Environmental effects on the Lamb shift according to stochastic electrodynamics

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The radiative energy shifts produced by environmental modifications of the electromagnetic vacuum are studied from the point of view of stochastic electrodynamics. The vacuum can be modified by the addition of external radiation or by the introduction of conducting objects that alter the distribution of modes. The general formulas obtained are applied to some simple but representative situations. The present results are shown to coincide with previous quantum calculations, and some conceptual differences between the two approaches are discussed.

I. INTRODUCTION

Of fundamental importance for quantum mechanics is the interaction of charged particles with ever-present radiation fields. Quantum electrodynamics (QED) has shown us how to interpret the spontaneous decay of excited states, most of the Lamb shift, the mass correction, and the anomalous magnetic moment, through the coupling of the atom to the vacuum field. Stochastic electrodynamics (SED) shares this view on the origin of radiative corrections.[1] In fact, since a starting point for this theory is precisely the existence of a real (stochastic) vacuum field, the radiative corrections are a natural outcome in SED. Specific SED calculations yielding results that are basically coincident with those of nonrelativistic QED support this view. There is, however, a difference (besides the formal and methodological ones) between the two theories, regarding the nature of the vacuum field and its influence on quantum behavior: While QED views it as a virtual field that is responsible only for minor effects (radiative *corrections*), for SED this vacuum is real; the main effect of its coupling to matter is quantum mechanics itself, and the radiative corrections are merely a second-order effect.

In either case, it is clear that some basic properties of the vacuum—such as the intensity of its fluctuations or its spectral distribution—are reflected in the radiative corrections. This means that a change in such properties can in principle lead to an observable modification of these corrections. The background field can be altered, for instance, by raising the temperature of the whole system, by adding external radiation, or by introducing metallic objects that alter the distribution of the normal modes of the field.

The possiblity of such environmental effects has been considered and analyzed for more than 40 years. As far back as 1946, Purcell[2] predicted the influence of a resonant cavity on the relaxation time of a spin system. A few years after the discovery of the Lamb shift, Auluck and Kothari[3] studied the influence of an external radiation field on the value of this energy shift. Since then numerous calculations have been published along similar lines, the great majority of them within the framework of quantum theory. The predicted effects are in general negligibly small, and their calculation was often considered merely an academic exercise. Indeed, for a long time these effects were hardly amenable to experimental proof, but the situation has changed in the last years, thanks to the development of new experimental techniques involving very small distances, very short times, and sharply defined frequencies. Since 1966, when Drexhage and Kuhn[4] reported a modulation of the fluorescence decay time of a dye molecule in front of a mirror, other authors have observed similar effects on the lifetime of excited states.[5]

Concerning environmental effects on the energy levels, in the 1960s a series of experiments was performed[6] to prove that external radiation produces a relative shift of the Zeeman sublevels of an atomic ground state. The advent of tunable lasers allowing selective population of Rydberg states has made it possible to observe not only the induced alterations of the decay rates mentioned above, but also atomic energy-level shifts produced by thermal radiation;[7] the fractional frequency shifts observed are of the order of 10^{-12}.

In view of the rapid development of small-size and high-precision experimental techniques, it seems convenient to develop also the formalism that leads to theoretical predictions that can be eventually subject to experimental test. One aim of the present work is to contribute to this effort. But we have a second intention, which is—at least to us—more motivating: namely, to elucidate the precise mechanism by which the modifications of the radiative corrections are induced; to trace the origin of these effects and explain their meaning. This task is facilitated by the use of SED, because in this theory the presence of the background radiation field is clear from the beginning and there is no need to think of virtual photons having real effects.

In this first paper we set out to study the shifts of the atomic energy levels induced by alterations of the background radition field. The results furnished by SED are compared with other results that have been derived previously through more orthodox procedures. The analysis of other induced modifications of the radiative effects (basically, of the lifetime of excited states) is left for a second paper.[8]

The paper has the following structure. Section II con-

tains a brief exposition of the SED description of a bound electron (specifically, a harmonic oscillator). In Sec. III we derive general formulas for the radiative energy shifts; these formulas are applied in Sec. IV to calculate the effects of external modifications on the energy levels, and the results are compared with previously reported quantum calculations. The general connection between the SED and the quantum formulas is analyzed in Sec. V; in Secs. V and VI, the main conceptual and methodological differences between the two approaches, concerning the treatment of the radiative energy corrections, are discussed.

II. THE HARMONIC OSCILLATOR ACCORDING TO SED

We should first of all mention that SED has not yet been able to offer a correct description of atomic behavior.[1] In many respects this has been a serious drawback of the theory, but for our present purposes there is a way around to this problem. We recall that SED provides a complete and correct description of the charged (nonrelativistic, spinless) harmonic oscillator; this description coincides formally with the quantum-mechanical one, including the Lamb shift and the lifetime of excited states.[1,9–11] Now the periodic motion of atomic electrons can be considered in a first instance—at least for qualitative purposes—as a spatial combination of harmonic oscillations with frequency corresponding to the period of the motion (this is strictly true, in particular, for circular orbits).[12] The main difference is that the frequency of the atomic electron depends on the energy level: the inner electrons have a higher frequency of motion than the outer ones. In view of the above, we shall treat the atomic electrons as harmonic oscillators with state-dependent frequency and calculate the environmental energy shifts of these oscillators using the formalism of SED.

