PHYSICAL REVIEW A VOLUME 37, NUMBER 6 MARCH 15, 1988

New critical point in smectic liquid crystals

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(Received 16 July 1987)

We study a new critical point which terminates a first-order transition line along which two smectic-A (SmA) liquid-crystal phases (or more generally any layered phases with uniaxial symmetry) with different layer spacing coexist. We call this new critical point C the SmA-SmA' critical point. We develop a model nonlinear elastic Hamiltonian to describe physical properties in the vicinity of C. We study this model using mean-field theory, one-loop-order perturbation theory, and the ϵ expansion. In mean-field theory, the SmA-SmA' transition is identical to the mean-field liquid-gas transition. In one-loop perturbation theory, critical corrections to the compressibility become important below an upper critical dimension of 6. In addition, there are important critical corrections to third-order vertex function between 6 and 8 dimensions indicating deviations from mean-field behavior below 8 dimensions. We determine a fixed point in $6-\epsilon$ dimensions describing C and calculate critical exponents to the first order in ϵ. This fixed point exhibits anisotropic scaling with different correlation length exponents $\nu_{\parallel}$ and $\nu_{\perp}$ parallel and perpendicular to the director. Scaling properties and x-ray scattering patterns in the vicinity of the SmA-SmA' critical point are also considered.

I. INTRODUCTION

Strongly polar smectogenic molecules give rise to a large number of smectic phases[1] which can be classified according to their symmetry into three groups. (1) The smectic-A (SmA) phases which are distinguished from the nematic phases by broken translational symmetry in the form of a periodic modulation of the density along the direction parallel to the equilibrium director $\mathbf{n}_0$ (z axis). These phases are subdivided into SmA_1, SmA_d, and SmA_2 phases where the indices 1, d, and 2 indicate that the wavelength of the periodic modulation is one, d ($1<d<2$), or two times the molecular length l. The associated fundamental wave number is $q_0=2\pi/l$, $2\pi/dl$, and $2\pi/2l$ in the three cases. In each case, there is liquidlike short-range order in the xy plane (plane of the layers) and quasi-long-range order (QLRO) along z. (2) The antiphases Sm$\tilde{A}$, SmA_{cre}, and Sm$\tilde{C}$ (Ref. 2) have an additional periodicity in a direction x in the plane of the layers. They, therefore, have two-dimensional long-range order in the xz plane and fluidlike short-range order in the y direction. Their hydrodynamic properties are the same as those of the columnar phases.[3] (3) The incommensurate SmA_i (Refs. 4–7) phases are characterized by spatial modulations along $\mathbf{n}$ with wavelengths l and l' whose ratio l/l' is irrational. $l<l'$ is of order the molecular length. If the ratio $l'/l=p/q$ (p and q integers), then, from the point of view of symmetry, these phases are equivalent to the smectic-A phases of (1).

Phase transitions between the smectic-A phases of (1) above are possible and warrant further comment. All three phases SmA_1, SmA_d, and SmA_2 have the same macroscopic symmetry, differing from each other only by the wavelength of their periodic modulations. (In fact the notation for these phases emphasizes their equivalence: The SmA_1 and SmA_2 phases are SmS_d phases with $d=1$ and $d=2$, respectively.) Thus it is possible to go from an SmA_1 to an SmA_d and from an SmA_d phase to an SmS_2 phase merely by continuously varying d. First-order transitions in which there is a discontinuous change in q_0 from q_{0+} to q_{0-}. (e.g., an SmA_1-SmA_d or an SmA_d-SmA_2 transition) are always possible. A line of such first-order transitions may terminate as shown in Fig. 1, in a critical point where the difference $\Delta q_0 \equiv q_{0+}-q_{0-}$ in wave vector goes to zero. Such a critical point provides a continuous path between coexisting SmA_1 (or SmA_2) and SmA_d phases.

The critical point C terminating a line of first-order transitions between smectics with different layer spacings was observed in a series of experiments by Shashidhar *et al.*[8] Previous experiments[9] providing evidence for its possible existence were not conclusive.[9] A reentrant cholesteric phase rather than a critical point has also been observed.[10] It was first discussed in Ref. 11 using a mean field theory. There, the particular transition studied was an SmA_2-SmA_d (rather than an SmA_1-SmA_d) transition with d less than 2 along the first-order line and tending toward 2 as C was approached. It was labeled an SmA_2-SmA_2' transition to emphasize the fact that the two phases have the same symmetry. In this paper, we will refer this transition as an SmA-SmA' transition since the particular value of d at the critical point is irrelevant.

Figure 2 shows the behavior as calculated in mean-field theory of the fundamental wave vector q_0 in the vicinity of C. The discontinuity Δq_0 in wave vector as the first-order line is crossed tends progressively to zero as C is ap-

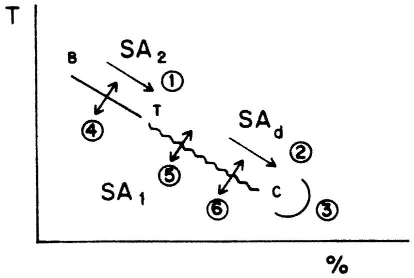

FIG. 1. Possible phase diagram with different smectic-A phases showing a second order SmA_1-SmA_2 line (BT), a tricritical point T, and a first order SmA_1-SmA_d line (TC) termination in a critical point C. A reentrant nematic or cholesteric phase occupying a finite area in the phase diagram could replace the critical point C. The paths indicated by the numbered arrows correspond to (1) continuous passage from SmA_2 to SmA_d, (2) continuous passage from SmA_d to Sm$A_{d'}$, with $d' > d$, (3) continuous passage from SmA_d to SmA_1, (4) a second-order SmA_1-SmA_2 transition, and (5) and (6) first-order SmA_1-SmA_d transitions.

proached. The slope of q_0 versus control parameter y_1 (which could be the temperature) is infinite at C indicating a disappearance of the compressional elastic constant $\overline{B}$ for the layers. The curves for q_0 shown in Fig. 2 are similar to the isotherms in the pressure volume plane near the liquid-gas critical point and to those observed by the experiments of Shashidhar *et al.*[8]

Even though all of the smectic-A phases have the same global symmetry, second-order transitions in which the size of the unit cell is multiplied by an integer are possible. There can exist a second-order SmA_1-SmA_2 transition in which the unit-cell size is doubled and in which the amplitude ψ_{k_1} of the mass density wave at wave vector $k_1 = 2\pi/2l$ grows continuously from zero. This transition has been observed experimentally.[12-15] In principle other period multiplying second-order transitions such as an Sm$A_{3/2}$-SmA_3 transition can also occur, but these involve fundamental periodicities that are larger than twice the length of a molecule and are not of interest to us at the moment. Figure 1 shows a possible phase diagram with SmA_1, SmA_d, and SmA_2 phases. Note that there is a line of second-order SmA_1- SmA_2 transitions terminating in a tricritical point T followed by a line of first-order SmA_1-SmA_d transitions terminating in a critical point C. The six different paths shown in Fig. 1 have been observed in different experimental systems and can be interpreted theoretically in terms of the frustration model of smectics.[4,16]

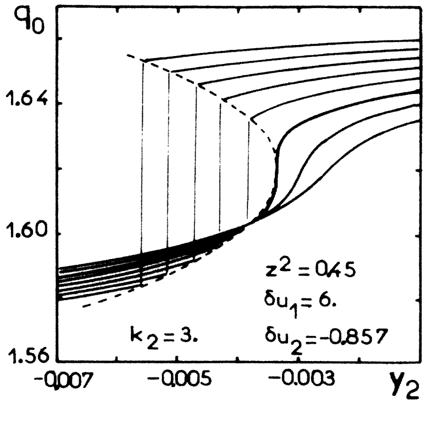

FIG. 2. This figure shows the dependence of the fundamental wave vector q_0 on the parameter y_1 as calculated in the mean-field theory. The discontinuity Δq_0 goes continuously to zero as C is approached.

In this paper we will develop a nonlinear elastic model to describe the critical point C and analyze its critical properties using ϵ expansion. Our principal results are as follows.

(1) Because the model Hamiltonian has two distinct third-order potentials, the upper critical dimension d_c below which singular corrections to $\overline{B}$ become important is six and not four as a strict analogy with the liquid-gas transition would imply.

(2) Mean-field theory applies above $d = 8$ dimensions and is identical to mean-field theory for the liquid-gas transition. Between 6 and 8 dimensions there are nonanalytic corrections to the third-order vertex, the consequences of which have not yet been fully analyzed. In all probability, they affect the shape of the mean-field coexistence curve in a nontrivial way.

(3) There is a fixed point describing C in $6-\epsilon$ dimensions. It leads to anisotropic scaling with different correlation length exponents $\nu_{\parallel}$ and $\nu_{\perp}$ for directions parallel and perpendicular to the director. It also implies that the third-order vertex diverges along the critical isochore (line of critical order parameter) in contrast to the liquid-gas transition where a similar vertex goes to zero. The strongest experimental signature of C is the discontinuity Δq_0 along the co-existence curve as C is approached. It would be very desirable to have theoretical calculations both of the shape of the coexistence curve (in a temperature-concentration plane) and the functional form Δq_0. We have found these calculations to be extremely complex and will not treat them here. We hope, however, to be able to discuss them in a publication in the near future.

We now summarize the content of the paper. In Sec. II we present an anisotropic model Hamiltonian H to describe the SmA-SmA' critical point C in terms of two order parameters: $S(\mathbf{x})$ describing the changes in the x-ray scattering intensity in the quasi-Bragg peaks and $u(\mathbf{x})$ describing the local dilation of the smectic layers. After integrating out the noncritical linear combination of $S(\mathbf{x})$ and $u(\mathbf{x})$, the resulting Hamiltonian $H[u(\mathbf{x})]$ will be written in terms of $u(\mathbf{x})$ only. In Sec. III we discuss Gaussian fluctuations near the mean-field critical point of the model Hamiltonian introduced in Sec. II. In Sec. IV we develop one-loop perturbation theory for H to identify the upper critical dimension to be six rather than four. We also show that the third-order vertex function has singular behavior for spatial dimension $6 < d < 8$, and the nth-order vertices are divergent throughout the smectic phase for $d < d_{nc} = 2n-1$. In Sec. V we derive renormalization-group recursion relation to locate a fixed

TABLE I. Fundamental exponents to the first order in ϵ.

Exponent	Description	$O(\epsilon)$ value
$\nu_\perp$	Perpendicular correlation length exponent	$0.5-0.1873\epsilon$
$\nu_{\parallel}$	Parallel correlation length exponent	$0.5-0.0534\epsilon$
$\eta_\perp$	Anomalous exponent	0.0364ϵ
$\mu_{\parallel}$	Anisotropy exponent	0.2679ϵ

TABLE II. Derived exponents to the first order in ϵ.

Exponent	Scaling relation	$O(\epsilon)$ value
α	$2-(d+\mu_{\parallel})\nu_\perp$	$-1+1.4899\epsilon$
β	$\frac{1}{2}(d-2+\eta_\perp+3\mu_{\parallel})\nu_\perp$	$1-0.4146\epsilon$
γ	$(2-\eta_\perp-2\mu_{\parallel})\nu_\perp$	$1-0.6607\epsilon$
Δ	$\frac{1}{2}(d+2-\eta_\perp-\mu_{\parallel})\nu_\perp$	$2-1.0753\epsilon$
ϕ_3	$\lambda_3\nu_\perp$	1.8388ϵ

point in $6-\epsilon$ dimensions and calculate to the first order in ϵ critical exponents summarized in Tables I and II. In Sec. VI we discuss scaling in the vicinity of C. Finally, in Sec. VII we review our results and suggest possible experiments. There are a number of appendices with calculational details.

II. DEVELOPMENT OF THE MODEL

As discussed in Refs. 5, two order parameters are necessary to describe the properties of frustrated smectics: the first $\rho(\mathbf{x})$ measures mass-density modulations familiar in traditional smectics A, and the second $P_z(\mathbf{x})$ measures the antiferroelectric ordering of the polar heads along the z axis (perpendicular to the plane of the layers). The nematic (N) and monolayer smectic-A (SmA_1) phases are characterized, respectively, by ($|\rho| = |P_z| = 0$) and ($\rho \neq 0$, $P_z = 0$). In the bilayer or SmA_2 phase, the two modulations coexist with

$$\begin{aligned}\rho(\mathbf{x})&=\psi_2(\mathbf{x})e^{2iq_0z}\ ,\\ P_z(\mathbf{x})&=\psi_1(\mathbf{x})e^{iq_0z}\ .\end{aligned}\tag{2.1}$$

Because of the asymmetry of the molecules, the antiferroelectric order also gives rise to a density modulation, and the structure of the SmA_2 phase can be described by the fundamental and the first harmonic of a periodic modulation of the density $m(\mathbf{x})$:

$$m(\mathbf{x})=\psi_1(\mathbf{x})e^{iq_0z}+\psi_2(\mathbf{x})e^{2iq_0z}+\text{c.c.}\tag{2.2}$$

It was shown in Refs. 4 and 5 that the value of the wave vector q_0 results from a compromise between elastic constraints favoring incommensurate wave vectors q_1 and q_2 for P_z and ρ (with $q_2 \approx 1.8q_1$) and a pinning potential favoring commensurate modulations with wave vectors q_0 and $2q_0$ with $q_2/2<q_0<q_1$. The SmA_d phase (sometimes called a partial bilayer phase) corresponds to the limiting case with $q_0 \approx q_1$. The existence of a line of first-order SmA_2- SmA_d transitions is a direct result of the frustration imposed by the necessity of the above compromise. Since the SmA_d and SmA_2 phases have the same symmetry, this first-order line can only terminate at some other phase boundary[10] or at a critical point C as shown in Fig. 1.

