Dielectronic-recombination rate coefficients of hydrogenlike ions in high-temperature low-density plasmas

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Dielectronic-recombination rate coefficients of selected hydrogenlike ions ($Z=10$, 14, 18, 20, 22, 26, and 28) have been calculated at various plasma temperatures ($kT_e=0.6-10$ keV) using the Hartree-Fock-Slater atomic model and assuming a Maxwellian velocity distribution for the free electrons. The effects of configuration interaction and spin-orbit coupling are included in all intermediate resonance states $|2lnl'SLJ\rangle$ for $n\leq4$. Configuration average rates have been used to calculate dielectronic-recombination rate coefficients for $n=5-8$, and $1/n^3$ scaling law for all higher-lying states. The results from the present calculations are compared with available theoretical and experimental values of dielectronic-recombination rate coefficients.

I. INTRODUCTION

In the dielectronic-recombination (DR) process a free electron is captured by an ion initially in the ground state $|g\rangle$ with the simultaneous excitation of one of the electrons of the ion, and the resulting autoionizing state $|s\rangle$ emits a photon leading to a final state $|f\rangle$ that cannot autoionize. For a hydrogenlike ion this process can be represented by

$$1s\,^2S_{1/2}+e\rightarrow|nln'l'SLJ\rangle\rightarrow|1sn''l''S'L'J'\rangle+h\nu\ . \tag{1}$$

The quasibound autoionizing state $|s\rangle\equiv|nln'l'SLJ\rangle$ is formed if $\varepsilon=E_s-E_g\pm\delta E$, where ε, E_g, and E_s are the energies, respectively, of the free electron, the ground-state hydrogenlike ion $|g\rangle$, and the excited-state heliumlike ions $|s\rangle$; and δE is the width of the autoionizing resonance state $|s\rangle$. The state $|s\rangle$ may either autoionize, reemitting the electron, or it may decay radiatively to some lower energy state $|f\rangle$. The former process represents the resonant electron-ion scattering; the latter is the dielectronic recombination if the final state $|f\rangle$ lies sufficiently low in energy and is stable against autoionization. The radiative transitions $|s\rangle\rightarrow|f\rangle$ give rise to dielectronic satellite lines with transition energies less than the Lyman-α resonance lines. These dielectronic x-ray satellites provide a convenient method for spectroscopic diagnostics of high-temperature plasmas.[1-5] DR is the most dominant recombination channel in a low-density and high-temperature plasma; in any mathematical modeling of fusion plasma DR rate coefficients are needed[6,7] because DR affects the ionization balance in the plasma and cools it radiatively.

High-resolution x-ray spectra of hydrogenlike ions have been observed in tokamaks,[4,8,9] laser-induced fusion devices,[10-12] and solar fares.[13-19] Recently Bitter *et al.*[4] have obtained x-ray spectra of Lyman-α lines and the associated dielectronic satellites from hydrogenlike titanium at Princeton Large Torus (PLT) with central temperatures $T_e=2$ keV and central electron densities $n_e=8\times10^{13}$ cm^{-3}. DR rate coefficients have been determined from the measured satellite-to-resonance line intensities, and theoretical electron excitation rate coefficients for $2p\rightarrow1s$ transitions. Källne *et al.* reported on DR satellite spectra of hydrogenlike and heliumlike argon,[8] and hydrogenlike sulfur[9] from the Alcator C tokamak. Spectra from hydrogenlike neon have been observed[10,11] from fusion microbaloons. Aglitski *et al.*[12] have reported on x-ray spectra from hydrogenlike and heliumlike sodium, magnesium, aluminum, silicon, phosphorus, sulfur, chlorine, potassium, calcium, and titanium from laser plasmas. X-rays from solar corona have been reported by Aglitski *et al.*[13] (hydrogenlike magnesium), Parmar *et al.*[14] (hydrogenlike iron), Tanaka *et al.*[15] (hydrogenlike and heliumlike iron), Doschek *et al.*[16-18] (hydrogenlike calcium and iron), and Parkinson *et al.*[19] (hydrogenlike silicon).