A usual starting point for the theory is the so-called Braffort-Marshall equation: a classical approximate stochastic equation of motion for the charged oscillator that includes both the Lorentz force produced by the random electromagnetic field acting on the particle, and the reaction force arising from the self-field radiated by the moving charge. In the nonrelativistic treatment the random Lorentz force can be reduced to its electric component, and the dependence of this electric force on the space coordinates can be omitted; the Braffort-Marshall equation reads then

$$\ddot{\mathbf{x}}+\omega_0^2\mathbf{x}-\tau\dddot{\mathbf{x}}=\frac{e}{m}\mathbf{E}(t)\ , \tag{1}$$

where $\tau=2e^2/3mc^3$, ω_0 is the natural frequency of the oscillator, and m represents the physical (renormalized) mass, since in Eq. (1) any effect of the self-field other than the reaction force $m\tau\dddot{\mathbf{x}}$ has already been taken into account. The statistical properties of the random electric field $\mathbf{E}$ are selected to insure that this field represents the classical, stochastic counterpart of the vacuum field of QED; in particular, it has an average energy $\mathbb{E}_0=\frac{1}{2}\hbar\omega$ per normal mode.

The appearance of noncausal solutions allowed for by the $\dddot{\mathbf{x}}$ term in Eq. (1) is avoided by rewriting this equation in the approximate form (to first order in τ),

$$\ddot{\mathbf{x}}+\omega_0^2\mathbf{x}+\tau\omega_0^2\dot{\mathbf{x}}=\frac{e}{m}E_m(t)\ , \tag{2}$$

where the modified electric field $\mathbf{E}_m$ is related to the vacuum field by

$$\mathbf{E}_m-\tau\dot{\mathbf{E}}_m=\mathbf{E}\ . \tag{3}$$

An approximate treatment of Eq. (2) to first order in τ leads to a correct description of the ground state of the quantum oscillator.[9] In particular, a straightforward calculation using the solution $\mathbf{x}(t)$ gives for the average energy of the oscillator in the stationary (long-time) limit,

$$\varepsilon=\tfrac{1}{2}m\langle\dot{\mathbf{x}}^2+\omega_0^2\mathbf{x}^2\rangle\ ,$$

the expression

$$\varepsilon=\frac{e^2}{2m\sigma}\int_0^\infty\langle\mathbf{E}_m(t)\cdot\mathbf{E}_m(0)\rangle e^{-\sigma t}\cos\omega_0 t\,dt\ , \tag{4}$$

where $\sigma=\frac{1}{2}\tau\omega_0^2$ and the angle brackets represent stochastic averages. To calculate Eq. (4) to first order in τ (or σ) it is convenient to express the integrand as a power series in σ, taking care to avoid the appearance of secular terms; we therefore write

$$e^{-\sigma t}\cos\omega_0 t\cong\cos\omega_0 t-\frac{\sigma}{\omega_0}\sin\omega_0 t\ .$$

The more rigorous treatment of Sec. III will show that this approximation is indeed valid. From Eq. (4), then,

$$\varepsilon=\varepsilon_0+\varepsilon_1\ , \tag{5}$$

with

$$\varepsilon_0=\frac{e^2}{m\tau\omega_0^2}\int_0^\infty\langle\mathbf{E}_m(t)\cdot\mathbf{E}_m(0)\rangle\cos\omega_0 t\,dt \tag{6}$$

and

$$\varepsilon_1=-\frac{e^2}{m\tau\omega_0^2}\int_0^\infty\langle\mathbf{E}_m(t)\cdot\mathbf{E}_m(0)\rangle\sin\omega_0 t\,dt\ . \tag{7}$$

It is clear from these expressions that ε_0 represents the main contribution to the energy and ε_1 is a correction of first order in τ. To calculate these terms we must know the field correlation, or alternatively, the spectral energy density, which is related to the former by the Wiener-Khintchine theorem,

$$\langle\mathbf{E}_i(t)\mathbf{E}_j(0)\rangle=\frac{2\pi}{3}\delta_{ij}\int_{-\infty}^{\infty}\rho(\omega)e^{i\omega t}d\omega\ . \tag{8}$$

In writing this equation we have taken into account the assumed isotropy of the background radiation field; in the more general case, the correlation tensor need not be diagonal.

The relationship between $\mathbf{E}$ and $\mathbf{E}_m$ expressed in Eq. (3) implies that the correlation of the modified field $\mathbf{E}$ is given by

$$\langle \mathbf{E}_{m,i}(t)\mathbf{E}_{m,j}(0)\rangle = \frac{2\pi}{3}\delta_{ij}\int_{-\infty}^{\infty}\frac{\rho(\omega)}{1+\tau^2\omega^2}e^{i\omega t}d\omega \ . \tag{9}$$

Since we are only interested in approximate results to order τ, we should for consistency omit the extra denominator appearing in Eq. (9); this we shall do, indeed, except in those instances where such omission can affect the convergence of the integral.