In order to describe fluctuations in the vicinity of the critical point C, it is convenient to express the order parameter $m(\mathbf{x})$ in the following way:

$$\begin{aligned}m(\mathbf{x})=&[S_{01}+a_1S(\mathbf{x})]e^{i\theta}e^{iq_0'[z+u(\mathbf{x})]}\\ &+[S_{02}-a_2S(\mathbf{x})]e^{2i\theta}e^{2iq_0'[z+u(\mathbf{x})]}+\text{c.c.}\ ,\end{aligned}\tag{2.3}$$

where S_{01} and S_{02} are, respectively, the amplitudes of the first and second density waves at the critical point. The Bragg peaks predicted by this order parameter occur at $q_0=q_0'(1+M_z)$ and $2q_0$ where $M_z=\langle\nabla_z u\rangle$. The choice of q_0' is arbitrary if nonzero values of M_z are allowed. There are two obvious choices for q_0' which we will use in the discussions that follow. The first is to choose q_0' so that M_z is always zero, i.e., to require q_0' always to be equal to the magnitude of the equilibrium wave vector. The second is to set $q_0'=q_{0c}$ where q_{0c} is the magnitude of wave vector at the critical point C. The latter form is the most convenient for discussing the transition. In this case $\Delta q_0=q_{0c}(M_z^+-M_z^-)$, where M_z^+ and M_z^- are the values of $\langle\nabla_z u\rangle$ in the coexisting smectic phases, becomes the order parameter which goes to zero as the critical point C is approached. $S(\mathbf{x})$ measures the deviation of the intensity of the q_0 and $2q_0$ quasi-Bragg peaks from their values at C. Its thermal average $\langle S(\mathbf{x})\rangle$ is zero at the critical point and non-zero and of opposite sign in the SmA and SmA' phases. S has Ising symmetry and fluctuates violently as C is approached. $u(\mathbf{x})$, unlike $\theta(\mathbf{x})$, is a hydrodynamic or elastic variable whose gradient describes local dilations of the layer spacing. Fluctuations in both $S(\mathbf{x})$ and $u(\mathbf{x})$ are long range at C, and both variables must be included in a Landau-Ginzburg-Wilson Hamiltonian determining the universality class of the critical point C.

The energy $H_{\text{sm}}[u(\mathbf{x})]$ (sm for smectic) of elastic deformations in smectic-A phases must be invariant with respect to both rigid translations and rigid rotations. The former invariance is assured if H_{sm} is a function only of gradients of $u(\mathbf{x})$, the latter by requiring that the compressional term, $\nabla_z u$, appear only in the rotationally invariant combination[17,18]

$$E[u(\mathbf{x})]=\nabla_z u+\tfrac{1}{2}(\nabla u)^2\ .\tag{2.4}$$

The "classical" elastic Hamiltonian is, thus,

$$H_{\text{sm}}(u)=\int d^dx\left[h_0E(u)+\tfrac{1}{2}B_0E^2(u)+\frac{1}{3!}w_0E^3(u)+\frac{1}{4!}v_0E^4(u)+\tfrac{1}{2}[K_1^0(\nabla_\perp^2u)^2+K_2^0(\nabla_z^2u)^2+2K_{12}^0(\nabla_z\nabla_\perp u)^2]\right]\ .\tag{2.5}$$

Here and in what follows, we use reduced units whereby all Hamiltonians and free energies are unitless by virtue of a division by the Boltzmann constant times temperature. The potentials h_0, B_0, etc., in this expression depend on the reference wave number q_0' and external control parameters such as temperature and concentration or pressure.

The part of the Hamiltonian depending on the order parameter $S(\mathbf{x})$ contains both even and odd powers of S:

$$H_S=\int d^dx\left[h_S S+\tfrac{1}{2}[rS^2+c(\nabla S)^2]+\frac{1}{3!}w_S S^3+\frac{1}{4!}u_S S^4\right] \tag{2.6}$$

where

$$r=a_S(T-T_S) \tag{2.7}$$

where T is the temperature and T_S is the mean-field transition temperature for S in the absence of coupling to $u(\mathbf{x})$. Because of the anisotropy of the smectic state, the gradient term in this equation should be written as $c_{\|}(\nabla_z S)^2+c_{\perp}(\nabla_{\perp} S)^2$. An anisotropic rescaling leads, however, to Eq. (2.6) which we will use without loss of generality in what follows. The Hamiltonian coupling S to u is

$$H_{\text{int}}=\int d^dx(\lambda_{11}SE+\lambda_{12}S^2E+\lambda_{21}E^2S+\lambda_{22}E^2S^2)\ . \tag{2.8}$$

The total Hamiltonian describing the critical point C is thus

$$H(S,u)=H_{\text{sm}}(u)+H_S(S)+H_{\text{int}}(S,u)\ . \tag{2.9}$$

We note that both S and $\nabla_z u$ change discontinuously as the coexistence line is crossed, and their fluctuations are described by $H[S,u]$. S and u are, however, coupled linearly in $H[S,u]$, and only a single linear combination is critical. The other independent combination is noncritical and can be removed from the problem. There are various ways to identify the critical and noncritical variables. We find it most convenient to remove the linear coupling between S and E via the transformation

$$S(\mathbf{x})=\sigma(\mathbf{x})-\lambda_{11}E[u(\mathbf{x})]/r. \tag{2.10}$$

Then the part of $H[S,u]$ harmonic in σ and E becomes

$$H^{(2)}(\sigma,u)=\int d^dx\ \tfrac{1}{2}(BE^2+r\sigma^2)\ , \tag{2.11}$$

where

$$B=B_0-\lambda_{11}^2/r\equiv B_0(r-r_c)/r\ , \tag{2.12}$$

and

$$r_c=\lambda_{11}^2/B_0\equiv a_s(T_c-T_S)\ . \tag{2.13}$$

Since r_c is positive, E becomes critical before σ as temperature decreases. Thus, E, or $\nabla_z u$ is critical at C, and σ is not. The full Hamiltonian $H[\sigma,u]$ becomes

$$H(\sigma,u)=H'_{\text{sm}}(u)+H_S(\sigma)+H'_{\text{int}}(u,\sigma), \tag{2.14}$$

where $H'_{\text{sm}}(u)$ has exactly the same form as Eq. (2.5) with the replacement $h_0\rightarrow h_0-\lambda_{11}h_S/r$, $B_0\rightarrow B_0-\lambda_{11}^2/r$, $w_0\rightarrow w_0-w_S(\lambda_{11}/r)^3$, $v_0\rightarrow v_0+u_S(\lambda_{11}/r)^4$, $K_1^0\rightarrow K_1^0$, $K_{12}^0\rightarrow K_{12}^0+c(\lambda_{11}/r)^2/2$, and $K_2^0\rightarrow K_2^0+c(\lambda_{11}/r)^2$. $H'_{\text{int}}(u,\sigma)$ involves terms that are at least trilinear in u and σ. Integration over the noncritical variable $\sigma(\mathbf{x})$ leads to a Hamiltonian in terms of $u(\mathbf{x})$ alone that again has exactly the same form as Eq. (2.5) but with renormalized potentials

$$\begin{aligned}H(u)&=H'_{\text{sm}}[u]-\ln\int D\sigma(\mathbf{x})e^{-[H_S(\sigma)+H'_{\text{int}}(u,\sigma)]}\\&\equiv\int d^dx\left[hE(u)+\tfrac{1}{2}BE^2(u)+\frac{1}{3!}wE^3(u)+\frac{1}{4!}vE^4(u)+\tfrac{1}{2}[K_1(\nabla_{\perp}^2u)^2+K_2(\nabla_z^2u)^2+2K_{12}(\nabla_z\nabla_{\perp}u)^2]\right]\ .\end{aligned} \tag{2.15}$$

This is the Hamiltonian we will use in the remainder of this paper. Since the independent variable is u not E, it is convenient for future analysis to reexpress $H[u]$ in terms of $\nabla_z u$ and $\nabla_{\perp} u$:

$$\begin{aligned}H=\int d^dx\Big[&h(\nabla_z u)+\tfrac{1}{2}B_1(\nabla_z u)^2+\tfrac{1}{2}B_2(\nabla_{\perp} u)^2+\tfrac{1}{2}[K_1(\nabla_{\perp}^2u)^2+K_2(\nabla_z^2u)^2+2K_{12}(\nabla_z\nabla_{\perp}u)^2]\\&+\frac{1}{3!}w_1(\nabla_z u)^3+\tfrac{1}{2}w_2(\nabla_z u)(\nabla_{\perp} u)^2+\frac{1}{4!}v_1(\nabla_z u)^4+v_2\frac{1}{4!}(\nabla_{\perp} u)^4+\tfrac{1}{12}v_{12}(\nabla_z u)^2(\nabla_{\perp} u)^2\Big]\end{aligned} \tag{2.16}$$

where

$$\begin{aligned}&B_1=B+h,\ \ w_1=w+3B,\ \ v_1=v+6w+3B\ ,\\&B_2=h,\ \ w_2=B,\ \ v_2=3B,\ \ v_{12}=3w+3B\ .\end{aligned} \tag{2.17}$$

In the absence of external fields ($h=B_2=0$). The quadratic terms to lowest order in ∇_z and $\nabla_{\perp}$ are responsible for the Landau-Peierls instability[19,20] and the divergence of $\langle\,|u(\mathbf{x})-u(0)|^2\rangle$ at large $|\mathbf{x}|$ calculated by Caillé.[21]

As discussed by Grinstein and Pelcovits,[17,18] the $(\nabla_z u)(\nabla_{\perp} u)^2$ and $(\nabla_{\perp} u)^4$ terms in Eq. (2.16) lead to a breakdown in the elastic gradient expansion in three dimensions. The renormalized elastic constants $B_1(q)$ and $K_1(q)$, respectively, vanish and diverge as powers of ln$q$ at small wave vectors q. These terms also lead to divergences of certain viscosities with inverse frequency.[22,23] The other anharmonic terms such as $(\nabla_z u)^3$, $(\nabla_z u)^4$, and $(\nabla_z u)^2(\nabla_{\perp} u)^2$ are irrelevant in the renormalization group sense in the smectic phase and do not lead to any singular renormalizations of elastic constants.

As discussed in the beginning of this section, the order parameter for the SmA-SmA' transition is $M_z=\langle\nabla_z u\rangle$. If $\nabla_{\perp}u$ is equal to zero in Eq. (2.16), the resulting Hamiltonian in terms of $\langle \nabla_z u \rangle$ alone is identical in form to that describing the liquid-gas transition as a function of its scalar order parameter $\phi = n_l - n_g$, the differences in densities of the liquid and gas phases. Thus, in mean-field theory, the SmA-SmA' transition occurs at $h = B_1 = w_1 = 0$ and is identical to the liquid-gas transition with susceptibility and order-parameter exponents $\gamma_{\mathrm{MF}} = 1$ and $\beta_{\mathrm{MF}} = 1/2$.

When fluctuations are included, $\langle \nabla_\perp u \rangle$ cannot be ignored. In the liquid-gas Hamiltonian, there is a single third-order invariant ϕ^3 which can be removed by shifting ϕ.[24] This implies that the liquid-gas transition belongs to the same universality as the Ising model with additional higher order irrelevant potentials. In the SmA-SmA' Hamiltonian, there are two distinct third-order potentials $(\nabla_z u)^3$ and $\nabla_z u (\nabla_\perp u)^2$. They cannot both be removed by shifting the order-parameter field $\langle \nabla_z u \rangle$. It is this fact that leads to an upper critical dimension of six and a new universality class for the SmA-SmA' transition.