Theoretical values of DR rate coefficients for hydrogenlike ions have been reported for titanium by Bitter *et al.*,[4] for chromium by Karim and Bhalla,[20] for iron by Dubau *et al.*,[21] and for neon by Karim and Bhalla.[22] The computations in Ref. 4 were performed using the Z-expansion technique,[23] and by using the multiconfiguration Thomas-Fermi model.[24] It is expected[4] that in the next generation of large tokamaks the electron and ion temperature at the core will be in excess of 10 keV, and hydrogenlike ions of elements with $Z\geq20$ will predominate. In this paper we present DR rate coefficients for selected hydrogenlike ions with $Z=10$, 14, 18, 20, 22, 26, and 28 at plasma temperatures of 0.6–10 keV. Calculations have been done in the intermediate coupling with the inclusion of effects of configuration interaction for all intermediate resonance states $|s\rangle=|2lnlSLJ\rangle$ up to $n=4$ with all possible angular quantum number l. Configuration average rates have been used for $n=5-8$, and $1/n^3$ scaling law for $n>8$. Effects of radiative cascades on DR rate coefficients are also investigated.

II. THEORY

The intensity of a dielectric satellite line is given by

$$I_d(s-f)=n_e n_g \alpha_d(s-f) , \tag{2}$$

where n_e and n_g are, respectively, the density of electrons and hydrogenlike ions in the ground state $|g\rangle$ in the plasma, and $\alpha_d(s-f)$ is the DR rate coefficients. Assuming a Maxwellian electron energy distribution one can obtain[25–28]

$$\begin{aligned}\alpha_d(s-f)&=(\tfrac{1}{2})(2\pi\hbar^2/mkT_e)^{3/2}F_2^*(s-f)\exp(-E_a/kT_e)\\ &\equiv 1.656\times10^{-22}(kT_e)^{-3/2}F_2^*(s-f)\\ &\quad\times\exp(-E_a/kT_e) ,\end{aligned} \tag{3}$$

where $E_a=E_s-E_g$ is the Auger electron energy and kT_e, the product of electron temperature T_e and Boltzman's constant k, is in eV. The satellite intensity factor $F_2^*(s\text{-}f)$ is defined as

$$F_2^*(s-f)=(g_s/g_g)\Gamma_a(s)\Gamma_r(s-f)/\Gamma(s) . \tag{4}$$

Here g_s and g_g are, respectively, the statistical weights of the autoionizing state $|s\rangle$ and the ground state $|g\rangle$; $\Gamma_a(s)$ is the total autoionization rate of $|s\rangle$, $\Gamma_r(s-f)$ is the rate for radiative transition for $|s\rangle\rightarrow|f\rangle$, and

$$\Gamma(s)=\Gamma_a(s)+\sum_f \Gamma_r(s-f) . \tag{5}$$

Computations of DR rate coefficients thus basically reduce to calculating radiative and nonradiative transition rates for a large number of autoionizing states. X-ray transition rates in atomic units can be written as[29]

$$\Gamma_r(S'L'J'-SLJ)=(\tfrac{4}{3})(\Delta E/c)^3\,|\langle S'L'J'\|D\|SLJ\rangle|^2 \tag{6}$$

where $|SLJ\rangle$ and $|S'L'J'\rangle$ are, respectively, the initial and final states of the system, ΔE is the energy difference between these states, c is the speed of light, D is the electric dipole operator, and $\langle S'L'J'\|D\|SLJ\rangle$ is the reduced matrix element. The reduced matrix element can be written as

$$\langle S'L'J'\|D\|SLJ\rangle=(-1)^{l_>-l}[(2J+1)(2J'+1)]^{1/2}(-1)^{S+L+J'+1}(l_>)^{1/2}\begin{Bmatrix}J' & 1 & J\\ L & S & L'\end{Bmatrix}I(n'l'-nl)R_{\text{mult}}(L'-L) , \tag{7}$$

where

$$I(n'l'-nl)=\int_0^\infty P(nl,r)r^3P(n'l',r)dr \tag{8}$$

and $l_>$ is the maximum of l and l'.