In free space and in absence of external radiation, $\rho(\omega)$ represents the zero-point vacuum field,

$$\rho_0(\omega)=\frac{\omega^2\mathbb{E}_0}{\pi^2c^3}=\frac{\hbar\omega^3}{2\pi^2c^3} \ . \tag{10}$$

In this case one obtains from Eqs. (6), (9), and (10),

$$\varepsilon_0=\tfrac{1}{2}\hbar\omega_0 \ . \tag{11}$$

This is the main contribution to the energy of the oscillator coupled to the vacuum field at zero temperature, i.e., the ground-state energy. For the first-order radiative correction one obtains from Eqs. (7), (9), and (10),

$$\varepsilon_1=\frac{3\hbar\tau}{2\pi}\left[\int_0^{\infty}\frac{\omega}{1+\tau^2\omega^2}d\omega + \omega_0^2\int_0^{\infty}\frac{\omega}{(1+\tau^2\omega^2)(\omega_0^2-\omega^2)}d\omega\right] . \tag{12}$$

We have separated the free-particle contribution (i.e., the value of ε_1 for $\omega_0=0$) to show that the logarithmic divergence of the integral comes from this term. A cutoff at high frequencies in the spectral density of the field reduces the free-particle first-order radiative correction to the value

$$\varepsilon_1^{\mathrm{FP}}=\frac{3\hbar}{4\pi\tau}\ln(1+\tau^2\omega_c^2) \ . \tag{13}$$

Even for ω_c as high as $mc^2/\hbar$ (corresponding to pair creation, in order of magnitude) we have $\tau\omega_c\sim\alpha=e^2/\hbar c$, and hence $\varepsilon_1^{\mathrm{FP}}\sim\alpha mc^2$. This energy has been interpreted as a mass correction[13] because it is independent of the state of motion and it exists even in the absence of forces. We shall come back to this point in Sec. IV.

The second term in Eq. (12) represents the radiative correction to the energy of the oscillator proper, i.e., the (nonrelativistic, spinless) Lamb shift. A straightforward calculation gives, always to lowest order in τ,

$$\varepsilon_1^{\mathrm{HO}}=\varepsilon_0(\tau\omega_0/\pi)\ln\tau\omega_0 \ . \tag{14}$$

For an oscillator in equilibrium with blackbody radiation at temperature T, whose spectral energy density is

$$\rho_T(\omega)=\rho_0(\omega)\frac{1+\epsilon}{1+\epsilon} \ , \tag{15}$$

with $\epsilon=\exp(-\hbar\omega/kT)$, Eqs. (11) and (12) transform into

$$\varepsilon_0(T)=\varepsilon_0(0)\left[1+\frac{2\epsilon(\omega_0)}{1-\epsilon(\omega_0)}\right] , \tag{16a}$$

$$\varepsilon_1(T)=\varepsilon_1(0)+\frac{3\hbar\tau}{\pi}\int_0^{\infty}\frac{\epsilon\omega^3}{(1-\epsilon)(\omega_0^2-\omega^2)}d\omega \ . \tag{16b}$$

The extra terms appearing in these equations represent thermal contributions to the main energy of the oscillator and to the radiative correction, respectively. To establish a connection between these results and the more familiar ones of quantum mechanics, it is convenient to recall the Fokker-Planck equation of SED and the more complete description of the oscillator furnished by this equation, which includes the excited states.[10,11] As we shall see in Sec. III, this description is directly related to the one given by quantum statistical mechanics.

III. GENERAL FORMULAS FOR THE ENERGY CORRECTIONS ACCORDING TO SED

Starting from the Braffort-Marshall equation a generalized Fokker-Planck equation has been derived in SED for the phase-space distribution $Q(\mathbf{x},\mathbf{p},t)$.[14] When applied to the harmonic oscillator in the long-time limit, this equation reads[10]

$$\frac{\partial Q}{\partial t}+\frac{\mathbf{p}}{m}\cdot\nabla_x Q-m\omega_0^2\nabla_p\cdot\left[\mathbf{x}+\frac{\tau}{m}\mathbf{p}\right]Q - \nabla_p\nabla_p:D^{pp}Q-\nabla_x\nabla_p:D^{xp}Q=0 \ , \tag{17}$$

where

$$D_{ij}^{xp}=-\frac{e^2}{m\omega_0}\int_0^{\infty}\langle E_{m,i}(t)E_{m,j}(0)\rangle\sin\omega_0 t\,dt \tag{18a}$$

and

$$D_{ij}^{pp}=e^2\int_0^{\infty}\langle E_{m,i}(t)E_{m,j}(0)\rangle\cos\omega_0 t\,dt \ . \tag{18b}$$

The time-independent solution of this equation represents the phase-space distribution for the oscillator in equilibrium with the background field. Let us consider that $\mathbf{E}$ is the electric part of the blackbody radiation field at temperature T. Then from Eqs. (9) and (18), the diffusion tensors are given by

$$D_{ij}^{xp}=-\frac{4\pi e^2}{3m}\delta_{ij}\int_0^{\infty}\frac{\rho_{\mathrm{T}}(\omega)}{(1+\tau^2\omega^2)(\omega_0^2-\omega^2)}d\omega \tag{19a}$$

and

$$D_{ij}^{pp}=\frac{2\pi^2e^2}{3m}\rho_{\mathrm{T}}(\omega)\delta_{ij}\equiv D\delta_{ij} \ , \tag{19b}$$

with $\rho_{\mathrm{T}}(\omega)$ given by Eq. (15), and hence

$$D=\frac{\hbar\tau m\omega_0^3}{2}\frac{1+\epsilon}{1-\epsilon} \ . \tag{20}$$

In the stationary situation, Eq. (17) separates to lowest order in τ into the equations[10]

$$\frac{\mathbf{p}}{m}\cdot\nabla_x Q-m\omega_0^2\mathbf{x}\cdot\nabla_p Q=0 \ ,$$

$$\tau\omega_0^2\mathbf{p}Q+D\nabla_p Q=0 \ ,$$

whose normalized solution is

$$Q=\frac{m\tau\omega_0^3}{2\pi D}\exp\left[-\frac{m\tau\omega_0^2}{D}\left(\frac{\mathbf{p}^2}{2m}+\frac{m\omega_0^2\chi^2}{2}\right)\right] .$$