III. GAUSSIAN FLUCTUATIONS

Before proceeding to a formal analysis of perturbation theory and the ϵ expansion, it is instructive to study Gaussian fluctuations near the mean-field critical point of the model Hamiltonian introduced in the preceding section. The $u-u$ and $\sigma - \sigma$ correlation function in Fourier space are

$$G_{uu}^{(2)}(\mathbf{q},r) = (Bq_z^2 + K_1 q_\perp^4 + K_2 q_z^4 + 2K_{12} q_z^2 q_\perp^2)^{-1} , \tag{3.1}$$

$$G_{\sigma\sigma}^{(2)}(\mathbf{q},r) = (r + cq^2)^{-1} , \tag{3.2}$$

$$G_{SS}^{(2)} = G_{\sigma\sigma}^{(2)}(\mathbf{q},r) + \frac{\lambda_{11}^2 q_z^2}{(r+cq^2)^2} G_{uu}^{(2)}(\mathbf{q},r) . \tag{3.3}$$

Thus, the order-parameter amplitude does exhibit critical fluctuation near C, but these are anisotropic and less divergent than those of u.

The director is normal to the smectic layers so that the director correlation function is $G_{n_\perp n_\perp}^{(2)} = q_\perp^2 G_{uu}^{(2)}$. At the critical point, $B=0$ so that

$$G_{n_\perp n_\perp}^{(2)}(\mathbf{q}) = (K_1 q_\perp^2 + 2K_{12} q_z^2 + K_2 q_z^4 q_\perp^{-2})^{-1} . \tag{3.4}$$

Apart from the term proportional to K_2, this correlation function is identical to that of the nematic phase.[25] Thus, a smectic at the critical point C is essentially a reentrant nematic.

The x-ray scattering intensity function, $I(\mathbf{k})$ at wave vector $\mathbf{k}$, is proportional to $\langle |m(\mathbf{k})|^2 \rangle$. Thus, from Eq. (2.3),

$$\begin{aligned} I(\mathbf{k}) &= \langle |m(\mathbf{k})|^2 \rangle \\ &= a_1^2 [G_1(\mathbf{k}-\mathbf{q}_0) + G_1(\mathbf{k}+\mathbf{q}_0)] \\ &\quad + a_2^2 [G_2(\mathbf{k}-2\mathbf{q}_0) + G_2(\mathbf{k}+2\mathbf{q}_0)] \end{aligned} \tag{3.5}$$

where $G_1(\mathbf{k})$ and $G_2(\mathbf{k})$ are the Fourier transforms of

$$G_1(\mathbf{x}) = \langle [S_{01} + a_1 S(\mathbf{x})] e^{i\theta(\mathbf{x})} e^{iq_0 u(\mathbf{x})} [S_{01} + a_1 S(0)] e^{-i\theta(0)} e^{-iq_0 u(0)} \rangle \tag{3.6}$$

and

$$G_2(\mathbf{x}) = \langle [S_{02} - a_2 S(\mathbf{x})] e^{2i\theta(\mathbf{x})} e^{i2q_0 u(\mathbf{x})} [S_{02} - a_2 S(0)] e^{-2\theta(0)} e^{-i2q_0 u(0)} \rangle . \tag{3.7}$$

In the vicinity of the critical point, $S(\mathbf{x})$ is small and fluctuations in $\theta(\mathbf{x})$ are important so that $G_p(\mathbf{x})$ can be approximated as

$$\begin{aligned} G_p(\mathbf{x}) &\approx S_{0p}^2 \langle e^{ipq_0[u(\mathbf{x})-u(0)]} \rangle \\ &= S_{0p}^2 e^{-p^2 q_0^2 g^{(2)}(\mathbf{x})/2} \end{aligned} \tag{3.8}$$

with $g^{(2)}(\mathbf{x}) = \langle [u(\mathbf{x}) - u(0)]^2 \rangle$. The second step follows because the Gaussian Hamiltonian is harmonic in u. $g^{(2)}(\mathbf{x})$ is simply related to $G_{uu}(\mathbf{q})$:

$$g^{(2)}(\mathbf{x}) = 2 \int \frac{d^d q}{(2\pi)^d} (1 - e^{i\mathbf{q}\cdot\mathbf{x}}) G_{uu}(\mathbf{q}) . \tag{3.9}$$

When $\overline{B}$ is nonzero, $g^{(2)}(\mathbf{x})$ reduces to the result obtained by Caillé:

$$g^{(2)}(\mathbf{x}) \sim q_0^{-2} \eta_c \begin{cases} \ln x_{\parallel} , & \mathbf{x}_\perp = 0, \\ 2\ln |\mathbf{x}_\perp| , & x_{\parallel} = 0 . \end{cases} \tag{3.10}$$

where $\eta_c = q_0^2 / [8\pi (K_1 \overline{B})^{1/2}]$ leading to $G_p(x_{\parallel}, 0) \sim x_{\parallel}^{-p^2 \eta_c}$ and $G_p(0, \mathbf{x}_\perp) \sim |\mathbf{x}_\perp|^{-2p^2 \eta_c}$. Away from the critical point where η_c is less than 2, the power-law decay of $G_p(\mathbf{x})$ implies a power-law divergence in the Fourier transform $G_p(\mathbf{k})$. As the critical point is approached, $\eta_c \to \infty$, and the simple power-law form of $G_p(\mathbf{x})$ breaks down in a complicated way.[26] At the critical point itself, $\overline{B} = 0$, and

$$\begin{aligned} g^{(2)}(\mathbf{x}) &= 2 \int \frac{d^3 q}{(2\pi)^3} \frac{1 - e^{i\mathbf{q}\cdot\mathbf{x}}}{Kq^4} \\ &\to \frac{|\mathbf{x}|}{4\pi K} \quad \text{as } |\mathbf{x}| \to \infty \end{aligned} \tag{3.11}$$

for $K = K_1 = K_2 = K_{12}$ in three dimensions. Hence, the correlation function $G(\mathbf{x})$ dies off exponentially as $|\mathbf{x}| \to \infty$, and the smectic order is destroyed at the critical point. There is no $|\mathbf{x}|^{-(d-2)}$ prefactor, however, so that in three dimensions $G_p(\mathbf{k})$ is a Lorentzian squared rather than a Lorentzian:

$$G_p(\mathbf{k}) = \frac{S_0^2}{K} \frac{p^2 q_0^2}{[|\mathbf{k}|^2 + (p^2 q_0^2 / 8\pi K)^2]^2} . \tag{3.12}$$

Thus, even though the director correlation function at C is very similar to that of the nematic phase, the order-parameter correlation function is quite different. Note that the width of the peaks in $G_p(\mathbf{k})$ increases with increasing p.

X-ray scattering provides the most direct measure of correlation lengths in the nematic phase above the nematic to smectic-A transition. Because of the complicated (and as yet not fully calculated) behavior of $G_p(\mathbf{k})$ in the vicinity of C, even in the Gaussian model, it is not clear how much information x-ray scattering will provide about correlation length critical exponents.

IV. PERTURBATION THEORY

A. Notation and vertex functions

Before proceeding to a discussion of perturbation theory, we need to introduce some notation. All thermodynamic properties and correlation functions of the model can be obtained from the partition function,

$$Z[h(\mathbf{x})]=\int Du(\mathbf{x})\exp\{-H_{\rm sm}[u(\mathbf{x})]-H_{\rm ext}\}\ , \tag{4.1}$$

in the presence of an external field h_e coupling to $u(\mathbf{x})$ via the Hamiltonian,

$$H_{\rm ext}=-\int d^dx\, h_e(\mathbf{x})u(\mathbf{x})\ . \tag{4.2}$$

In particular, the average displacement $\langle u(\mathbf{x})\rangle$ satisfies

$$\langle u(\mathbf{x})\rangle=Z^{-1}\int Du\ u(\mathbf{x})e^{-H_{\rm sm}-H_{\rm ext}}\ . \tag{4.3}$$

Alternatively, thermodynamic functions and correlation functions can be determined from the Legendre transformed potential

$$\Gamma[\langle u(\mathbf{x})\rangle]=-\ln Z[h(\mathbf{x})]+\int d^dx\, h_e(\mathbf{x})\langle u(\mathbf{x})\rangle\ . \tag{4.4}$$

The vertex functions of this potential are defined via

$$\Gamma^{(n)}(\mathbf{x}_1,\dots,\mathbf{x}_n)=\frac{\delta^n\Gamma}{\delta\langle u(\mathbf{x}_1)\rangle\cdots\delta\langle u(\mathbf{x}_n)\rangle}\ . \tag{4.5}$$

Since $H_{\rm sm}$ depends only on the gradients of u, it is useful to consider a restricted set of external potentials $h_e=-\nabla\cdot\mathbf{h}_e$ so that

$$H_{\rm ext}=-\int d^dx\,\mathbf{h}_e\cdot\nabla u(\mathbf{x}) \tag{4.6}$$

and to introduce reduced vertex functions

$$f^{(n)}_{i_1\cdots i_n}=\frac{1}{V}\frac{\partial^n\Gamma}{\partial M_{i_1}\cdots\partial M_{i_n}}\ , \tag{4.7}$$

where V is the volume of the system and

$$M_i\equiv\langle\nabla_i u(\mathbf{x})\rangle\ . \tag{4.8}$$

The small wave number part of the Fourier transforms of the vertex functions defined in Eq. (4.5) are controlled by the reduced vertices $f^{(n)}_{i_1\cdots i_n}$. $f^{(0)}$ is the free-energy density. In addition, the Fourier transform of the second- and third-order vertices are

$$\Gamma^{(2)}(\mathbf{q})=f^{(2)}_{ij}q_iq_j+O(q^4) \tag{4.9}$$

and

$$\Gamma^{(3)}(\mathbf{q}_1,\mathbf{q}_2,-\mathbf{q}_1-\mathbf{q}_2)=f^{(3)}_{ijk}q_{1i}q_{2j}(-q_{1k}-q_{2k})+O(q^5) \tag{4.10}$$

where the Einstein convention is understood and where momentum conserving delta functions have been removed as usual. By symmetry, $f^{(2)}_{ij}$ and $f^{(3)}_{ijk}$ each have only two independent components:

$$f^{(2)}_{ij}=f^{(2)}_{zz}n_{0i}n_{0j}+f^{(2)}_{\perp\perp}\delta^T_{ij} \tag{4.11}$$

and

$$f^{(3)}_{ijk}=f^{(3)}_{zzz}n_{0i}n_{0j}n_{0k}+f^{(3)}_{z\perp\perp}(n_{0i}\delta^T_{jk}+n_{0j}\delta^T_{ik}+n_{0k}\delta^T_{ij}) \tag{4.12}$$

where $\delta^T_{ij}\equiv\delta_{ij}-n_{0i}n_{0j}$. The vertex $f^{(2)}_{zz}(\mathbf{q}=0)$ is the physical compressibility modulus $\overline{B}$ of the system. Because of the repeated use in what follows, we introduce a compact notation

$$f_n(M_z)\equiv f^{(n)}_{z\cdots z}(M_z) \tag{4.13}$$

for the vertices associated with the z component of the order parameter.

As discussed in Sec. II, $H_{\rm sm}$ is invariant under uniform rotations of the coordinated system. This invariance leads to Ward identities[27] among the various vertex functions that are discussed in Appendix A. Of particular importance is the identity

$$f^{(2)}_{\perp\perp}=f_1=0\ \ , \tag{4.14}$$

which is valid in the absence of symmetry breaking external fields and in equilibrium where the equation of state $f_1=0$ is satisfied. It says there is no $(\nabla_\perp u)^2$ term in a renormalized free energy. This identity is necessarily satisfied to all orders in perturbation theory.

B. One-loop calculations and the upper critical dimension

In this section we will calculate the vertex functions just introduced to one-loop order in perturbation theory. We will find this perturbation theory becomes singular at C below $d=d_c=6$ rather than below $d=4$ as in the liquid-gas transition.

We begin with some general observations about the critical point C. The equation of state

$$f_1(M_z)=0 \tag{4.15}$$

is always satisfied in equilibrium at any point in the phase diagram. At the critical point, the compressibility diverges so that

$$f_2\equiv\overline{B}=0\ . \tag{4.16}$$

In addition, the critical point terminates a coexistence line along which two phases with respective order parameters M_z^+ and M_z^- coexist. This means that the free energy has two branches $f_0^+(M_z)$ and $f_0^-(M_z)$ which are equal when evaluated at the equilibrium order parameters M_z^+ and M_z^-. Since the critical point terminates the coexistence line, it must be that

$$f_0^+(M_z)-f_0^-(M_z)\to 0 \tag{4.17}$$

as C is approached. Equations (4.15) to (4.17) impose three constraints on the potentials in $H_{\rm sm}$. When an analytic expansion of f_0 is possible, as it is in mean-field theories, the last condition is equivalent to the condition

$$f_3=0\ . \tag{4.18}$$

As discussed earlier, all potentials depend on the reference wave number q_0' which can be freely varied. If the critical point is to be reached, the potentials should depend on two other external control parameters, say temperature T

and pressure or concentration P, i.e., $h=h(q_0',T,P)$, $B=B(q_0',T,P)$, $w=w(q_0',T,P)$, etc. The critical point $(q_0'=q_{0c},T_c,P_c)$ is determined by Eqs. (4.15) to (4.17). After q_{0c}, T_c, P_c have been determined, the vicinity of the critical point can be studied by fixing $q_0'=q_{0c}$ and setting $T=T_c+\Delta T$ and $P=P_c+\Delta P$. Variations in the wave number are determined by the order parameter M_z. Equations (4.15) to (4.17) impose three constraints on the order parameter M_z and the potentials in H_{sm}.