A two-electron ion in configuration $nln'l'$ can decay radiatively by any of the following transitions:

$$\begin{aligned}&\text{(i) } nln'l'\rightarrow n_0l_0n'l',\quad n_0\le n\le n' ,\\ &\text{(ii) } nln'l'\rightarrow n_0l_0nl,\quad n_0\le n\le n' ,\\ &\text{(iii) } nln'l'\rightarrow nln''l'',\quad n\le n''\le n' .\end{aligned} \tag{9}$$

The multiplet factors $R_{\text{mult}}(L'-L)$ for these transitions are, respectively,

$$R(\text{i})=(-1)^{l_0+l'+L}[(2L+1)(2L'+1)]^{1/2}\begin{Bmatrix}l_0 & L' & l'\\ L & l & 1\end{Bmatrix} , \tag{10}$$

$$R(\text{ii})=(-1)^{l_0+l'-S}[(2L+1)(2L'+1)]^{1/2}\begin{Bmatrix}l_0 & L' & l\\ L & l' & 1\end{Bmatrix} , \tag{11}$$

and

$$R(\text{iii})=(-1)^{l+l'+L'}[(2L+1)(2L'+1)]^{1/2}\begin{Bmatrix}L' & l & l''\\ l' & 1 & L\end{Bmatrix} . \tag{12}$$

The Auger transition rate in atomic units for the decay of an excited state $|s\rangle=nln'l'SLJ\rangle$ to a state $|g\rangle=|n_0l_0n_cl_cS'L'J'\rangle$ is given by

$$\begin{aligned}\Gamma_a(s-g)=&\,2\pi(2l_1+1)(2l_2+1)(2l_0+1)(2l_c+1)\\ &\times\left[(-1)^p\sum_k a_kR^k(n_1l_1n_2l_2;n_0l_0n_cl_c)+(-1)^q\sum_k b_kR^k(n_1l_1n_2l_2;n_cl_cn_0l_0)\right]^2 ,\end{aligned} \tag{13}$$

where $R^k(n_1l_1n_2l_2;n_3l_3n_4l_4)$ are Slater integrals,

$$\begin{aligned}p&=l_1+l_0+L ,\\ q&=l_1+l_0+S ,\\ a_k&=\begin{Bmatrix}l_1 & l_2 & L\\ l_c & l_0 & k\end{Bmatrix}\begin{pmatrix}l_1 & k & l_0\\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix}l_2 & k & l_c\\ 0 & 0 & 0\end{pmatrix} ,\end{aligned} \tag{14}$$

and b_k is obtained from a_k by interchanging the quantum numbers l_c and l_0.

III. NUMERICAL CALCULATION

Hartree-Fock-Slater[30] atomic orbitals were used in calculating transition energies and transition-matrix elements. Doubly excited heliumlike configurations considered were $2lnl'$ with $n=2$, 3, and 4, and with all allowed values of l and l'. The uncorrelated configuration state functions ϕ_i obtained from angular momentum coupling served as the basis set by constructing a matrix representation of the total Hamiltonian. The diagonal elements $\langle \phi_i \mid H \mid \phi_i \rangle$ were corrected for relativistic effects. The mixing was allowed among configurations belonging to the same complex. Atomic-state functions $\psi_i = \sum_j C_{ij}\phi_j$ were obtained for diagonalizing the Hamiltonian. These wave functions were then used to calculate the transition rates and other atomic parameters through the formulations of Sec. II.