This is the Wigner distribution for the quantum oscillator in equilibrium with blackbody radiation; by eliminating the temperature parameter one arrives at the usual description of the oscillator in terms of quantum states and discrete energies. A perturbative treatment in terms of the small parameter τ gives for the first-order radiative correction to the energy,[10]

$$\varepsilon_1=\langle n \mid D_{ij}^{xp}x_i\partial_{xj} \mid n\rangle=-\tfrac{1}{2}\,\mathrm{Tr}D^{xp}\ . \tag{21}$$

Note that for the harmonic oscillator the value of ε_1 is independent of the state of motion, which means that there is no relative shift between levels and hence the transition frequency is not affected by this energy correction. For an arbitrary (nonisochronous) bounded system, however, the frequency ω_0 depends on the energy level, and so does the expectation value in Eq. (21). This happens, in particular, in the atomic case; the radiative correction produces then an observable frequency shift for transitions between bound states.

In order to compare Eq. (21) with quantum mechanics we take Eq. (18a) for D^{xp}, use $\mathbf{E}=-(1/c)\dot{\mathbf{A}}$ in the long-wavelength approximation, and perform a double integration by parts, using the stationarity of the field, $\langle \mathbf{E}(t)\cdot\mathbf{E}(0)\rangle=\langle \mathbf{E}(t+s)\cdot\mathbf{E}(s)\rangle$. Once more, we shall recall the difference between $\mathbf{E}$ and $\mathbf{E}_m$ only where it is necessary for the convergence of the integrals. We thus obtain

$$\varepsilon_1=\frac{e^2}{2mc^2}\langle \mathbf{A}^2\rangle-\frac{e^2\omega_0}{2mc^2}\int_0^\infty\langle \mathbf{A}(t)\cdot\mathbf{A}(t-s)\rangle\sin\omega_0 s\,ds\ . \tag{22}$$

Note that the first term is independent of ω_0, whereas the second one vanishes for $\omega_0=0$. This allows us to identify the first term as the free-particle contribution to the energy correction and the second one as the Lamb shift proper, i.e.,

$$\varepsilon_1^{\mathrm{FP}}=(e^2/2mc^2)\langle \mathbf{A}^2\rangle\ , \tag{23a}$$

$$\varepsilon_1^{\mathrm{HO}}=-\frac{e^2\omega_0}{2mc^2}\int_0^\infty\langle \mathbf{A}(s)\cdot\mathbf{A}(0)\rangle\sin\omega_0 s\,ds\ . \tag{23b}$$

When $\mathbf{A}$ represents the zero-point radiation field these equations reproduce Eqs. (13) and (14). But more generally, Eqs. (23) give the radiative corrections (for the free particle and for the oscillator, respectively) produced by any radiation field represented by the vector potential $\mathbf{A}$.

The same equations (23) can be used to calculate the variations of the energy shift produced by a modification of the external conditions. Let us denote with a subindex e the modified field and with a subindex 0 the vacuum field; then the variations of the (first-order) radiative corrections are

$$\delta\varepsilon_1^{\mathrm{FP}}=\frac{e^2}{2mc^2}(\langle \mathbf{A}^2\rangle_e-\langle \mathbf{A}^2\rangle_0)\ , \tag{24a}$$

$$\delta\varepsilon_1^{\mathrm{HO}}=-\frac{e^2\omega_0}{2mc^2}\int_0^\infty[\langle \mathbf{A}(s)\cdot\mathbf{A}(0)\rangle_e-\langle \mathbf{A}(s)\cdot\mathbf{A}(0)\rangle_0]\times\sin\omega_0 s\,ds\ . \tag{24b}$$

Equation (24a) coincides with the result obtained by Barton[15] for the (first-order) correction of the energy for a free electron (called δ_1 in Ref. 15). In quantum mechanics, this correction comes naturally from the $\mathbf{A}^2$ term in the Schrödinger equation when the electromagnetic field is introduced as a perturbation.

The discussion of the connection between Eq. (24b) and the corresponding quantum formula is left for Sec. V. In the following we shall use Eqs. (24) to calculate the energy variations in some simple situations where the field is externally modified.

IV. APPLICATIONS TO SIMPLE PROBLEMS

As Eqs. (24) show, an energy variation is always produced when the radiation field is modified. Such a modification can be produced (a) by changing the energy content per normal mode (for instance, by raising the temperature of the field, or adding external radiation), or (b) by altering the spectral distribution of the field (for instance, by introducing a metallic cavity that eliminates the low-frequency modes).