M_z measures the deviation of the wave number of the density modulation of the smectic phase from some reference value q_{0c}. Away from the critical point, the equation,

$$f_1(M_z=0)=0 \tag{4.19}$$

determines a line intersecting C in the T-P plane along which M_z is zero. This is the analog of the critical isochore in the liquid-gas system. In this section, we will consider the approach to the critical point along this line so that M_z will be zero in all vertex functions and propagators.

The reduced free energy to one-loop order is obtained in the usual way by calculating Gaussian fluctuations about the mean-field solution:

$$\Gamma=H_{\text{sm}}(M_z)-\tfrac{1}{2}V\int_{\mathbf{q}}\ln G(\mathbf{q})\equiv Vf_0 \tag{4.20}$$

with

$$\begin{aligned}G^{-1}={}&(B_1+w_1M_z+\tfrac{1}{2}v_1M_z^2+\tfrac{1}{6}v_{12}M_\perp^2)q_z^2\\&+(B_2+w_2M_z+\tfrac{1}{2}v_{12}M_z^2+\tfrac{1}{6}v_2M_\perp^2)q_\perp^2\\&+(w_2\mathbf{M}_\perp+\tfrac{1}{3}v_2M_z\mathbf{M}_\perp)\cdot\mathbf{q}_\perp q_z\\&+(K_1q_\perp^4+K_2q_z^4+K_{12}q_z^2q_\perp^2)\,,\end{aligned} \tag{4.21}$$

where $\int_{\mathbf{q}}\equiv\int d^dq/(2\pi)^d$. From Eq. (3.20) it is straightforward to obtain other vertex functions by differentiation:

$$\begin{aligned}f_1={}&h+B_1M_z+\tfrac{1}{2}w_1M_z^2+\frac{1}{3!}v_1M_z^3+\tfrac{1}{6}v_2M_\perp^2M_z\\&-\tfrac{1}{2}\int_{\mathbf{q}}G_0^{-1}\frac{\partial G_0}{\partial M_z}\\&\to h+\tfrac{1}{2}\int_{\mathbf{q}}w_z(\theta)q_z^2G_0\quad\text{for } M_z\to 0,\end{aligned} \tag{4.22}$$

$$\begin{aligned}f_2(M_z=0)=B_1-\Big[&\tfrac{1}{2}\int_{\mathbf{q}}w_z^2(\theta)q^4G_0^2(\mathbf{q})\\&-\tfrac{1}{2}v_1\int_{\mathbf{q}}q_z^2G_0(\mathbf{q})\\&-\tfrac{1}{6}v_{12}\int_{\mathbf{q}}q_\perp^2G_0(\mathbf{q})\Big]\,,\end{aligned} \tag{4.23}$$

$$\begin{aligned}f_3(M_z=0)={}&w_1+\int_{\mathbf{q}}w_z^3(\theta)q^6G_0^3(\mathbf{q})\\&-\tfrac{3}{2}v_1\int_{\mathbf{q}}w_z(\theta)q_z^2q^2G_0^2(\mathbf{q})\\&-\tfrac{1}{2}v_{12}\int_{\mathbf{q}}w_z(\theta)q_\perp^2q^2G_0^2(\mathbf{q})\,,\end{aligned} \tag{4.24}$$

where

$$w_z(\theta)\equiv w_1\cos^2\theta+w_2\sin^2\theta\,. \tag{4.25}$$

Alternatively, these results could have been obtained by direct evaluation of the graphs shown in Fig. 3. The full one-loop contribution to $\Gamma^{(2)}(\mathbf{q})$ is given in Appendix B.

In higher dimension where an analytic expansion of $f_0(M_z)$ is possible, Eqs. (4.15), (4.16), and (4.18) can be used to locate C. Here we will keep the fourth-order potential v fixed and choose the three potentials h, B, and w to be functions of q_0', T, and P. Eqs. (4.15) and (4.18) determine the critical values B_c and w_c. These in turn can be used in Eq. (4.15) to determine h_c. h_c, B_c, and w_c then determine q_{0c}, T_c, and P_c. Near C, h, B, and w can be expressed as linear functions of $\Delta P=P-P_c$ and $\Delta T=T-T_c$. We fix $q_0'=q_{0c}$ and measure all variations in q_0 with M_z.

When $v=0$, conditions Eqs. (4.15), (4.16), and (4.17) along which Eqs. (4.17) and (4.22) to (4.24) imply that $B_c=w_c=h_c=0$ or equivalently that $B_{1c}=w_{1c}=w_{2c}=h_c=0$ as in mean-field theory. When $v\neq 0$ the critical values of the potentials can be calculated in a perturbation series in v. It is clear from Eqs. (4.22)—(4.24), that

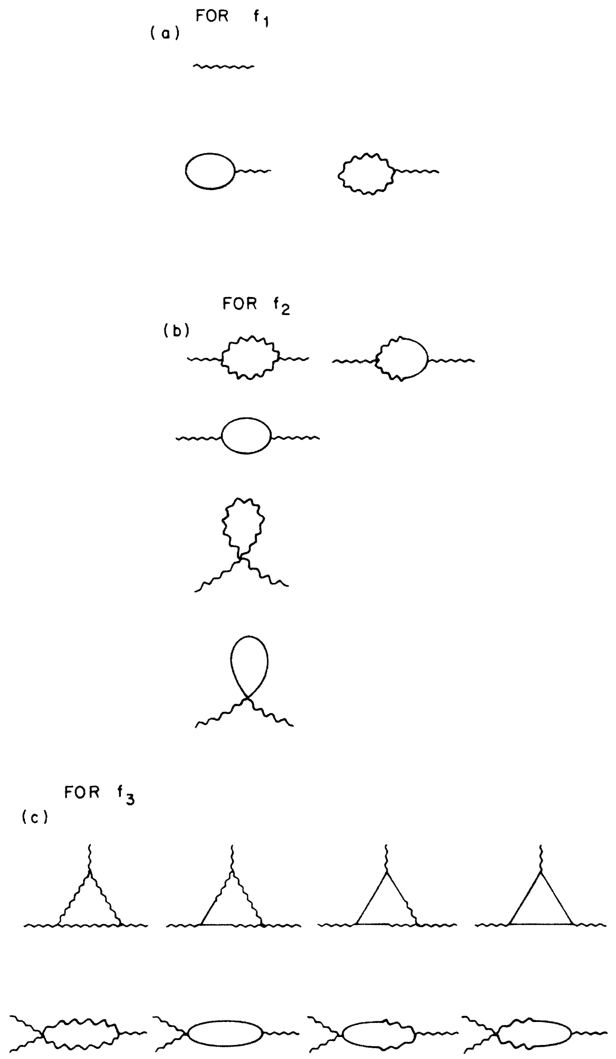

FIG. 3. One-loop contribution to vertex functions in Eqs. (4.22)—(4.24). The wavy lines indicate a factor of k_z arising from an adjacent vertex and the solid lines a factor of $k_\perp$. (a), (b), (c), respectively, show diagrams for f_1, f_2, f_3.

$B_{1c}=B_c+h_c$ and $w_{2c}=B_c$ are of order v and $w_{1c}=w_c+3B_c$ of order v^2 for small v. To leading order in v, we obtain

$$B_{1c}=\frac{vI_{201}^{c}(1-I_{021}^{c}/2)}{15I_{201}^{c}+3I_{021}^{c}-2}\ , \tag{4.26}$$

$$w_{2c}=vI_{201}^{c}/(15I_{201}^{c}+3I_{021}^{c}-2)\ , \tag{4.27}$$

where

$$I_{mnp}^{c}=\int_{\mathbf{q}} q_z^m q_\perp^n (K_1 q_\perp^4+K_2 q_z^4+K_{12}q_z^2q_\perp^2)^{-p}\ . \tag{4.28}$$

From these equations, we obtain

$$h_c=B_{2c}=-\frac{v}{2}\frac{I_{021}^{c}I_{201}^{c}}{15I_{201}^{c}+3I_{021}^{c}-2}\ . \tag{4.29}$$

Thus when v is nonzero, all potentials including the third-order potentials w_{1c} and w_{2c} are nonzero. This is in contrast to the liquid-gas transition where the unique third-order potential is zero at the critical point. As we shall now see, this leads to an upper critical dimension of 6 rather than 4.

Treating Eq. (4.23) as a self-consistent equation for f_2, we obtain

$$\begin{aligned}\overline{B}\equiv f_2=B_1-B_{1c}&+\tfrac{1}{2}\int_{\mathbf{q}} w_{zc}(\theta)q^4[G_{0c}^2(\mathbf{q})-G_0^2(\mathbf{q})]\\&+\tfrac{1}{2}\int_{\mathbf{q}}[w_{zc}^2(\theta)-w_z^2(\theta)]q^4G_0^2(\mathbf{q})\\&-\tfrac{1}{2}v_1\int_{\mathbf{q}} q_z^2[G_{0c}(\mathbf{q})-G_0(\mathbf{q})]\\&-\tfrac{1}{6}v_2\int_{\mathbf{q}} q_\perp^2\,[G_{0c}(\mathbf{q})-G_0(\mathbf{q})]\end{aligned} \tag{4.30}$$

where

$$w_{zc}(\theta)\equiv w_1^*\cos^2\theta+w_2^*\sin^2\theta\ , \tag{4.31}$$

$$G_0(\mathbf{q})=(\overline{B}q_z^2+K_1q_\perp^4+K_2q_z^4+K_{12}q_z^2q_\perp^2)^{-1}\ , \tag{4.32}$$

and

$$G_{0c}(\mathbf{q})=G_0(\mathbf{q};\overline{B}=0)\ . \tag{4.33}$$

Equation (4.30) determines $\overline{B}$ as a function of B_1-B_{1c} Above spatial dimension $d=6$, the right-hand side (rhs) of this equation is analytic in $\overline{B}$, and $\overline{B}$ goes linearly to zero with B_1-B_{1c} or with ΔT and ΔP since B_1 is an analytic function of ΔT and ΔP. At the upper critical dimension of $d_c=6$, the second term of the rhs of Eq. (4.30) becomes nonanalytic in $\overline{B}$ signaling a breakdown of mean-field theory. Because w_{1c} and w_{2c} are nonzero, it is clear that the upper critical dimension d_c is 6. If w_{1c} and w_{2c} were zero, the rhs of Eq. (4.30) would become nonanalytic in $\overline{B}$ only below four dimensions.

C. Higher-order vertex functions at the one-loop order

In this subsection, we will discuss higher-order vertex functions in one-loop perturbation theory. We begin with the third-order vertex and find that it goes to zero linearly in ΔT and ΔP for $d<8$. For $d<8$, however, it goes to zero as $\overline{B}^{(d-6)/2}$. The one-loop expansion of f_3 becomes

$$\begin{aligned}f_3=w_1-w_{1c}&+\int_{\mathbf{q}}[w_z^3(\theta)G_0^3(\mathbf{q})-w_{zc}^3(\theta)G_{0c}^3(\mathbf{q})]q^6\\&-\tfrac{3}{2}v_1\int_{\mathbf{q}}[w_z(\theta)G_0(\mathbf{q})-w_{zc}(\theta)G_{0c}(\mathbf{q})]q_z^2q^2\\&-\tfrac{1}{2}v_{12}\int_{\mathbf{q}}[w_z(\theta)G_0(\mathbf{q})-w_{zc}(\theta)G_{0c}]q_\perp^2q^2\ .\end{aligned} \tag{4.34}$$

We note that (w_1-w_{1c}) goes to zero linearly with ΔT and ΔP. After the integrals in Eq. (4.27) are performed, f_3 can be written

$$f_3=(w_1-w_{1c})+A_0\overline{B} \tag{4.35}$$

for $d>8$, where A_0 is a constant. Thus for $d>8$, f_3 goes to zero linearly in ΔT and ΔP. But, for $6<d<8$,

$$f_3=(w_1-w_{1c})+\overline{B}^{\epsilon'/2}(w_{1c}A_1+w_{2c}A_2)^3+\cdots\ , \tag{4.36}$$

where $\epsilon'\equiv d-6$ and A_1 and A_2 are nonzero constants implying f_3 goes to zero at C more slowly than linearly in ΔT and ΔP. The singular dependence of f_3 on $\overline{B}$ will, in all probability, lead to non-mean-field behavior for the shape of the coexistence curve and for the temperature dependence of the coexisting order parameter M_z^+ and M_z^-. This question is currently under investigation.