IV. RESULTS AND DISCUSSION

DR rate coefficients for individual intermediate resonance states $| s \rangle$ are calculated from Eq. (3). These partial DR rate coefficients are summed to obtain the total DR rates. In Table I we list the total DR rate coefficients for selected hydrogenlike ions ($Z=10$, 14, 18, 20, 22, 26, and 28). Figure 1 gives the total DR rates as a function of electron temperature T_e. The following features are apparent from the Fig. 1: (i) The maximum value of DR rate coefficients decreases with atomic number Z, (ii) the position of the maxima shifts to the higher temperature as Z increases, and (iii) at lower temperatures, DR rates for lighter elements are always greater than those for heavier elements; at very high temperatures, however, this trend is reversed. These can be understood as follows: At a particular plasma temperature T_e, partial DR rate coefficients are proportional to the product of satellite intensity factors $F_2^*(s-f)$ and the exponential functions $\exp(-E_a/kT_e)$. Rearranging Eq. (4) it can be seen that

$$F_2^*(s-f)=(g_s/g_g)\omega(s-f)a(s)/T(s)\ , \qquad (15)$$

where $\omega(s-f)=\Gamma_r(s-f)/\Gamma(s)$ is the fluorescence yield, $a(s)=\Gamma_a(s)/\Gamma(s)$ is the Auger yield for the state $| s \rangle$,

TABLE I. Total dielectronic-recombination (DR) rate coefficients for hydrogenlike ions with $Z=10$, 14, 18, 20, 22, 26, and 28 as a function of electron temperature T_e. The plasma temperature are in keV and rate coefficients are in units of $10^{-13}\ \text{cm}^3\,\text{s}^{-1}$.

kT_e (keV) \ $Z=$	10	14	18	20	22	26	28
0.60	11.20	5.46	1.65	0.76	0.33	0.04	0.01
0.80	10.68	7.19	3.18	1.82	0.99	0.21	0.09
1.00	9.62	7.89	4.41	2.89	1.80	0.54	0.26
1.20	8.54	8.00	5.25	3.76	2.59	0.97	0.53
1.40	7.57	7.79	5.74	4.40	3.26	1.43	0.86
1.60	6.73	7.44	5.99	4.83	3.77	1.88	1.20
1.80	6.01	7.04	6.06	5.09	4.14	2.27	1.54
2.00	5.40	6.62	6.03	5.22	4.40	2.60	1.84
2.20	4.89	6.21	5.92	5.26	4.56	2.88	2.11
2.40	4.44	5.82	5.76	5.24	4.65	3.09	2.34
2.60	4.06	5.46	5.58	5.17	4.68	3.26	2.53
2.80	3.73	5.12	5.39	5.07	4.67	3.38	2.68
3.00	3.43	4.82	5.19	4.95	4.63	3.47	2.81
3.20	3.18	4.53	4.99	4.82	4.57	3.53	2.90
3.40	2.95	4.27	4.79	4.68	4.49	3.56	2.97
3.60	2.75	4.04	4.60	4.54	4.40	3.57	3.02
3.80	2.57	3.82	4.42	4.39	4.30	3.57	3.05
4.00	2.41	3.62	4.25	4.25	4.20	3.55	3.07
4.20	2.27	3.43	4.08	4.11	4.09	3.52	3.08
4.40	2.13	3.26	3.92	3.98	3.99	3.49	3.07
4.60	2.01	3.11	3.77	3.85	3.88	3.44	3.06
4.80	1.91	2.96	3.63	3.72	3.78	3.39	3.04
5.00	1.81	2.83	3.49	3.60	3.67	3.34	3.01
5.40	1.63	2.58	3.24	3.37	3.47	3.23	2.95
5.80	1.48	2.37	3.02	3.17	3.29	3.11	2.88
6.20	1.36	2.19	2.82	2.97	3.11	3.00	2.79
6.80	1.20	1.95	2.55	2.72	2.87	2.82	2.66
7.20	1.11	1.82	2.39	2.56	2.72	2.71	2.57
7.60	1.03	1.70	2.25	2.42	2.58	2.60	2.49
8.00	0.96	1.59	2.13	2.29	2.46	2.50	2.40
8.40	0.89	1.50	2.01	2.18	2.34	2.40	2.32
9.00	0.81	1.37	1.85	2.02	2.18	2.26	2.20
9.60	0.74	1.26	1.71	1.87	2.03	2.14	2.09
10.00	0.70	1.19	1.63	1.79	1.95	2.06	2.02