The simplest kind of modification of the field, from the mathematical point of view, consists in a homogeneous and isotropic change of its spectral energy density. Since for an isotropic field, we have from Eq. (8),

$$\langle \mathbf{A}(s)\cdot\mathbf{A}(0)\rangle=4\pi c^2\int_0^\infty\rho(\omega)\omega^{-2}\cos\omega s\,d\omega\ , \tag{25}$$

Eqs. (24) reduce in this case to

$$\delta\varepsilon_1^{\mathrm{FP}}=\frac{2\pi e^2}{m}\int_0^\infty(\rho_e-\rho_0)\omega^{-2}d\omega\ , \tag{26a}$$

$$\delta\varepsilon_1^{\mathrm{HO}}=-\frac{2\pi e^2\omega_0^2}{m}\int_0^\infty\frac{\rho_e-\rho_0}{\omega^2}\mathbb{P}\frac{1}{\omega_0^2-\omega^2}d\omega\ , \tag{26b}$$

where $\mathbb{P}$ denotes the principal part. Let us study the two cases separately.

(a) First consider a change of the energy $\mathbb{E}$ of the field modes. If, for instance, the temperature of the field is raised, then $\rho(\omega)$ represents blackbody radiation,

$$\rho_{\mathrm{T}}(\omega)=\omega^2\mathbb{E}_{\mathrm{T}}/\pi^2c^3\ , \tag{27}$$

with

$$\mathbb{E}_{\mathrm{T}}=\tfrac{1}{2}\hbar\omega(1+\epsilon)/(1-\epsilon)\ ,$$

according to Eq. (15). From Eqs. (26), the energy variations are then given by

$$\delta\varepsilon_1^{\mathrm{FP}}=\frac{\pi\alpha}{3}\frac{(kT)^2}{mc^2}\ , \tag{28a}$$

$$\delta\varepsilon_1^{\mathrm{HO}}=\frac{2\alpha\hbar^2\omega_0^2}{\pi mc^2}\int_0^\infty\frac{\omega\epsilon}{1-\epsilon}\mathbb{P}\frac{1}{\omega_0^2-\omega^2}d\omega\ . \tag{28b}$$

Equation (28a) shows that any particle (even a free particle) acquires an extra energy as the temperature of the background field is raised. This result coincides with Δ_1 derived by Knight[16] on a quantum basis. As discussed in Ref. 17, from the point of view of SED it is natural that $\varepsilon_1^{\mathrm{FP}}$ depends on temperature, because it represents the kinetic energy of the random motion of the particle im-

pressed on it by the blackbody radiation field. Avan *et al.*[18] arrive within the quantum scheme at the same conclusion about the kinetic origin of ε_1^{FP}; if this shift is interpreted in its turn as a mass correction, as these authors propose, it will affect the value of $g-2$ for the spinning electron; this point, however, is outside the scope of the present paper.

At room temperatures the frequency shift calculated from Eq. (28a) amounts to $\delta\nu=2.4$ kHz. Such a small shift has been recently observed[7] in highly excited Rydberg states, in which the electron is nearly a free particle.

To evaluate the thermal contribution to the Lamb shift we rewrite Eq. (28b) in the form

$$\delta\varepsilon_1^{HO}=(2\alpha\hbar^2/\pi mc^2)\omega_0^2 f(y)\ , \tag{29}$$

where

$$f(y)=\int_0^\infty \frac{x}{e^x-1}\mathbb{P}\frac{1}{x^2-y^2}dx$$

and $y=\hbar\omega_0/kT$. For $y\ll 1$, the integral gives approximately $f(y)=-C/y$, with C of order unity; hence,

$$\delta\varepsilon_1^{HO}\cong -(2\alpha k^2T^2/\pi mc^2)(\hbar\omega_0/kT)\ .$$

This is a correction of first order in y to the free-particle energy (28a). The energy shift for a loosely bound electron is therefore correctly given by (28a) in a first approximation.

For $y\gg 1$, on the other hand, $f(y)$ has the approximate value

$$f(y)\cong -(\pi^2/6y^2)-(\pi^4/15y^4)\ ,$$

which introduced in Eq. (29) gives

$$\delta\varepsilon_1^{HO}\cong -\delta\varepsilon_1^{FP}-(2\pi^3\alpha\hbar^2\omega_0^2/15mc^2)(kT/\hbar\omega_0)^4\ .$$

Thus we see that for a tightly bound particle, the Lamb shift is almost cancelled by the free-particle shift: The small thermal correction that survives is

$$\delta\varepsilon_1=-(2\pi^3/15)\Pi_0(kT)^4/(\hbar c)^3\ ,$$

where $\Pi_0=e^2/m\omega_0^2$ is the polarizability of the oscillator. In the quantum-mechanical analog of this result—which looks formally identical to the above—$\delta\varepsilon_1$ is interpreted as a dynamic Stark shift due to blackbody radiation.[16]

It is interesting to note that the strong binding of the electron has the effect of suppressing the kinetic energy otherwise induced by the thermal radiation field. In more orthodox terms, binding of the electron to an atom suppresses the scattering amplitude for (low-frequency) photons.[19] It is precisely the difference between $\delta\varepsilon_1$ for tightly bound electrons and for nearly free electrons which has made it possible to observe the thermal Lamb shift by measuring the transition frequencies between the ground state and highly-excited atomic states.[7]

