Brief Reports

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Two-photon Jaynes-Cummings model interacting with the squeezed vacuum

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We consider squeezed light described as an SU(1,1) coherent state interacting with a two-photon Jaynes-Cummings model of a two-level atom. We study the time variation of the mean photon number and also the variance of the field quadratures with particular regard to the squeezing.

Since it is now possible to produce squeezed electromagnetic fields in the laboratory[1,2] it would seem to be of some interest to study the interaction of this radiation with various media. Some studies along this line have been performed. For example, Los Terreros and Bermejo[3] have shown that when squeezed single-mode light interacts with a two-photon amplifier the squeezing is eventually revoked. A similar result was found by the present author in a study of squeezed light described as an SU(1,1) coherent state (CS) interacting with a nonlinear nonabsorbing medium modeled as an anharmonic oscillator.[4] The SU(1,1) CS's have been shown[5,6] to be the squeezed vacuum states that are essentially equivalent to the two-photon coherent states of Yuen.[7] In particular, the prototype Hamiltonian for the production of the two-photon states may be written in terms of the generators of SU(1,1) so it becomes clear that the vacuum state evolves into an SU(1,1) CS.[4,5]

In this paper we study the interaction of squeezed light, again described as an SU(1,1) CS, with a single two-level atom. Since the squeezed light used here is of the two-photon type we consider the two-photon generalization[5,6] of the Jaynes-Cummings model (JCM), whose interaction terms can be written in terms of the SU(1,1) generators. Multiphoton generalizations have previously been considered. Sukumar and Buck[8,9] have studied the atomic dynamics of such models interacting with coherent light. In particular, they showed that these models exhibit periodic decay and revival of atomic coherence. On the other hand, Singh[10] has studied the effect of the interaction on the field statistics and has shown that the mean photon number may also exhibit periodic decay and revival.

In the present work we study the time evolution of the field statistics, and in particular the time evolution of the mean photon number and the variance of the field quadratures for a squeezed vacuum state, described as an SU(1,1) CS, interacting with a two-photon Jaynes-Cummings model of a two-level atom. (Previously, Meystre and Zubairy[11] showed that coherent light interacting with the one-photon JCM can become squeezed.) For a summary of the SU(1,1) CS description of the squeezed vacuum state we refer the reader to Refs. 5 and 6.

The Hamiltonian for the Jaynes-Cummings model of the two-level atom generalized to include two-photon interactions[5,6] is

$$H=\tfrac{1}{2}\hbar\omega_0\sigma_3+\hbar\omega a^\dagger a+\hbar\lambda(a^{\dagger 2}\sigma_-+a^2\sigma_+)\ , \tag{1}$$

where $\sigma_3,\sigma_\pm$ are the Pauli matrices. Since the SU(1,1) Lie algebra for a single-mode photon system may be realized as

$$\begin{aligned}&K_+=\tfrac{1}{2}a^{\dagger 2},\quad K_-=\tfrac{1}{2}a^2\ ,\\&K_0=\tfrac{1}{4}(a^\dagger a+aa^\dagger)=\tfrac{1}{2}(N+\tfrac{1}{2})\ ,\end{aligned} \tag{2}$$

where $N=a^\dagger a$ is the number operator, the Hamiltonian may be written as

$$H=\tfrac{1}{2}\hbar\omega_0\sigma_3+2\hbar\omega(K_0-\tfrac{1}{2})+2\lambda\hbar(K_+\sigma_-+K_-\sigma_+)\ . \tag{3}$$

Assuming the radiation field to be at resonance with the two-level atom, $\omega_0=2\omega$, we may rewrite Eq. (3) as

$$H=\hbar\omega N_0+\hbar C\ , \tag{4}$$

where

$$\begin{aligned}&N_0=a^\dagger a+\sigma_3\ ,\\&C=2\lambda(K_+\sigma_-+K_-\sigma_+)\ .\end{aligned} \tag{5}$$

It can be shown that[10]

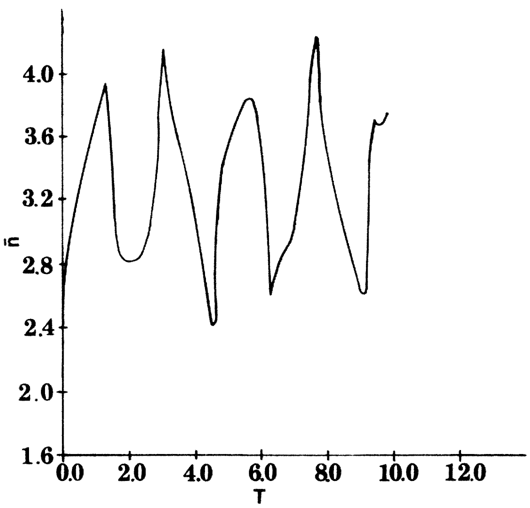

FIG. 1. Mean photon number as a function of time where $\tau=\lambda t$ and $|\xi|=0.8$.