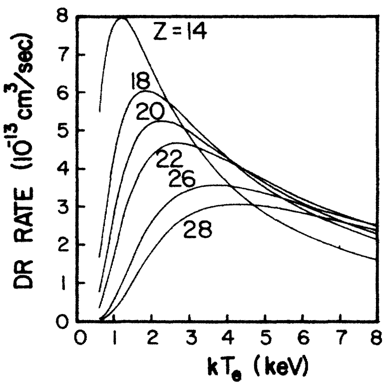

FIG. 1. Total dielectronic-recombination (DR) rate coefficients for hydrogenlike ions with $Z=14$, 18, 20, 22, 26, and 28 as a function of electron temperature.

and $T(s)$ is the lifetime of the state $|s\rangle$. Since $a(s)+\sum_f \omega(s-f)=1$, Eq. (15) suggests that $F_2^*(s-f)$, and hence DR rate coefficients at a particular T_e, are expected to be large when the Auger and fluorescence yields are comparable. For hydrogenlike ions, radiative transition rates increase rapidly with Z whereas the nonradiative rates remain nearly the same. Autoionization is the dominant decay channel for $Z\leq 20$; for $Z=22$–26 the strengths of the two channels become comparable. For higher Z the radiative channels start to dominate. Accordingly, the satellite intensity functions for the important dielectronic satellite lines increase with Z, reach a plateau near $Z=24$, and slowly falls off.[31] To understand the overall dependence of DR rate coefficients with Z and T_e, we have to impose on this trend the effects of the exponential function $\exp(-E_a/kT_e)$ which is related to the probability of a certain state $|s\rangle$ with energy E_a of being populated. For a particular autoionizing state $|s\rangle=|2lnlSLJ\rangle$, the Auger energy E_a increases with Z; hence the DR rate coefficients are expected to decrease. At very high temperatures the accessible states $|s\rangle$ for low-Z elements starts to disappear and the total DR rates for heavier elements exceed the rates of lower-Z elements. The shift of maxima to higher temperature with Z is readily explainable; by differentiating Eq. (3) with respect to kT_e and equating it to zero it can be easily seen that the DR rate coefficient is maximum at $kT_e=(\frac{2}{3})E_a$. Since Auger energy of a particular state increases with Z, the maximum shifts to higher temperatures with Z. At the temperature corresponding to the maximum DR rates the exponential function $\exp(-E_a/kT_e)$ has the same value $[=\exp(-\frac{3}{2})]$ for all elements and the factor $(1/kT_e)^{3/2}=(3/2E_a)^{3/2}$ together with the satellite intensity functions $F_2^*(s-f)$ determine the value of DR rate coefficients. For the elements considered here, the effects of the $(3/2E_a)^{3/2}$ term overweigh any variation of $F_2^*(s-f)$ functions with Z and is responsible for the decrease of maxima of DR rate coefficients as Z increases.