Higher-order vertices can be calculated by differentiating Eq. (4.20) with respect to M_z. The most divergent contribution to f_n is

$$f_n=\tfrac{1}{2}(-1)^{n-1}(n-1)!\int_{\mathbf{q}} w_z^n(\theta)q^{2n}G_0^n\ . \tag{4.37}$$

In the absence of external fields, B_2 is zero, and Eq. (4.32) for $G_0(\mathbf{q})$ applies. There is no constant (or mass) term in G_0, and f_n is infinite for all $\overline{B}$ (not just for $\overline{B}=0$) for all d less than $d_{nc}=2n-1$. Thus, f_4 is infinite, in one-loop perturbation theory, below 7 dimensions and f_3 below 5 dimensions. These divergences are direct consequence of the rotational invariance of H and are analogous to coexistence curve singularities in Heisenberg ferromagnet.[28] When $n=2$, they lead to the nonanalytic properties in $\overline{B}$ and K_1 discussed by Grinstein and Pelcovits.[17,18] The above considerations imply that an analytic expansion of f_n in powers of M_z breaks down at order n for all $d\le d_{nc}$ even away from C, implying that extreme caution must be exercised in calculating equation of state by matching on to perturbative solutions.

V. THE ϵ EXPANSION

A. Recursion relations

We are now in a position to calculate critical exponents for the critical point C using an ϵ expansion about the upper critical dimension of 6. As in the preceding section, we will use the Hamiltonian $H[u]$. To simplify our calculations, we rescale lengths and u so that

$$K_1=1,\ K_{12}=1,\ K_2=1-c\ . \tag{5.1}$$

The fourth-order potentials v_i are irrelevant in the renormalization-group sense near six dimensions and will not be treated in detail in this article. v_1, however, stablizes the Hamiltonian at large values of $\nabla_z u$. It is, therefore, a dangerous irrelevant variable[29] that will play an important role in determining the behavior of the order parameter along the coexistence curve, as will be discussed in a future publication. The Hamiltonian with all relevant potentials is thus

$$H=\int\left[h\nabla_z u+\tfrac{1}{2}B_1(\nabla_z u)^2+\tfrac{1}{2}B_2(\nabla_\perp u)^2+\frac{1}{3!}w_1(\nabla_z u)^3+\tfrac{1}{2}w_2\nabla_z u(\nabla_\perp u)^2\right.$$
$$\left.+\tfrac{1}{2}[(\nabla_\perp^2 u)^2+2(\nabla_z\nabla_\perp u)^2+(1-c)(\nabla_z^2 u)^2]\right]d^dx\ . \tag{5.2}$$

We will now develop momentum shell recursion relations, requiring K_1 and K_{12} to remain at unity at all stages of the calculation. In order to satisfy this constraint, it is necessary to introduce an anisotropic form[30,31] of the renormalization group such that degrees of freedom with wave vector $\mathbf{k}$ in the domain D lying between the unit sphere and the ellipsoid $b^{2(1+\mu_{||})}k_z^2+b^2k_\perp^2=1$ with ($b\equiv e^{\delta l}$) are removed. Wave vector and fields are then rescaled according to

$$\begin{aligned}&k_\perp'=bk_\perp\ ,\quad k_z'=b^{1+\mu_{||}}k_z\ ,\quad u(b^{-1-\mu_{||}}k_z\ ,\ b^{-1}k_\perp)=\zeta^2u'(\mathbf{k})\ ,\\&\zeta^2=b^{d+4+\mu_{||}-\eta_\perp}\ .\end{aligned}\tag{5.3}$$

The resulting recursion relations are

$$\frac{dh(l)}{dl}=[4-\tfrac{1}{2}(\epsilon+\mu_{||}+\eta_\perp)]h+\tfrac{1}{2}w_1I_{2,4,1}+\tfrac{1}{2}w_2I_{0,6,1}\ , \tag{5.4}$$

$$\frac{dB_1(l)}{dl}=(2-2\mu_{||}-\eta_\perp)B_1-\tfrac{1}{2}w_1^2I_{4,4,2}-\tfrac{1}{2}w_2^2I_{0,8,2}-w_1w_2I_{2,6,2}\ , \tag{5.5}$$

$$\frac{dB_2(l)}{dl}=(2-\eta_\perp)B_2-\tfrac{2}{5}w_2^2I_{2,6,2}\ , \tag{5.6}$$

$$\frac{dK_1}{dl}=-\eta_\perp K_1(l)-R(1)w_2^2\ , \tag{5.7}$$

$$\frac{dK_{12}}{dl}=(-2\mu_{||}-\eta_\perp)K_{12}-\tfrac{1}{2}[R(2)w_1^2+R(3)w_2^2+R(4)w_1w_2]\ , \tag{5.8}$$

$$\frac{d(1-c)}{dl}=(-4\mu_{||}-\eta_\perp)(1-c)-[R(5)w_1^2+R(6)w_2^2+R(7)w_1w_2]\ , \tag{5.9}$$

$$\frac{dw_1}{dl}=(\tfrac{1}{2}\epsilon-\tfrac{7}{2}\mu_{||}-\tfrac{3}{2}\eta_\perp)w_1+w_1^3R(8)+w_1^2w_2R(9)+w_1w_2^2R(10)+w_2^3R(11)\ , \tag{5.10}$$

$$\frac{dw_2}{dl}=(\tfrac{1}{2}\epsilon-\tfrac{3}{2}\mu_{||}-\tfrac{3}{2}\eta_\perp)w_2+R(12)w_1w_2^2+w_2{}^3R(13)\ , \tag{5.11}$$

where the coefficients $R(p)$ are listed in Appendix C as functions of the integral

$$I_{m,n,p}=\frac{1}{\delta l}\int_{q\in D}\frac{d^dq}{(2\pi)^d}\frac{q_z^mq_\perp^{n-4}}{(B_1q_z^2+B_2q_\perp^2+q^4-cq_z^4)^p}\ . \tag{5.12}$$

The constraints that K_1 and K_{12} remain unity yield

$$\eta_\perp=-R(1)w_2^2\ , \tag{5.13}$$

$$\mu_{||}=\tfrac{1}{4}\{-R(2)w_1^2+[2R(1)-R(3)]w_2^2-R(4)w_1w_2\}\ . \tag{5.14}$$

Then the remaining recursion relations for B_1, c, w_1, w_2 can be written as

$$\frac{dB_1}{dl}=2B_1+\tfrac{1}{2}B_1[R(2)w_1^2+R(3)w_2^2+R(4)w_1w_2]-\tfrac{1}{2}w_1^2I_{4,4,2}-\tfrac{1}{2}w_2^2I_{0,8,2}-w_1w_2I_{2,6,2}\ , \tag{5.15}$$

$$\frac{dB_2}{dl}=2B_2+B_2R(1)w_2^2-\tfrac{2}{5}w_2^2I_{2,6,2}\ , \tag{5.16}$$

$$\frac{dc}{dl}=[R(5)-(1-c)R(2)]w_1^2+[R(7)-(1-c)R(4)]w_1w_2+\{R(6)+(1-c)[R(1)-R(3)]\}w_2^2\ , \tag{5.17}$$

$$\frac{dw_1}{dl}=\tfrac{1}{2}\epsilon w_1+w_1^3[\tfrac{7}{8}R(2)+R(8)]+w_1^2w_2[\tfrac{7}{8}R(4)+R(9)]+w_1w_2^2[\tfrac{7}{8}R(3)-\tfrac{1}{4}R(1)+R(10)]+w_2^3R(11)\ , \tag{5.18}$$

$$\frac{dw_2}{dl}=\tfrac{1}{2}\epsilon w_2+\tfrac{3}{8}w_1^2w_2R(2)+w_1w_2^2[\tfrac{3}{8}R(4)+R(12)]+w_2^3[\tfrac{3}{8}R(3)+\tfrac{3}{4}R(1)+R(13)]\ . \tag{5.19}$$

In Eqs. (5.15) and (5.16), we retained only terms up to linear order in B_1 and B_2.

Before analyzing the nontrivial fixed point in Eqs. (5.16) to (5.20), we should recall how many relevant operators the fixed point must have in order to describe the critical point C. Since three parameters, q_0', T, and P are varied to reach the critical point, there should be three relevant potentials. In the case of the liquid-gas transition, there are also three relevant potentials: the linear and quadratic potentials (the analogue of h and B_1) and a third-order potential w. The third-order potential is redundant operator[32] with an exponent identical to that $[\omega=\frac{1}{2}(d-2+\eta)]$ for the order parameter. In the present case, we must remember that the Ward identity Eq. (4.14), $f^{(2)}_{\perp\perp}=f^{(1)}_{z}=0$ imposes a constraint on a relevant potential. We, therefore, expect that the critical point describing C to have four rather than three relevant operators: h, two linear combinations of B_1 and B_2, and one operator coming from the third-order potentials w_1 and w_2.

There is clearly a fixed point with $w_1^*=w_2^*=0$. This has two relevant third-order potentials (i.e., two positive eigenvalues of order ϵ) and is thus not a candidate for describing C. We have located a fixed point with $w_2^*=0$ and $w_1^*\neq 0$; this fixed point also has two positive eigenvalues. There is no fixed point with $w_1=0$ and $w_2\neq 0$ as can be seen from Eq. (5.19). We, therefore, seek a fixed point with $w_1^*\sim w_2^*\sim\epsilon^{1/2}$, $c^*\sim 1$, and $B_1^*\sim\epsilon$ implying $\eta_\perp\sim\epsilon$ and $\mu_{||}\sim\epsilon$. Thus, in the search for fixed points B_1, B_2, and $\mu_{||}$ can be set equal to zero in the integrals $I_{m,n,p}$. And we have

$$I_{m,m,p}=\frac{K_5}{2\pi}\int_0^{\pi}\frac{\cos^m\theta\sin^n\theta}{(1-c\cos^4\theta)^p}d\theta+O(\epsilon)\ , \tag{5.20}$$

where $K_5=\Gamma(5/2)(2\pi)^{-5/2}$.

Equations (5.17) to (5.19) determine the three fixed point potentials w_1^*, w_2^*, and c^*. Equation (5.17) is a homogeneous function of w_1 and w_2 and can be rewritten

$$\begin{aligned}\frac{dc}{dl}=w_2^{*2}\{&[R(5)-(1-c)R(2)](w_1/w_2)^2\\&+[R(7)-(1-c)R(4)](w_1/w_2)\\&+R(6)+(1-c)[R(1)-R(3)]\}\ .\end{aligned} \tag{5.21}$$

At the fixed point $dc/dl=0$, and Eq. (5.21) can be solved analytically to yield (w_1/w_2) as a function of c. From Eq. (5.18) and (5.19), we can obtain a linear combination $w_2(dw_1/dl)-w_1(dw_2/dl)$:

$$\begin{aligned}w_2\frac{dw_1}{dl}-w_1\frac{dw_2}{dl}=w_2^2\{&[-\tfrac{1}{8}R(2)+R(8)+R(11)](w_1/w_2)^3+[\tfrac{1}{2}R(4)+R(9)-R(12)](w_1/w_2)^2\\&+[\tfrac{1}{2}R(3)-R(1)+R(10)-R(13)](w_1/w_2)\}\ .\end{aligned} \tag{5.22}$$

The right hand side of this equation is zero at the fixed point. With (w_1/w_2) expressed as the function of c obtained from Eq. (5.21), c^* can be determined numerically:

$$c^*=0.9628,\quad w_2^*/w_1^*=-0.7680\ . \tag{5.23}$$

These results can now be used in Eq. (5.18) or (5.19) to yield

$$\begin{aligned}&w_1^*=-1.4283(K_5/2\pi)^{1/2}\epsilon^{1/2}\ ,\\&w_2^*=1.0970(K_5/2\pi)^{1/2}\epsilon^{1/2}\ ,\end{aligned} \tag{5.24}$$

and

$$\eta_\perp=0.0364\epsilon\ ,\ \mu_{||}=0.2679\epsilon. \tag{5.25}$$

From Eqs. (5.15) and (5.16), we obtain

$$B_1^*=0.2569\epsilon\ ,\ B_2^*=0.0439\epsilon\ . \tag{5.26}$$

B. Fixed-point stability and critical exponents

The recursion relations of Eqs. (5.15) to (5.19) can be linearized about the fixed point of Eqs. (5.23)–(5.26) to yield the matrix stability equation which determines the critical exponent $\nu_\perp$ and crossover exponents . With the change of variables, $y_1=B_1-B_1^*$, $y_2=B_2-B_2^*$, $y_1=w_1-w_2^*$, $y_2=w_2-w_2^*$, and $y_3=c-c^*$, the stability equation can be expressed in the matrix form

$$\frac{dy_\alpha}{dl}=A_{\alpha\beta}y_\alpha\ , \tag{5.27}$$

where summation over repeated indices is understood. The eigenvalues of $M_{\alpha\beta}$ are the stability exponents $\lambda_p(p=1,2,\ldots,5)$. They are most easily calculated with the change of variables

$$\begin{aligned}&x_1=\epsilon^{-1}y_1\ ,\ x_2=\epsilon^{-1}y_2,\ x_3=\epsilon^{-1/2}y_3\ ,\\&x_4=\epsilon^{-1/2}y_4,\ x_5=y_5.\end{aligned} \tag{5.28}$$

In terms of these variables, we have

$$\frac{dx_\alpha}{dl}=2\delta_{\alpha 1}x_\alpha+2\delta_{\alpha 2}x_\alpha+a_{\alpha\beta}x_\beta\ . \tag{5.29}$$

The components of the matrix $a_{\alpha\beta}$ can be calculated using

$$\partial I_{m,n,p}/\partial B_1=-pI_{m+2,n,p+1}\ , \tag{5.30}$$

$$\partial I_{m,n,p}/\partial B_2=-pI_{m,n+2,p+1}\ , \tag{5.31}$$

$$\partial I_{m,n,p}/\partial c=pI_{m+4,n,p+1}\ , \tag{5.32}$$

and are listed in Appendix D.