There are of course other means of modifying the average energy content of the normal modes $\mathbb{E}(\omega)$, thereby affecting $\rho(\omega)$; this can be achieved, for instance, by optical irradiation. The energy shift thus produced—called "Lamp shift" by Kastler[6]—can be readily calculated from Eqs. (26) and (27), if $\mathbb{E}(\omega)$ is known. Knight[16] considers an admittedly artificial situation in which a constant number $\bar{n}$ of photons of every frequency ω is added to the vacuum within the range of frequencies $a<\omega/c<b$; this amounts to $\mathbb{E}(\omega)=(\bar{n}+\frac{1}{2})\hbar\omega$ and, hence, from Eq. (27), $\rho_e=(2\bar{n}+1)\rho_0$ for $a<\omega/c<b$ and $\rho_e=\rho_0$ outside this range. Equations (26) then give

$$\delta\varepsilon_1^{FP}=\bar{n}\alpha\hbar^2(b^2-a^2)/\pi m\ ,$$

$$\delta\varepsilon_1^{HO}=\bar{n}\frac{\alpha(\hbar\omega_0)^2}{\pi mc^2}\ln\left|\frac{b^2-(\omega_0/c)^2}{a^2-(\omega_0/c)^2}\right|\ ,$$

in accordance with the quantum results reported in Ref. 16. (To compare $\delta\varepsilon_1^{HO}$, apply the quantum result specifically to the harmonic oscillator.) It is evident from the above results that the number $\bar{n}$ and the spectral width $(b-a)$ must be quite large to obtain a significative shift.

(b) The distribution of modes of the radiation field can be affected by introducing conducting objects that impose new boundary conditions on the electric and magnetic components of the field. The modified spectral distribution will of course be, in general, anisotropic and inhomogeneous, which means that instead of Eq. (25), one must use

$$\langle A_i(s)A_j(0)\rangle=\frac{4\pi c^2}{3}\int_0^\infty\frac{\rho_{ij}(\omega,\mathbf{x})}{\omega^2}\cos\omega s\,d\omega \tag{30}$$

in Eqs. (24). When the distribution of modes becomes discrete—for instance, in the space between two parallel metallic plates—the integral over ω transforms into a sum over all possible modes. Such a case has been considered by Barton,[15] who obtains for the free-particle correction

$$\delta\varepsilon_1^{FP}=\frac{\alpha\pi\hbar^2}{mL^2}\left[\frac{1}{12}+\frac{1}{4\sin^2(\pi z/L)}\right] \tag{31}$$

for $L>z>\hbar/mc=\lambda_c$, where L is the separation between plates and z is the distance of the atom to one of the plates. [Note that for $L\sim\lambda_c$, Eq. (31) already gives $\delta\varepsilon_1^{FP}\sim\alpha mc^2$ which is too large to be considered a perturbation; a more reasonable limit for the validity of this result would therefore be $L>\lambda_c/\alpha$. Moreover, at such small distances one would have to consider the effect of the plates on the distribution $|\psi|^2$ of the electron, since the mean radius of the electronic orbits is larger than λ_c/α.]

Since the above result was derived by Barton from Eq. (24a)—although using a quantum language—there is no point in repeating the derivation here. In the calculations that led to Eq. (31) the radiation field was considered in its ground state; it is clear that when the energy content of the modes is raised, the formula for $\delta\varepsilon_1^{FP}$ has to be modified accordingly.

It is interesting to observe that the modification of the field-mode distribution results in a repulsive force on the particle from the nearest one of the plates; the energy correction attains its minimum value at $z=L/2$.

Already in 1965, Marshall[20] had shown that the Casimir force—the attractive force between two parallel

neutral metal plates—usually considered to be of quantum origin, can be explained as a consequence of the effect of the boundary conditions on the random zero-point radiation field; what Eq. (31) expresses is another consequence of this effect, now on a charged particle located between the plates.

When the distribution of modes is changed without altering the isotropy of the field, one can use Eqs. (26) to evaluate the energy shifts. Consider, for instance, a waveguide that has the effect of eliminating all modes of frequencies $\omega<\omega_m$, without altering significantly the other modes. As a result, the free-particle energy is lowered by the amount

$$\delta\varepsilon_1^{\mathrm{FP}}=-(\alpha/2\pi)(\hbar\omega_m)^2/mc^2 ,$$

and the oscillator energy is shifted by

$$\delta\varepsilon_1^{\mathrm{HO}}=-\frac{\alpha\hbar^2\omega_0^2}{2\pi mc^2}\ln|1-(\omega_m/\omega_0)^2| .$$

Once more we see that for very high oscillator frequencies ($\omega_0\gg\omega_m$), $\delta\varepsilon_1^{\mathrm{HO}}$ essentially cancels $\delta\varepsilon_1^{\mathrm{FP}}$: A tightly bound particle does not "feel" the presence of the walls. The reason for this is that the field modes that contribute most to the kinetic energy ε_1 of the particle (those with $\omega\sim\omega_0$) were not altered by the waveguide. For an almost free particle, on the other hand ($\omega_0\ll\omega_m$), one has

$$\delta\varepsilon_1^{\mathrm{HO}}=-\frac{\alpha\hbar^2\omega_0^2}{\pi mc^2}\ln\frac{\omega_m}{\omega_0} ,$$

which represents a small correction to $\delta\varepsilon_1^{\mathrm{FP}}$. In this case the field modes with $\omega\sim\omega_0$ are absent, and their contribution to the kinetic energy ε_1 is therefore subtracted.