The DR rate coefficients from the present calculation can be compared with the calculated and experimental values reported by Bitter *et al.*[14] for hydrogenlike titanium, and the theoretical predictions by Dubau *et al.*[21] for hydrogenlike iron. From Table I, the maximum DR rate coefficient for hydrogenlike titanium occurs at $kT_e\approx 2.6$ keV and is equal to 4.68×10^{13} cm^3 s^{-1}; the corresponding values from Thomas-Fermi model and the Z-expansion technique are about 3.6×10^{13} cm^2 s^{-1} and 4.4×10^{13} cm^3 s^{-1}, respectively. The total DR rate coefficient at 2.1 keV, obtained from observed total relative satellite-to-resonance line intensity and theoretical collisional excitation rate coefficients for the resonance transition,[4] is about 4.3×10^{13} cm^3 s^{-1}. This is in excellent agreement with our calculated value of 4.45×10^{13} cm^3 s^{-1}. The maximum DR rate coefficient for hydrogenlike iron was calculated by Dubau *et al.*[21] to be about 2.1×10^{13} cm^3 s^{-1} which is about 40% less than our value of 3.57×10^{13} cm^3 s^{-1}. This discrepancy comes from (i) the neglect of all higher-lying states $2lnl'$ with $n\geq 4$ by Dubau *et al.*, and (ii) a systematic disagreement in the calculated values of $F_2^*(s-f)$ functions. It has been pointed by Bhalla and Karim[32] that for the prominent satellite lines of hydrogenlike titanium, the satellite intensity factors $F_2^*(s-f)$ calculated by Vainshtein and Safronova[23] using the Z-expansion technique are systematically larger than the Hartree-Fock-Slater values by about 1–20% whereas $F_2^*(s-f)$ functions calculated by Bely-Dubau *et al.*[24] using the multiconfiguration Thomas-Fermi model are smaller from the Hartree-Fock-Slater value by about 2–9%. For satellite lines originating from states belonging to $2l3l'$ and $2l4l'$ configurations, this discrepancy was even larger.

The difficulty in calculating total DR rate coefficients lies in the enormous number of resonance states $|2lnl'SLJ\rangle$ over which summation has to made. These states lie close in energy; inclusion of configuration interaction is therefore essential. The complexity of a calculation with configuration interaction increases progressively with n, the contribution to the total DR rate coefficients from these higher-lying autoionizing states, however, decreases rapidly. As a compromise between accuracy and computational efforts, we calculated partial DR rates from configurations with $n=5$–8 by an approximate configuration average method.[33] In Table II we list separately the DR rate coefficients obtained from summing over all possible states of configurations $2lnl'$ for $n=2$, 3, and 4 for hydrogenlike iron at different plasma temperatures. Column 5 of Table II includes contributions from all configurations with $n\geq 5$; these were obtained using configuration average scheme for $n=5$–8, and from $1/n^3$ scaling law for $n\geq 9$. These partial DR rate coefficients are plotted in Fig. 2. for hydrogenlike iron. Curves A, B, and C give, respectively, the contributions from $2l2l'$, $2l3l'$, and $2l4l'$ configurations. Curve D represents contributions from all configurations with $N\geq 5$. It can be seen that the contribution from $n\geq 5$ states is approximately equal to that from $2l4l'$ configurations and is about 11% of the total rate at $kT_e=3.5$ keV.

If autoionizing states $|i\rangle$, $|j\rangle$, $|k\rangle$, $\cdots$ lie above the state $|s\rangle$, DR rate coefficients including the effects of all radiative cascades can be written as

TABLE II. Rate coefficients for dielectronic-recombination processes that proceed via intermediate resonance states belonging to configuration $2lnl'$ with $n=2$, 3, and 4 for hydrogenlike iron at various plasma temperatures T_e. Column 5 includes contributions from all configurations with $N \geq 5$. DR rate coefficients are in units of 10^{-13} $cm^3 s^{-1}$.