It is clear from Eq. (5.28) that the exponent $\lambda_1=\nu_\perp^{-1}$ and λ_2 associated with x_1 and x_2 are 2 plus corrections of order ϵ. The other three exponents $\lambda_\alpha\equiv\epsilon\kappa_\alpha$, $\alpha=3,4,5$, determined by Eq. (5.28) are of order ϵ. This allows us to decouple x_1 and x_2 from x_3,x_4, and x_5 to obtain

$$\lambda_1=2+0.7490\epsilon\equiv\nu_\perp^{-1}\ ,\quad \nu_\perp=0.5-0.1873\epsilon\ , \tag{5.33}$$

$$\lambda_2=2-0.0769\epsilon\ . \tag{5.34}$$

We note that from the structure of recursion relations,

$$\lambda_2=\tfrac{1}{2}(d-2+\eta_\perp+3\mu_{||})\equiv\omega\ , \tag{5.35}$$

where ω is the order-parameter exponent governing the flow of $M_z(l)=e^{\omega l}M_z(0)$. Thus, the field associated with λ_2, like the third-order potential in the liquid-gas problem, in a redundant operator.[32] The detailed calculations leading to Eq. (5.35) are shown in Appendix E. The other three exponents are then the eigenvalues of the matrix $\epsilon a_{\alpha\beta}$ which were calculated numerically:

$$\lambda_3=3.6776\epsilon\ ,\ \lambda_4=-1.7400\epsilon\ ,\ \lambda_5=-1.0000\epsilon\ . \tag{5.36}$$

We close this section with some observations about the calculation of the exponent λ_{v_1} associated with the dangerous irrelevant variable v_1. λ_{v_1} is -2 at $d=6$. To calculate order ϵ corrections to v_1, it is necessary[33] to treat simultaneously all fields with exponents equal to -2 at $d=6$. These include not only fourth-order potentials v_1, v_{12}, and v_2, but also the potentials associated with third-order terms with six gradients of the form $(\nabla_z^2u)(\nabla_z u)^2$, etc. There are at least eight such potentials making the calculation of order ϵ calculation to v_1 a formidable task.

VI. SCALING NEAR THE CRITICAL POINT

The analysis of the previous section leads to scaling forms for vertex functions near the critical point. There are three relevant fields $\delta h=h-h_c$, t, and g associated with three positive exponents λ_h and ν^{-1} and λ_3. t is essentially B_1, but includes order ϵ contributions from B_2. g is a linear contribution of w_1, w_2, and c. As discussed in Sec. III, the critical point can be reached by controlling two parameters, say temperature T, and pressure (concentration) P. All relevant variables must be zero at C, and we expand δh, t, g to linear order in ΔT and ΔP:

$$\delta h=a_h\Delta T+b_h\Delta P\ , \tag{6.1}$$

$$t=a_t\Delta T+b_t\Delta P\ , \tag{6.2}$$

$$g=a_g\Delta T+b_g\Delta P\ . \tag{6.3}$$

All other potentials have finite values at C. The singular parts of nth-order vertex functions satisfies

$$f_{n,\mathrm{sing}}(t,g,v_1,M_z)=e^{-(d+\mu_{||})l^*+n\omega l^*}f_{n,\mathrm{sing}}(e^{\nu_\perp^{-1}l^*}t,e^{\lambda_3 l^*}g,e^{-\lambda_{v_1}l^*}v_1,e^{\omega l^*}M_z) \tag{6.4}$$

where ω is the order parameter exponent given in Eq. (5.35). In addition, the wave-number-dependent two-point vertex satisfies

$$f_2(\mathbf{q})=e^{-(2-\eta_\perp-2\mu_{||})l^*}f_2(e^{\nu^{-1}l^*}t,e^{\lambda_3 l^*}g,e^{\omega l^*}M_z,e^{l^*}q_\perp,e^{(1+\mu_{||})l^*}q_z) \tag{6.5}$$

where we have dropped dependence on v_1. With $e^{\nu_\perp^{-1}l^*}=|t|$, these equations imply

$$f_{n,\mathrm{sing}}=|t|^{(d+\mu_{||}-n\omega)\nu_\perp}X_n(g|t|^{-\phi_3},|t|^{|\phi_{v_1}|}v_1,M_z|t|^{-\beta})\ , \tag{6.6}$$

$$f_2(\mathbf{q})=|t|^{\gamma}X_2(g|t|^{-\phi_3},M_z|t|^{-\beta},q_{||}\xi_{||},q_\perp\xi_\perp)\ , \tag{6.7}$$

where X_n is a scaling function and where

$$\beta=\omega\nu_\perp=1-0.4145\epsilon\ , \tag{6.8}$$

$$\phi_3=\lambda_3\nu_\perp=1.8388\epsilon\ , \tag{6.9}$$

$$\gamma=(2-\eta_\perp-2\mu_{||})\nu_\perp=1-0.6617\epsilon\ . \tag{6.10}$$

The correlation lengths parallel and perpendicular to the layer normals are

$$\xi_{||}\sim|t|^{-\nu_{||}}\ ,\ \ \xi_\perp\sim|t|^{-\nu_\perp} \tag{6.11}$$

with

$$\nu_{||}=(1+\mu_{||})\nu_\perp=0.5-0.0534\epsilon\ . \tag{6.12}$$

As C is approached, $g|t|^{-\phi_3}$ goes to zero, and will only contribute corrections to dominant singularities. We will therefore, drop g in our discussion to follow.

M_z is determined by the equation of state

$$f_1=f_{1,\mathrm{reg}}+f_{1,\mathrm{sing}}=0\ . \tag{6.13}$$

δh is $(-f_{1,\mathrm{reg}})$ so that

$$\delta h=|t|^{\Delta}X_1(v_1|t|^{|\phi_{v_1}|},M_z|t|^{-\beta})\ , \tag{6.14}$$

where

$$\Delta\equiv\lambda_h\nu_\perp=2-1.0751\epsilon \tag{6.15}$$

with $\lambda_h=(d+\mu_{||}-\omega)=4-0.6522\epsilon$. This equation can be solved for M_z to yield

$$M_z=|t|^{\beta}Y_x(\delta h|t|^{-\Delta},v_1|t|^{|\phi_{v_1}|})\ , \tag{6.16}$$

where Y_z is an as yet undetermined scaling function. Finally, the displacement correlation function satisfies

$$G_{uu}(\mathbf{q})=e^{(4-\eta_\perp)l^*}\times X_{uu}(e^{\nu^{-1}l^*}t,e^{\lambda_3 l^*}g,e^{\omega l^*}M_z,e^{l^*}q_\perp,e^{(1+\mu_{||})l^*}q_z)\ . \tag{6.17}$$

We can now discuss routes to C not crossing the coexistence curve. The line $M_z=0$ is the analogue of the critical isochore of the liquid-gas transition. Along this line,

$$f_2(\mathbf{q})=|t|^{\gamma}f_2(q_{||}\xi_{||},q_\perp\xi_\perp)\ , \tag{6.18}$$

and the physical compressional constant $\overline{B}$ goes to zero as $|t|^{\gamma}$. In addition, the specific heat C_m for the constant layer spacing[34]

$$C_m \sim \frac{d^2 f_0}{dT^2} \sim |t|^{-\alpha} \tag{6.19}$$

diverges with exponent

$$\alpha = 2-(d+\mu_{||})\nu_{\perp} = -1+1.148\,98\epsilon \tag{6.20}$$

obeying anisotropic hyperscaling. The third-order vertex,

$$f_3 \sim |t|^{\gamma-\beta} \tag{6.21}$$

diverges as $|t| \rightarrow 0$ since $\gamma-\beta=-0.2462\epsilon<0$. This is in contrast to the liquid-gas case where f_3 goes to zero (since $\gamma-\beta>0$).

The line $M_z=0$, unlike the critical isochore of the liquid-gas transition, is not a natural or easy path to follow experimentally. A much easier approach to C is along the critical concentration (or pressure) line ($\Delta P=0$). In this case

$$\delta h\,|t|^{-\Delta} \sim |\Delta T|^{1-\Delta} \rightarrow \infty\ , \tag{6.22}$$

and

$$\overline{B} \sim |\Delta T|^{\gamma/\Delta}\ , \tag{6.23}$$

$$f_3 \sim |\Delta T|^{(\gamma-\beta)/\Delta}\ , \tag{6.24}$$

$$C_P \sim |\Delta T|^{-\gamma/\Delta}\ , \tag{6.25}$$

$$\xi_{||} \sim |\Delta T|^{-\nu_{||}/\Delta}\ ,\quad \xi_{\perp} \sim |\Delta T|^{-\nu_{\perp}/\Delta}\ . \tag{6.26}$$

Thus, the singularities are less violent along this line than they are along $M_z=0$. Note, however, that f_3 is still divergent.

At the critical point, δh, g, and t are zero. Then from Eqs. (6.5) and (6.17), the two-point vertex function $\gamma_2(\mathbf{q})$ and $u-u$ correlation function $G_{uu}(\mathbf{q})$ satisfy the homogeneity relations

$$f_2(\mathbf{q}) = b^{-(2-\eta_{\perp}-2\mu_{||})} f_2(bq_{\perp}, b^{1+\mu_{||}} q_{||})\ , \tag{6.27}$$

$$G_{uu}(\mathbf{q}) = b^{4-\eta_{\perp}} X_{uu}(bq_{\perp}, b^{1+\mu_{||}} q_{||})\ . \tag{6.28}$$

With $b=q_{\perp}^{-1}$, these equations imply

$$f_2(\mathbf{q}) = q_{\perp}^{(2-2\eta_{\perp}-2\mu_{||})} \overline{f}_2(q_{||}/q_{\perp}^{1+\mu_{||}})\ , \tag{6.29}$$

$$\begin{aligned} G_{uu}(\mathbf{q}) &= b^{4-\eta_{\perp}} X_{uu}[(b^{1+\mu_{||}} q_z)^2] \\ &= q_{\perp}^{-(4-\eta_{\perp})} \overline{X}_{uu}(q_z^2/q_{\perp}^{2(1+\mu_{||})}) \\ &= q_z^{-(4-\eta_{||})} X'_{uu}(q_{\perp}^{2(1+\mu_{||})}/q_z^2) \end{aligned} \tag{6.30}$$

where $\overline{f_2}$, $\overline{X}_{uu}$, and X'_{uu} are the scaling functions and

$$\eta_{||} = (\eta_{\perp}+4\mu_{||})/(1+\mu_{||}) = 1.1160\epsilon\ . \tag{6.31}$$

Note that with this form of $\eta_{||}$, $\gamma=(2-\eta_{||})\nu_{||}$. The function $\overline{X}_{uu}(\mathbf{x})$ and

$$X'_{uu}(x) = x^{-(4-\eta_{\perp})/2(1+\mu_{||})} \overline{X}_{uu}(1/x)$$

tend to constants as $x \rightarrow 0$. The large $|\mathbf{x}|$ behavior of the function $g^{(2)}(\mathbf{x})$ controlling the smectic-order-parameter correlation function can be calculated, using Eq. (6.30) and methods discussed in Ref. 35, we find

$$g^{(2)}(x_{\perp}\neq 0, z=0) \sim |\mathbf{x}|^{1-\eta_{\perp}-\mu_{||}}\ , \tag{6.32}$$

$$g^{(2)}(x_{\perp}=0, z\neq 0) \sim z^{3-\eta_{||}-2/(2+\mu_{||})} \tag{6.33}$$

as $|\mathbf{x}| \rightarrow \infty$. We note that the strongly nonanalytic form of $g^{(2)}(\mathbf{x})$ makes it difficult to perform the Fourier transform of $G(\mathbf{x})$. Presumably, it will be highly anisotropic and approximately Lorentzian-squared in shape.

