., neglecting in $\eta(t)$ all terms proportional to $1/n_0$, $(1/n_0)^2$, etc.] using the stationary phase method to sum the series (we replace $p_n$ by a Gaussian distribution). Note that both $E_0$ and $E_0^2$ exhibit collapses and revivals. Squeezing occurs in the region of the first collapse and may be viewed as a consequence of a phase mismatch between the two components. To this end we obtain $[\tau = t/(n_0)^{1/2}]$:

$$\eta(t) = 1(2n_0+1)\{[1+\cos(\tau+\tau/8n_0)e^{-\tau^2/8n_0}]-e^{-\tau^2/16n_0}[1+\cos(\tau+3\tau/8n_0)]\}\ . \qquad \text{(A2)}$$

Keeping only terms of order 1 we obtain Eq. (5).

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