kT_e (keV) \ $n=$	2	3	4	$5-\infty$	Total
0.60	0.038	0.003	0.001	0.000	0.042
0.80	0.179	0.024	0.007	0.005	0.215
1.00	0.423	0.076	0.026	0.020	0.544
1.20	0.714	0.157	0.057	0.047	0.974
1.40	1.001	0.255	0.097	0.082	1.435
1.60	1.255	0.357	0.141	0.123	1.876
1.80	1.466	0.454	0.184	0.164	2.269
2.00	1.633	0.542	0.225	0.203	2.602
2.20	1.759	0.617	0.261	0.239	2.876
2.40	1.850	0.680	0.292	0.270	3.093
2.60	1.913	0.732	0.319	0.297	3.260
2.80	1.952	0.772	0.340	0.319	3.384
3.00	1.972	0.803	0.358	0.338	3.471
3.20	1.978	0.827	0.371	0.352	3.528
3.40	1.972	0.843	0.382	0.364	3.560
3.60	1.957	0.854	0.389	0.373	3.572
3.80	1.935	0.860	0.394	0.379	3.568
4.00	1.908	0.862	0.397	0.384	3.550
4.20	1.877	0.860	0.398	0.386	3.522
4.40	1.844	0.856	0.398	0.387	3.485
4.60	1.808	0.850	0.397	0.387	3.442
4.80	1.771	0.842	0.395	0.386	3.394
5.00	1.734	0.833	0.392	0.384	3.343
5.40	1.658	0.811	0.384	0.378	3.231
5.80	1.584	0.787	0.375	0.370	3.115
6.20	1.511	0.761	0.364	0.360	2.997
6.60	1.442	0.735	0.353	0.350	2.880
7.00	1.376	0.709	0.342	0.340	2.766
7.40	1.313	0.683	0.330	0.329	2.656
7.80	1.254	0.658	0.319	0.319	2.551
8.20	1.199	0.634	0.308	0.308	2.450
8.60	1.147	0.611	0.298	0.298	2.354
9.00	1.098	0.589	0.288	0.289	2.263
9.40	1.053	0.567	0.278	0.279	2.177
10.00	0.989	0.537	0.264	0.265	2.055

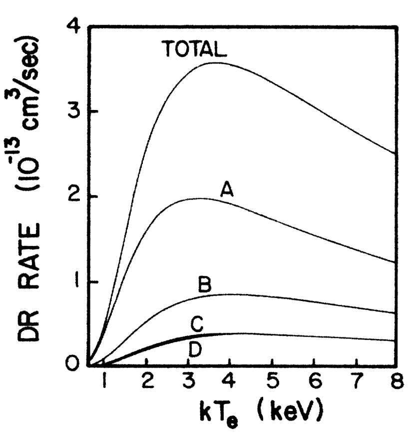

FIG. 2. Dielectronic-recombination (DR) rate coefficients of hydrogenlike iron. Curves A B, and C, represent total rate coefficient of DR processes that proceed, respectively, via autoionizing states of $2l2l'$, $2l3l'$, and $2l4l$ configurations. Curve D is obtained by summing over contributions from all configurations with $n \geq 5$.

$$\alpha_d(s)=\alpha_1(s-f)+\sum_i \alpha_d(i-s)\omega(s-f) + \sum_{\substack{i,j \\ (j>i)}} \alpha_d(j-i)\omega(i-s)\omega(s-f)+\cdots , \qquad (16)$$

where $\omega(k-j)$ are the fluorescence yields. The effect of radiative cascades to total DR rate coefficients was estimated to be less than 3%. The contributions to DR rates of $2l2l'$ configurations via radiative cascades from the $2l3l'$ complex is an order of magnitude greater than the $2l4l'$ complex. Contributions from other higher-lying states is negligible. The contribution from the third term of the above Eq. (16) is estimated to be about two orders of magnitude less than is obtained from the second term.

In conclusion, DR rate coefficients are presented as a function of electron temperature for hydrogenlike ions with $Z=10$, 14, 18, 20, 22, 26, and 28. For the case of hydrogenlike titanium the DR rate coefficient from the present Hartree-Fock-Slater calculation agrees with the experimental value of Bitter *et al.*[4] within experimental uncertainty, and differs by about 6% and 23%, respectively, from the theoretical values obtained using the Z-expansion technique and the multiconfiguration Thomas-Fermi model.

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