VII. CONCLUSIONS

In this paper we developed a nonlinear elastic model to describe the critical point C terminating a line of coexistence of two smectic-A phases with different layer spacing. This model is universal in that it applies regardless of the microscopic origin of the SmA-SmA' phases. It should apply, for example, to SmA-SmA' consolute points that are likely to be found in polymeric binary mixtures. Mean-field theory for this model is identical to mean-field theory for the liquid-gas transition. Critical fluctuations, however, lead to a breakdown of mean-field theory for the compressibility below an upper critical dimension of 6 rather than 4. In addition, deviations from liquid-gas mean-field behavior are expected below 8 dimensions because of one-loop nonanalyticities in the third-order vertex. We derived renormalization-group recursion relations and located a fixed point with the correct number of relevant potentials to describe C. To first-order in ϵ, exponents for this fixed point predict anisotropic scaling with different correlation length exponents $\nu_{||}$ and $\nu_{\perp}$ for directions parallel and perpendicular to the director. In addition, the third-order vertex is predicted to diverge at C.

We believe that the fixed point we have located in $6-\epsilon$ dimensions evolves continuously as dimension is lowered to one which describes the experimentally observed critical point in three dimensions. If so our analysis leads to a number of predictions that should be valid in three dimensions. The most salient prediction is that the compressional elastic constant $\overline{B}$ vanishes at the critical point in exact analogy with the divergence of the bulk compressibility at the liquid-gas critical point. Indeed preliminary experiments[36] do reveal a sharp drop in $\overline{B}$ in the vicinity of the critical point, but no exponents have yet been extracted. The specific heat C_P at constant pressure and $\overline{B}$ along the critical pressure line yield the exponent γ/Δ. Along the critical isochore, which can be determined independently by measurements of $q_0(P,T)$, C_P, and $\overline{B}$ yield the exponent γ. In general the product $C_P\overline{B}$ should be independent of temperature as the critical point is approached. The constant layer spacing specific heat C_m has a singularity determined by the exponent α. An experiment measuring C_m would be of some interest. The exponent $\eta_{\perp}\nu_{\perp}$ can be determined, in principle, from the divergence of K_1 ($\sim t^{-\eta_{\perp}\nu_{\perp}}$ on the critical isochore and $\sim t^{-\eta_{\perp}\nu_{\perp}/\Delta}$ along the critical concentration curve).[37] It should be noted that the divergence of K_1 is a characteristic feature of the SmA-SmA' critical point not found, for example, at the nematic to smectic-A (N-SmA) critical point. Unlike the case of the N-SmA transition, x-rays do not couple directly to the order parameter so that the x-ray line shape does not provide a direct measure of the correlation length exponents $\nu_{\|}$ and $\nu_{\perp}$. Nevertheless, x-ray experiments in the vicinity of C should be interesting in that they might show the crossover in intensity from power law in wave vector to nonanalytic size dependence as η_c passes through one.[26] Since the amplitude of the smectic modulation is finite at C, there may be more hope of seeing this crossover than in the vicinity of the N-SmA transition where the amplitude approaches zero.

The coexistence curve and Δq_0 are of some interest and have been measured experimentally.[8] Unfortunately we have not been able to calculate either the shape of the coexistence curve or the order parameters along it. The calculation of these quantities is both subtle and complex. All nth-order vertices with $n \geq 4$ are infinite in one-loop order perturbation theory throughout smectic phases when $d<7$. Such infinities are analogous to coexistence curve singularities in Heisenberg ferromagnet[28] and must be treated with great caution in calculating the equation of state. The dangerous irrelevant potential v_1 will play an important role in determining the shape of the coexistence curve. It is one of at least eight potentials with exponents of order $-2+O(\epsilon)$ and calculation of its dominant exponent will be difficult. Nevertheless, we hope to have some predictions about the shape of the coexistence curve and the order parameters along it in the near future.

ACKNOWLEDGMENTS

This work is supported in part by the National Science Foundation under Grant No. DMR-8540332 and under Grant No. DMR-8216718 at the Laboratory for Research in the Structure of Matter at the University of Pennsylvania. J.P. acknowledges stimulating discussions on the subtleties of the specific heat measurements with W. Van Dael and J. Thoen. Y.P. and T.C.L. acknowledge a useful discussion with Michael Fisher.

APPENDIX A: WARD IDENTITIES

In this appendix, we derive Ward identities due to the invariance of H_{sm} under the rotation $\mathbf{x}'=R_{ij}^{-1}\mathbf{x}$ where R_{ij} is a rotation matrix. The interesting rotations are those about an axis perpendicular to $\mathbf{n}_0$ (the z axis), that rotate $\mathbf{n}_0$ in the $(z-\perp)$ plane where $\perp$ is a direction perpendicular to the z axis. In this case, the component of R_{ij} in the $(z-\perp)$ plane for a rotation through angle θ are

$$R_{ij}=\begin{bmatrix}\cos\theta & -\sin\theta \\ \sin\theta & \cos\theta\end{bmatrix} . \tag{A1}$$

This uniform rotation transforms the phase $q_0[u(\mathbf{x})+\mathbf{n}_0\cdot\mathbf{x}]$ to $q_0[u(\mathbf{x}')+\mathbf{n}_0\cdot\mathbf{x}']$. This transformation can alternatively be interpreted as a change of the displacement variable $u(\mathbf{x}')$ to $u'(\mathbf{x})$ such that $q_0[u'(\mathbf{x})+\mathbf{n}_0\cdot\mathbf{x}]=q_0[u(\mathbf{x}')+\mathbf{n}_0\cdot\mathbf{x}']$ or

$$u'(\mathbf{x})=u(\mathbf{x}')+\mathbf{x}\cdot(R\mathbf{n}_0-\mathbf{n}_0) . \tag{A2}$$

The elastic energy is invariant under Eq. (A2) since

$$\begin{aligned}E(u'(\mathbf{x}))&\equiv\nabla_z u'(\mathbf{x})+\tfrac{1}{2}[\nabla_{\perp} u'(\mathbf{x})]^2\\ &=\nabla_{z'} u(\mathbf{x}')+\tfrac{1}{2}[\nabla_{\perp'} u(\mathbf{x}')]^2\\ &\equiv E(u(\mathbf{x}'))\end{aligned} \tag{A3a}$$

and

$$\int d^dx' E(u(\mathbf{x}'))=\int d^dx E(u'(\mathbf{x})) . \tag{A3b}$$

Thus

$$\begin{aligned}H_{\mathrm{sm}}[u(\mathbf{x})]&=H_{\mathrm{sm}}[u'(\mathbf{x})]\\ &=H_{\mathrm{sm}}[u(R^{-1}\mathbf{x})+\mathbf{x}\cdot(R\mathbf{n}_0-\mathbf{n}_0)] .\end{aligned} \tag{A4}$$

Then the partition function $Z(h_z,h_{\perp})$ in the presence of a constant external field $\mathbf{h}_e=(h_z,h_{\perp})$ satisfies

$$Z(h_z,h_{\perp})=\int Du(\mathbf{x})\exp\{H_{\mathrm{sm}}[u(\mathbf{x})]\}\exp\left[\int_{\mathbf{x}}[h_z\nabla_z u(\mathbf{x})+h_{\perp}\nabla_{\perp}u(\mathbf{x})]\right] \tag{A5a}$$

$$=\int Du'(\mathbf{x})\exp\{H_{\mathrm{sm}}[u'(\mathbf{x})]\}\exp\left[\int_{\mathbf{x}}[h_z\nabla_z u'(\mathbf{x})+h_{\perp}\nabla_{\perp}u'(\mathbf{x})]\right] \tag{A5b}$$

$$\begin{aligned}&=\int Du(\mathbf{x}')\exp\{H_{\mathrm{sm}}[u(\mathbf{x}')]\}\exp\left[\int_{\mathbf{x}}{}' h_z[\nabla_z u(\mathbf{x}')\cos\theta+\sin\theta\nabla_{\perp}u(\mathbf{x}')]\right]\\ &\quad\times\exp\left[\int_{\mathbf{x}}h_{\perp}[\nabla_{\perp}u(\mathbf{x}')\cos\theta mi\nabla_z u(\mathbf{x}')\sin\theta]\right]\exp\{V[(\cos\theta-1)h_z-h_{\perp}\sin\theta]\}\end{aligned} \tag{A5c}$$

$$=Z[(\cos\theta h_z-\sin\theta h_{\perp}),(\cos\theta h_{\perp}+\sin\theta h_z)]\exp\{V[(\cos\theta-1)h_z-h_{\perp}\sin\theta]\} \tag{A5d}$$

where V is the volume of the system. By defining average order parameters $M_i\equiv\langle\nabla_{i'}u(\mathbf{x}')\rangle$ and $M_i'\equiv\langle\nabla_i u'(\mathbf{x})\rangle$, the Legendre transformed potential can be written as

$$\Gamma[M_z(\mathbf{x}),M_{\perp}(\mathbf{x})]=-\ln Z[h_z,h_{\perp}]+\int d^dx(h_zM_z+h_{\perp}M_{\perp}) \tag{A6a}$$

$$\begin{aligned}&=-\ln Z[(h_z\cos\theta-h_{\perp}\sin\theta),(h_{\perp}\cos\theta+h_z\sin\theta)]+\int d^dx(h_z\cos\theta-h_{\perp}\sin\theta)M_z\\ &\quad+\int d^dx(h_{\perp}\cos\theta+h_z\sin\theta)M_{\perp} .\end{aligned} \tag{A6b}$$

Then using Eq. (A5d), we have

$$\Gamma[M_z,M_{\perp}]=\Gamma[(\cos\theta M_z+\sin\theta M_{\perp}+\cos\theta-1),(\cos\theta M_{\perp}-\sin\theta M_z-\sin\theta)] . \tag{A7}$$

Since the right-hand side of this equation is independent of θ, $\partial\Gamma/\partial\theta=0$, and when $\theta=0$,

$$M_\perp \frac{\partial\Gamma}{\partial M_z}-(M_z+1)\frac{\partial\Gamma}{\partial M_\perp}=0 \tag{A8}$$

which can be expressed in terms of $f_i^{(1)}$ as

$$M_\perp f_z^{(1)}=(M_z+1)f_\perp^{(1)} . \tag{A9}$$

Differentiating Eq. (A9) with respect to $M_\perp$ and using the equation of state, $f_z^{(1)}=0$, we obtain

$$f_{\perp\perp}^{(2)}=0 \tag{A10}$$

in equilibrium in the absence of the external field.

APPENDIX B: ONE-LOOP CONTRIBUTION TO $\Gamma^{(2)}(\mathbf{k})$

One can express $\Gamma^{(2)}(\mathbf{k})$ as

$$\Gamma^{(2)}=B_1k_z^2+B_2k_\perp^2+K_1k_\perp^4+K_2k_z^4+2K_{12}k_z^2k_\perp^2-\Sigma^{(2)}(\mathbf{k}) , \tag{B1}$$

where $\Sigma^{(2)}(\mathbf{k})$ is given as

$$\begin{aligned}\Sigma^{(2)}=&\tfrac{1}{2}w_1^2k_z^2\int_{\mathbf{q}}q_z^2(k_z+q_z)^2G(\mathbf{q})G(\mathbf{k}+\mathbf{q})+\tfrac{1}{2}w_2^2k_z^2\int_{\mathbf{q}}[q_\perp\cdot(k_\perp+q_\perp)]^2G(\mathbf{q})G(\mathbf{q}+\mathbf{k})\\&+w_2^2k_{\perp i}k_{\perp j}\int_{\mathbf{q}}(k_z+q_z)^2q_{\perp i}q_{\perp j}G(\mathbf{q})G(\mathbf{q}+\mathbf{k})+w_2^2k_{\perp i}k_{\perp j}\int_{\mathbf{q}}(k_z+q_z)(k_{\perp i}+q_{\perp i})q_zq_{\perp j}G(\mathbf{q})G(\mathbf{q}+\mathbf{k})\\&+2w_2^2k_zk_{\perp i}\int_{\mathbf{q}}q_\perp\cdot(k_\perp+q_\perp)q_z(k_{\perp i}+q_{\perp i})G(\mathbf{q})G(\mathbf{q}+\mathbf{k})+w_1w_2k_z^2\int_{\mathbf{q}}(k_z+q_z)q_zq_\perp\cdot(k_\perp+q_\perp)G(\mathbf{q})G(\mathbf{q}+\mathbf{k})\\&+2w_1w_2k_zk_{\perp i}\int_{\mathbf{q}}(k_z+q_z)^2q_zq_{\perp i}G(\mathbf{q})G(\mathbf{q}+\mathbf{k})mi\tfrac{1}{2}v_1k_z^2\int_{\mathbf{q}}q_z^2G(\mathbf{q})mi\tfrac{1}{6}v_{12}k_z^2\int_{\mathbf{q}}q_\perp^2G(\mathbf{q})\\&-\tfrac{1}{6}v_{12}k_\perp^2\int_{\mathbf{q}}q_z^2G(\mathbf{q})-\tfrac{1}{6}v_3\left[\frac{2}{d-1}+1\right]k_\perp^2\int_{\mathbf{q}}q_\perp^2G(\mathbf{q}) ,\end{aligned} \tag{B2}$$

which is calculated by the Feynman diagrams shown in Fig. 3.