$$[H,N_0]=[H,C]=[N_0,C]=0\ . \tag{6}$$

Thus the evolution operator can be factored as

$$U(t,0)=e^{-i\omega N_0 t}e^{-iCt}\ . \tag{7}$$

Evidently, the evolution operator in the interaction picture is

$$U_I(t,0)=e^{-iCt}\ . \tag{8}$$

In the two-dimensional subspace of the atom this becomes

$$U_I(t,0)=\begin{bmatrix} \cos(\lambda\sqrt{\nu}t) & \dfrac{-2iK_-}{\sqrt{\nu'}}\sin(\lambda\sqrt{\nu'}t) \\ \dfrac{-2iK_+}{\sqrt{\nu}}\sin(\lambda\sqrt{\nu}t) & \cos(\lambda\sqrt{\nu'}t) \end{bmatrix}, \tag{9}$$

where $\nu=4K_-K_+$ and $\nu'=4K_+K_-$. Assuming the atom to be initially in the excited state we have the density matrix of the field as

$$\rho_f(t)=\mathrm{Tr}_{\mathrm{atom}}U_I(t,0)\begin{bmatrix}\rho_f(0) & 0\\ 0 & 0\end{bmatrix}U_I^\dagger(t,0)$$

$$=\cos(\lambda\sqrt{\nu}t)\rho_f(0)\cos(\lambda\sqrt{\nu}t)$$

$$+4K_+\frac{\sin(\lambda\sqrt{\nu}t)}{\sqrt{\nu}}\rho_f(0)\frac{\sin(\lambda\sqrt{\nu}t)}{\sqrt{\nu}}K_-\ . \tag{10}$$

We assume the initial state to be an SU(1,1)CS,[12] which, for the squeezed vacuum, is

$$|\xi\rangle=(1-|\xi|^2)^{1/4}\sum_{m=0}^{\infty}\left[\frac{\Gamma(m+\frac{1}{2})}{m!\Gamma(\frac{1}{2})}\right]^{1/2}\xi^m|m\rangle\ . \tag{11}$$

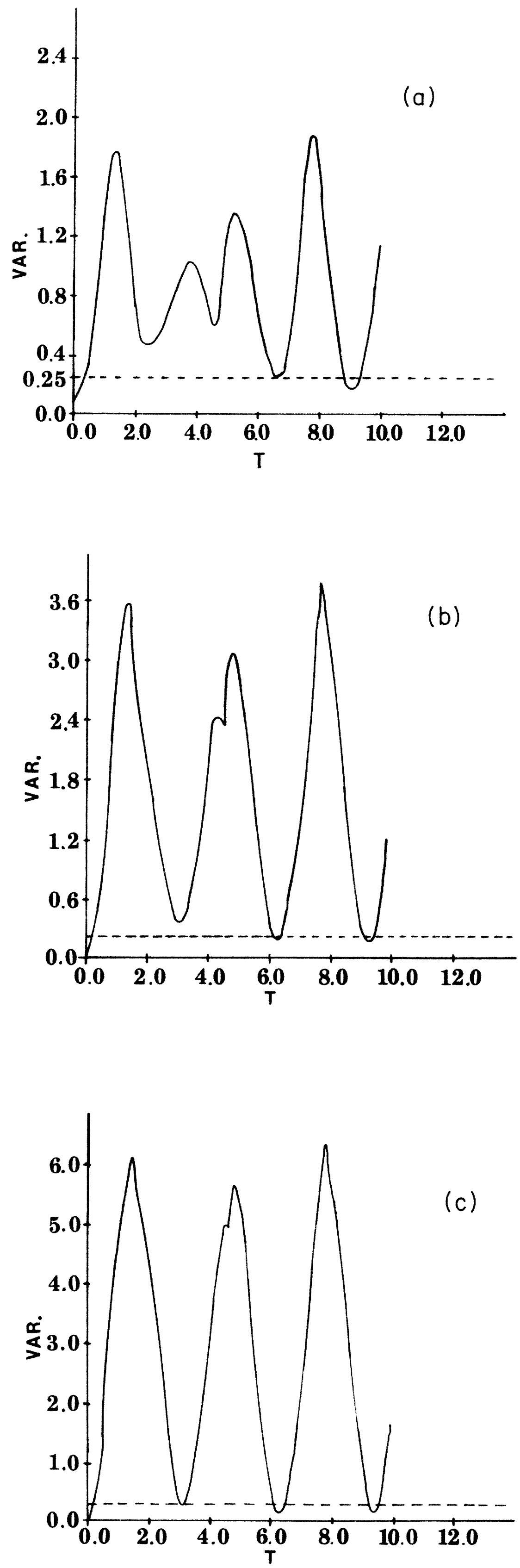

FIG. 2. Time development of the variance of X_1 for (a) $|\xi|=0.4$, (b) $|\xi|=0.8$, and (c) $|\xi|=0.9$.