One should be aware that in practice ω_m cannot be arbitrarily high: Conducting walls are transparent to electromagnetic fields of frequencies higher than $10^{-2}mc^2/\hbar$ (or even lower, if the walls are very thin) and the cavities are therefore not effective at these frequencies. Since the main contribution to ε_1 comes from the high frequencies, only a small fraction of this energy [of order $(\omega_m/\omega_c)^2$, where ω_c is the relativistic cutoff frequency $mc^2/\hbar$] can be removed by means of the cavity.

V. CONNECTION WITH THE GENERAL QUANTUM FORMULAS

The specific calculations of Sec. IV serve to show that the SED formulas for the energy variations induced by environmental changes of the radiation field, Eqs. (24), give correct results. It seems therefore opportune to analyze the connection between these general formulas and those derived from the quantum formalism.

With this purpose in mind let us go back to Eq. (21), which gives the total radiative correction to the energy, and rewrite it in a quantum language; with D^{xp} given by Eq. (18a), we obtain

$$\varepsilon_1=\frac{i}{\hbar}D^{xp}\sum_k\langle n|x_j|k\rangle\langle k|p_i|n\rangle$$
$$=\frac{ie^2}{\hbar m}\sum_k\int_0^\infty\langle E_i(s)E_j(0)\rangle\langle n|x_j|k\rangle\times\langle k|p_i|n\rangle\omega_{nk}^{-1}\sin\omega_{nk}s\,ds ,$$

where k refers to the intermediate states of the particle. In making this transition to the quantum language we have taken advantage of the fact that the oscillator of SED has been shown to coincide with the quantum one. In writing the last equality we have taken into account that $\omega_{nk}=\pm\omega_0$. After a double integration by parts one obtains

$$\varepsilon_1=\frac{ie^2}{\hbar mc^2}\sum_k\langle n|x_j|k\rangle\langle k|p_i|n\rangle\left[-\langle A_iA_j\rangle+\omega_{nk}\int_0^\infty\langle A_i(s)A_j(0)\rangle\sin\omega_{nk}s\,ds\right] .$$

Finally, applying the Thomas-Reiche-Kuhn sum rule to the first term and using

$$\langle n|p_j|k\rangle=im\omega_{nk}\langle n|x_j|k\rangle ,$$

one gets

$$\varepsilon_1=\varepsilon_1^{\mathrm{FP}}+\varepsilon_1^{\mathrm{HO}} ,$$

with

$$\varepsilon_1^{\mathrm{FP}}=(e^2/2mc^2)\langle\mathbf{A}^2\rangle , \tag{32a}$$

and

$$\varepsilon_1^{\mathrm{HO}}=\frac{e^2}{\hbar m^2c^2}\sum_k\int_0^\infty\langle n|\mathbf{p}\cdot\mathbf{A}(s)|k\rangle\langle k|\mathbf{p}\cdot\mathbf{A}(0)|n\rangle\times\sin\omega_{nk}s\,ds . \tag{32b}$$

Equation (32a) coincides with (23a), and Eq. (32b) is an alternative expression for the Lamb shift of the oscillator, equivalent to (23b). To perform the time integration in Eq. (32b), we expand the vector potential as a Fourier series,

$$\mathbf{A}(t)=\sum_\alpha[\mathbf{A}_\alpha\exp(-i\omega_\alpha t)+\mathbf{A}_\alpha^*\exp(i\omega_\alpha t)] , \tag{33}$$

where $\omega_\alpha>0$. The amplitudes corresponding to different normal modes are uncorrelated: $\langle A_{i\alpha}A_{j\alpha'}^*\rangle=\langle A_{i\alpha}A_{j\alpha}^*\rangle\delta_{\alpha\alpha'}$, whence Eq. (32b) transforms into

$$\varepsilon_1^{\mathrm{HO}}=\frac{e^2}{\hbar m^2c^2}\sum_k\sum_\alpha\langle n|\mathbf{p}\cdot A_\alpha^*|k\rangle\langle k|\mathbf{p}\cdot\mathbf{A}_\alpha|n\rangle\times\mathbb{P}\left[\frac{1}{\omega_{nk}+\omega_\alpha}+\frac{1}{\omega_{nk}-\omega_\alpha}\right] . \tag{34}$$

Although not explicitly stated, Eqs. (32b) and (34) imply statistical averaging of $A_{i\alpha}A^{*}_{j\alpha}$.