APPENDIX C: COEFFICIENTS $R(p)$ APPEARING IN EQS. (5.7)–(5.11)

$$R(1)=-\tfrac{8}{5}I_{2,6,3}-\tfrac{24}{35}I_{2,8,3}+\tfrac{96}{35}I_{2,8,4} , \tag{C1}$$

$$R(2)=-I_{4,4,3}+\tfrac{8}{5}I_{4,6,4}-\tfrac{2}{5}I_{4,6,3} , \tag{C2}$$

$$\begin{aligned}R(3)=&\tfrac{1}{5}[\tfrac{3}{2}I_{0,6,2}-9I_{0,8,3}-2I_{0,10,3}+8I_{0,10,4}-64I_{2,6,3}+(72c-8)I_{4,6,3}+32I_{4,6,4}+32c^2I_{8,6,4}-64cI_{6,6,4}\\&-16I_{2,8,3}+64I_{2,8,4}-64cI_{4,8,4}] ,\end{aligned} \tag{C3}$$

$$R(4)=\tfrac{1}{5}(-30I_{2,6,3}-4I_{2,8,3}+16I_{2,8,4}-16I_{4,6,3}+64I_{4,6,4}-64cI_{6,6,4}) , \tag{C4}$$

$$R(5)=\tfrac{1}{2}I_{2,4,2}-5I_{4,4,3}-(2-7c)I_{6,4,3}+8I_{6,4,4}-16cI_{8,4,4}+8c^2I_{10,4,4} , \tag{C5}$$

$$R(6)=-I_{0,8,3}-(2-3c)I_{2,8,3}+8I_{2,8,4}-16cI_{4,8,4}+8c^2I_{6,8,4} , \tag{C6}$$

$$R(7)=-6I_{2,6,3}-(4-10c)I_{4,6,3}+16I_{4,6,4}+16c^2I_{8,6,4}-32cI_{6,6,4} , \tag{C7}$$

$$R(8)=I_{6,4,3} , \tag{C8}$$

$$R(9)=3I_{4,6,3} , \tag{C9}$$

$$R(10)=3I_{2,8,3} , \tag{C10}$$

$$R(11)=I_{0,10,3} , \tag{C11}$$

$$R(12)=\tfrac{4}{5}I_{4,6,3} , \tag{C12}$$

$$R(13)=\tfrac{4}{5}I_{2,8,3} . \tag{C13}$$

APPENDIX D: MATRIX ELEMENTS $a_{\alpha\beta}$ IN EQ. (5.29)

$$a_{11}=\{w_1^{*2}[\tfrac{1}{2}R^*(2)+I_{6,4,3}^*]+w_2^{*2}[\tfrac{1}{2}R^*(3)+I_{2,8,3}^*]+w_1^*w_2^*[\tfrac{1}{2}R(4)+2I_{4,6,3}]\}\epsilon^{-1}=0.5810 , \tag{D1}$$

$$a_{12}=(w_1^{*2}I^*_{4,6,3}+2w_1^*w_2^*I^*_{2,8,3}+w_2^{*2}I^*_{0,10,3})\epsilon^{-1}=0.8565\ , \tag{D2}$$

$$a_{21}=(\tfrac{4}{5}w_2^{*2}I^*_{4,6,3})\epsilon^{-1}=0.1290\ , \tag{D3}$$

$$a_{22}=(\tfrac{4}{5}w_2^{*2}I^*_{2,8,3})\epsilon^{-1}=0.0911\ , \tag{D4}$$

$$a_{33}=\{\tfrac{1}{2}\epsilon+3w_1^{*2}[R^*(8)+\tfrac{7}{8}R^*(2)]+w_2^{*2}[-\tfrac{1}{4}R^*(1)+R^*(10)+\tfrac{7}{8}R^*(3)]+2w_1^*w_2^*[R^*(9)+\tfrac{7}{8}R^*(4)]\}\epsilon^{-1}=2.5073\ , \tag{D5}$$

$$a_{34}=\{w_1^{*2}[R^*(9)+\tfrac{7}{8}R^*(4)]+3w_2^{**}R(11)+2w_1^*w_2^*[\tfrac{7}{8}R^*(3)-\tfrac{1}{4}R^*(1)+R^*(10)]\}\epsilon^{-1}=4.5664\ , \tag{D6}$$

$$\begin{aligned}a_{35}=\Bigg[&w_1^{*3}\left[\frac{\partial R^*(8)}{\partial c}+\frac{7}{8}\frac{\partial R^*(2)}{\partial c}\right]+w_1^{*2}w_2^*\left[\frac{\partial R^*(9)}{\partial c}+\frac{7}{8}\frac{\partial R^*(4)}{\partial c}\right]\\&+w_2^{*3}\frac{\partial R^*(11)}{\partial c}+w_1^*w_2^{*2}\left[\frac{\partial R^*(10)}{\partial c}+\frac{7}{8}\frac{\partial R^*(3)}{\partial c}-\frac{1}{4}\frac{\partial R^*(1)}{\partial c}\right]\Bigg]\epsilon^{-1.5}=24.6859\ ,\end{aligned} \tag{D7}$$

$$a_{43}=\{w_2^{*2}[R(12)+\tfrac{3}{8}R(4)]+\tfrac{3}{4}w_1^*w_2^*R(2)\}\epsilon^{-1}=0.2821\ , \tag{D8}$$

$$a_{44}=\{\tfrac{1}{2}\epsilon+w_1^{*2}[\tfrac{3}{8}R^*(2)]+3w_2^{*2}[\tfrac{3}{4}R^*(1)+\tfrac{3}{8}R^*(3)+R^*(13)]+2w_1^*w_2^*[\tfrac{3}{8}R^*(4)+R^*(12)]\}\epsilon^{-1}=-0.6327\ , \tag{D9}$$

$$\begin{aligned}a_{45}=&\left[\tfrac{3}{8}w_1^{*2}w_2^*\frac{\partial R^*(2)}{\partial c}+w_1^*w_2^{*2}\left[\frac{\partial R^*(12)}{\partial c}+\frac{3}{8}\frac{\partial^* R(4)}{\partial c}\right]+w_2^{*3}\left[\frac{\partial R^*(13)}{\partial c}+\frac{3}{4}\frac{\partial R^*(1)}{\partial c}+\frac{3}{8}\frac{\partial R^*(3)}{\partial c}\right]\right]\epsilon^{-1.5}\\&=3.93745\ ,\end{aligned} \tag{D10}$$

$$a_{53}=\{2w_1^*[R^*(5)-(1-c^*)R^*(2)]+w_2^*[R^*(7)-(1-c^*)R^*(4)]\}\epsilon^{-0.5}=0.1212\ , \tag{D11}$$

$$a_{54}=\Big[2w_2^*\{R^*(6)+(1-c^*)[R^*(1)-R^*(3)]\}+w_1^*[R^*(7)-(1-c^*)R^*(4)]\Big]\epsilon^{-0.5}=0.1577\ , \tag{D12}$$

$$\begin{aligned}a_{55}=\Bigg\{&w_2^{*2}[R^*(3)-R^*(1)]+w_1^{*2}R^*(2)+w_1^*w_2^*R^*(4)+w_1^{*2}\left[\frac{\partial R^*(5)}{\partial c}-(1-c^*)\frac{\partial R^*(2)}{\partial c}\right]\\&+w_2^{*2}\left[\frac{\partial R^*(6)}{\partial c}+(1-c^*)\left[\frac{\partial^* R(1)}{\partial c}-\frac{\partial R^*(3)}{\partial c}\right]\right]+w_1^*w_2^*\left[\frac{\partial R^*(7)}{\partial c}-(1-c^*)\frac{\partial R^*(4)}{\partial c}\right]\Bigg\}\epsilon^{-1}=-0.9175\ .\end{aligned} \tag{D13}$$

APPENDIX E: DERIVATION OF EQ. (5.35)

Recursion relation of $B_1(l)$, $B_2(l)$, $w_1(l)$, and $w_2(l)$ can be written in the following compact forms:

$$\frac{dB_1(l)}{dl}=(2-2\mu_{\|}-\eta_{\perp})B_1(l)mi\tfrac{1}{2}\int_{\Omega}\frac{\overline{w}_z^2}{[1+B(l,\theta)]^2}\ , \tag{E1}$$

$$\frac{dB_2(l)}{dl}=(2-\eta_{\perp})B_2(l)mi\tfrac{1}{2}\int_{\Omega}\frac{\overline{w}_{\perp}^2}{[1+B(l,\theta)]^2}\ , \tag{E2}$$

$$\frac{dw_1(l)}{dl}=\tfrac{1}{2}(\epsilon-7\mu_{\|}-3\eta_{\perp})w_1+\int_{\Omega}\frac{\overline{w}_z^3(l,\theta)}{[1+B(l,\theta)]^3}\ , \tag{E3}$$

$$\frac{dw_2(l)}{dl}=\tfrac{1}{2}(\epsilon-3\mu_{\|}-3\eta_{\perp})w_2+\int_{\Omega}\frac{\overline{w}_z(l,\theta)\overline{w}_{\perp}^2(l,\theta)}{[1+B(l,\theta)]^3}\ , \tag{E4}$$

where $\int_{\Omega}\equiv(2\pi)^{-6}\int d\Omega$ and

$$\overline{w}_z\equiv[w_1(l)\cos\theta+w_1(l)\sin\theta][1-c(l)\cos^4\theta]^{-1}\ , \tag{E5}$$

$$\overline{w}_{\perp}\equiv\frac{2}{5^{1/2}}w_2(l)\cos\theta\sin\theta[1-c(l)\cos^4\theta]^{-1}\ , \tag{E6}$$

$$B(l,\theta)\equiv[B_1(l)\cos^2\theta+B_2(l)\sin^2\theta][1-c(l)\cos^4\theta]^{-1}\ . \tag{E7}$$

Then the stability matrix for B_1 and B_2 becomes

$$\frac{\delta(B_i - B_i^*)}{dl} = \sum_{j=1}^{j=2} A_{ij}(B_j(l) - B_j^*) \quad (i=1,2)\,, \tag{E8}$$

where the matrix element can be written in the following form:

$$A_{11} = 2 - 2\mu_{||} - \eta_\perp + \int_\Omega \frac{\overline{w}_z^{*2}(\theta)\cos^2\theta}{1 - c^*\cos^4\theta} \tag{E9}$$

$$A_{12} = \int_\Omega \frac{\overline{w}_z^{*2}(\theta)\sin^2\theta}{1 - c^*\cos^4\theta}\,, \tag{E10}$$

$$A_{21} = \int_\Omega \frac{\overline{w}_\perp^{*2}(\theta)\cos^2\theta}{1 - c^*\cos^4\theta}\,, \tag{E11}$$

$$A_{22} = 2 - \eta_\perp + \int_\Omega \frac{\overline{w}_\perp^{*2}(\theta)\sin^2\theta}{1 - c^*\cos^4\theta}\,. \tag{E12}$$

Since the order-parameter exponent is $\omega = (d - 2 + \eta_\perp + 3\mu_{||})/2$, the difference between ω and A_{11} becomes

$$\begin{aligned} A_{11} - \omega = \tfrac{1}{2}(\epsilon - 7\mu_{||} - 4\eta_\perp) + \int_\Omega \frac{\overline{w}_z^{*2}(\theta)\cos^2\theta}{1 - c^*\cos^4\theta} &= \frac{1}{w_1^*}\left[\frac{dw_1}{dl} mi \int_\Omega \frac{\overline{w}_z^{*3}}{1 - c^*\cos^4\theta}\right] + \int_\Omega \frac{\overline{w}_z^{*2}\cos^2\theta}{1 - c^*\cos^4\theta} \\ &= -\frac{w_2^*}{w_1^*}\int_\Omega \frac{\overline{w}_z^{*2}}{1 - c^*\cos^4\theta} = mi\frac{w_2^*}{w_1^*}A_{12} \end{aligned} \tag{E13}$$

where we used the recursion for w_1 in Eq. (E3). Similar calculation shows

$$A_{22} - \omega = mi\frac{w_1^*}{w_2^*}A_{21}\,. \tag{E14}$$

Hence the matrix A_{ij} can be expressed as $A_{ij} = \delta A_{ij} + \omega\delta_{ij}$ with

$$\delta A_{ij} \equiv A_{ij} - w\delta_{ij} = \begin{bmatrix} -A_{12}w_2^*(w_1^*)^{-1} & A_{21} \\ A_{21} & -A_{21}w_1^*(w_2^*)^{-1} \end{bmatrix}. \tag{E15}$$

Since δA_{ij} is the matrix whose second column is a constant times the first column, one of its eigenvalues must be zero. Hence one of eigenvalues of the matrix A_{ij} must be equal to ω, which proves the statement Eq. (5.35).

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