The states $\{|m\rangle\}$ are not strictly number states but actually contain even numbers of photons, i.e., $2m$ photons where $m=0,1,2,\ldots$. The parameter ξ is related to the average photon number $n=\langle 2K_0-\frac{1}{2}\rangle$ according to $|\xi|=[n/(n+1)]^{1/2}$ and $\xi=-|\xi|$. With the above assumption

$$\rho_f(0)=|\xi\rangle\langle\xi| \tag{12}$$

and we obtain the matrix elements of $\rho_f(t)$ as

$$\langle m|\rho_f(t)|m'\rangle=\frac{(1-|\xi|^2)^{1/2}}{\Gamma(\frac{1}{2})}\left[\frac{\Gamma(m+\frac{1}{2})\Gamma(m'+\frac{1}{2})}{m!m'!}\right]^{1/2}\xi^{*m}\xi^{m'} \times\Bigg[\cos[2\lambda t\sqrt{(m+1)(m+\tfrac{1}{2})}]\cos[2\lambda t\sqrt{(m'+1)(m'+\tfrac{1}{2})}] + \left[\frac{mm'}{(m-\frac{1}{2})(m'-\frac{1}{2})}\right]^{1/2}(\xi^*\xi)^{-1}\sin[2\lambda t\sqrt{m(m-\tfrac{1}{2})}]\sin[2\lambda t\sqrt{m'(m'-\tfrac{1}{2})}]\Bigg]. \tag{13}$$

We first calculate the effect of the field statistics on the mean photon number. Using $N=2K_0-\frac{1}{2}$

$$\bar{n}(t)=\mathrm{Tr}_{\mathrm{field}}[N\rho_f(t)] =\sum_{m=0}^{\infty}2mp_m(t), \tag{14}$$

where we have used the fact that K_0 is diagonal with eigenvalues $m+\frac{1}{4}$ and we have set $p_m(t)=\langle m|\rho_f(t)|m\rangle$. We have considered the case for $|\xi|=0.8$ for which the initial photon number is $N(0)=1.78$. In Fig. 1 we display our results. The apparently random behavior is reminiscent of the overlapping decays and revivals observed for the atomic inversion in a field of thermal light. In fact, the squeezed vacuum state has statistical properties similar to thermal light. From the coefficients of $|m\rangle$ in Eq. (11) one has the distribution

$$p_m=\left[\frac{1}{\bar{n}+1}\right]^{1/2}\left[\frac{\bar{n}}{\bar{n}+1}\right]^{m}\frac{\Gamma(m+\frac{1}{2})}{m!\Gamma(\frac{1}{2})}, \tag{15}$$

which indeed is similar to the distribution for thermal light. However, p_m is the probability that there are $2m$ photons in the state. The behavior of the atomic inversion for a squeezed state interacting with the one-photon JCM has been discussed by Milburn.[13]

We next consider the time development of the variances of the quadrature operators

$$X_1=\tfrac{1}{2}(a+a^\dagger), \quad X_2=\frac{1}{2i}(a-a^\dagger). \tag{16}$$

The variances for the SU(1,1) states may be written as[5,6]

$$(\Delta X_{1,2})^2=\langle K_0\rangle\pm\tfrac{1}{2}\langle K_+ + K_-\rangle. \tag{17}$$

The time evolution of the variances is given by

$$(\Delta X_{1,2})^2(t)=\mathrm{Tr}_{\mathrm{field}}\{\rho_f(t)[K_0\pm\tfrac{1}{2}(K_+ + K_-)]\} =\sum_{m=0}^{\infty}\{(m+\tfrac{1}{4})\langle m|\rho_f(t)|m\rangle\pm\tfrac{1}{2}[\langle m|\rho_f(t)|m+1\rangle\sqrt{(m+1)(m+\tfrac{1}{2})} +\langle m|\rho_f(t)|m-1\rangle\sqrt{m(m-\tfrac{1}{2})}]\}, \tag{18}$$

where we have used the fact that

$$K_+|m\rangle=[(m+1)(m+\tfrac{1}{2})]^{1/2}|m+1\rangle, \quad K_-|m\rangle=[m(m-\tfrac{1}{2})]^{1/2}|m-1\rangle. \tag{19}$$

In Fig. 2 we show results for $|\xi|=0.4$, 0.8, and 0.9 for the X_1 quadrature. (The higher the $|\xi|$ is, the higher the initial squeezing.) We notice that the squeezing $[(\Delta X_1)^2<\frac{1}{4}]$ is initially revoked; however, squeezing recurs at later times. The higher the initial squeezing is, the more regular the oscillations become.

In summary, we have investigated the effects of squeezed vacuum light on the average photon number and the variance of the field quadratures. It is worthwhile to point out that the generalized JCM is phenomenological and, as pointed out by Alsing and Zubairy,[14] results based on such models ignore the dynamic Stark effect. This may be taken into account by considering an effective two-level atom interacting with the field through intermediate states. We hope to discuss this approach elsewhere.

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