The above equation applies to any kind of random radiation field represented by **A**, whether in free space or inside a cavity, whether in its ground state or in an excited state. In particular, when **A** represents the zero-point vacuum field, the sum over α transforms into an integral over ω, and Eq. (34) reduces, using Eqs. (9), (10), and (33), to

$$\varepsilon_1^{\mathrm{HO}}=\frac{e^2}{3\pi m^2c^3}\sum_k |\langle k|\mathbf{p}|n\rangle|^2 I_{nk} , \tag{35}$$

with

$$I_{nk}=-\omega_{nk}\int_0^\infty \mathbb{P}\left[\frac{1}{\omega_{nk}+\omega}-\frac{1}{\omega_{nk}-\omega}\right]d\omega .$$

The logarithmic divergence of this integral can be eliminated by recovering the denominator $1+\tau^2\omega^2$ which we have omitted throughout; but in order to maintain the similarity with the usual quantum procedure we choose rather to make the integral converge by replacing the infinite upper limit by the finite cutoff $\omega_c\sim mc^2/\hbar\gg\omega_{nk}$. The above equation becomes then

$$\varepsilon_1^{\mathrm{HO}}=-\frac{2e^2}{3\pi m^2c^3}\sum_k |\langle k|\mathbf{p}|n\rangle|^2\omega_{nk}\ln\left|\frac{\omega_c}{\omega_{nk}}\right| . \tag{36}$$

This result coincides with the Bethe formula for the Lamb shift.[21] In conclusion, both the free-particle energy correction, Eq. (32a), and the Lamb shift, Eq. (35), are correctly given by SED in the nonrelativistic, spinless approximation.

It seems convenient to point out some differences between the procedures used in SED and in QED to arrive at these formulas. In the quantum case, second-order perturbation theory is used, with the interaction Hamiltonian given by $H_{\mathrm{int}}=-(e/mc)\mathbf{p}\cdot A+(e^2/2mc^2)\mathbf{A}^2$. The quadratic term gives rise to the free-particle energy correction. But the energy derived from the linear term,

$$-\frac{2e^2}{3\pi m^2c^3}\sum_k |\langle k|\mathbf{p}|n\rangle|^2\int_0^\infty\frac{\omega}{\omega-\omega_{nk}}d\omega , \tag{37}$$

still contains a (linearly divergent) free-particle contribution that must be subtracted to obtain the Lamb shift proper. The formula thus derived,[21]

$$\varepsilon_Q^{\mathrm{HO}}=-\frac{2e^2}{3\pi m^2c^3}\sum_k |\langle k|\mathbf{p}|n\rangle|^2\omega_{nk}\int_0^\infty\frac{d\omega}{\omega-\omega_{nk}} ,$$

still contains a logarithmic divergence, which calls for the introduction of a cutoff.

In the SED case, the term that gives rise to the radiative energy shift is present from the very beginning [see Eqs. (17) and (21)]; there is no need to introduce it as an additional interaction. In the explicit calculation of the energy shift, the free-particle and bound-state contributions are clearly identified and the latter gives a finite result without the need to renormalize: The appearance of the two terms in the integral I_{nk} in Eq. (35)— in contrast with the single term under the integral sign in the quantum expression (37)—eliminates the presence of a free-particle ($\omega_{nk}=0$) contribution in the SED case. Moreover, as mentioned above, the convergence of I_{nk} is assured by the extra factor $(1+\tau^2\omega^2)^{-1}$; the cutoff ω_c has been introduced just for convenience.

VI. CONCLUSIONS

We have seen that SED and QED give the same formulas for the radiative energy corrections of the oscillator in the nonrelativistic, spinless approximation, and hence they also predict the same results for the energy variations produced by environmental modifications of the radiation field. But we observe that in SED there is no risk of an incorrect or undue interpretation of the various energy terms, because the physical origin and nature of these terms is clear from the beginning. Moreover, the fact that the coupling with the radiation field is used as a starting point in SED, and not introduced *ad hoc*, guarantees that no part of this interaction will be omitted, thus avoiding the risk of arriving at misleading results such as the partly incorrect quantum calculations discussed in Refs. 13 and 14. In addition, by keeping track of a second-order effect of radiation reaction, SED arrives at a convergent expression for the Lamb shift, in contrast with Bethe's formula.

The SED theory as presented here does not enable us to calculate the external effects on the observable mass, because our starting point, Eq. (1), is already written in terms of the physical (renormalized) mass. In a second paper[8] we show that a Hamiltonian treatment of the particle-plus-field system, leading to the Braffort-Marshall equation (1), yields a formula for the mass correction δm, and that this correction does indeed depend on external conditions.

Barton[15] asserts that the long-range effects on the atom placed between the metallic plates depend wholly on the fact that the field is quantized. This was also usually said of the Casimir and Van der Waals forces before they could be shown to be a consequence of the random zero-point radiation field, without the need of quantization.[1,20] The calculations presented in this paper support the SED conclusion: The appearance of long-range effects that depend on Planck's constant $\hbar$ (or the fine-structure constant α) can be explained as a natural consequence of the coupling of matter to the random zero-point field.

This allows for a clearer explanation of the energy variations due to environmental changes: there is no need to think that the "mode structure of virtual photons" is being changed; there is, in fact, no need to think at all of "virtual photons having real effects," as is said, e.g., in Ref. 12; the zero-point radiation field is as real as the effects it produces. By the same token, there is no need to invoke "virtual transitions" to explain the contribution of nonresonant frequencies $\omega\neq\omega_{nk}$ to the Lamb shift, as is customary in quantum calculations: Instead of being a product of energy-non-conserving emission and absorp-

tion of virtual photons, for SED the Lamb shift results from the permanent interaction of the stochastic electron with background radiation of all frequencies.

Although the explicit calculations presented in this paper refer only to the free particle and to the harmonic oscillator, the meaning of the results and all our above conclusions are directly applicable to other bounded systems. When SED eventually offers a correct and complete description of the atomic problem, it will also serve to calculate the radiative energy corrections and their environmental changes more rigorously.

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