Surface-wave excitation at the interface between diffusive Kerr-like nonlinear and linear media

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Numerical calculations have been performed which show the nature of energy coupling between an incident, two-dimensional Gaussian beam at small glancing angle onto the interface between a linear medium and a nonlinear diffusive medium, and a surface wave which can exist at the interface under certain conditions. The coupling is shown to be related to the boundary conditions at the interface that are imposed upon the diffusion equation governing the nonlinearity. This surface wave is postulated as the mechanism which causes the previously unexplained experimental result of "missing" energy in the partially transmissive state of the interface.

INTRODUCTION

The problem of reflection and refraction of light at an interface between linear and nonlinear Kerr-like media has been considered by a number of authors. The first theoretical analysis by Kaplan[1] concerned plane-wave solutions. It demonstrated that for a low intensity index of refraction, slightly lower in the nonlinear medium than in the linear medium, and for a positive Kerr coefficient, at small glancing angles total internal reflection (TIR) was obtained. This continued until an angle-dependent intensity threshold was reached and an abrupt switch to a partially transmissive state occurred. As the intensity increased further, the index of refraction in the nonlinear medium also increased until at some particular intensity the indices of refraction in the two media were equal and complete transparentization of the interface was achieved. Further increase of the beam intensity resulted in an increase in the absolute value of the reflection coefficient. As the intensity was decreased below the switching threshold discussed above, however, the interface did not return to the TIR mode at the same switch point, but instead at a lower intensity, resulting in a hysteresis of the reflection coefficient.

Experiments were performed[2] by Smith *et al.* on a nonlinear interface comprised of Schott glass as the linear medium and CS_2 (a Kerr liquid) as the nonlinear medium. These experiments provided some evidence of hysteresis of the reflection coefficient, as well as some transmission, although complete transparentization of the interface was not achieved. Perhaps the most striking result of these experiments, though, was the "missing" energy. In the partially transmissive state, the reflected and transmitted beams were both observed and the sum of the two did not add up to the total input power provided to the interface.

A second series of experiments was done[3] by Smith and Tomlinson, this time using an artificially nonlinear liquid medium consisting of $\sim$800-Å spheres of SiO_2 colloidally suspended in an aqueous solution. The medium exhibited a positiver Kerr-like nonlinearity due to radiation pressure on the spheres induced by the more or less intense light beam. The second experiment possessed a large enough nonlinearity that cw operation of the interface could be undertaken. In this case, however, due to the large scattering losses resulting from the dielectric spheres, no measure of the transmitted beam was reported. Thus, it is unclear how much missing energy exists in this case.

Marcuse performed the first numerical simulation[4] of the nonlinear interface and his results seemed to indicate the presence of a surface wave excited by the Gaussian beam incident upon the interface. However, the surface wave was later shown to result from a mesh size that was too coarse. Subsequent calculations[5] using a much finer mesh by Tomlinson *et al.* indicated that for a strictly Kerr nonlinearity, no energy is coupled from the incident beam into the surface wave. This result concurs with theoretical arguments[6] by Kaplan based upon soliton theory. However, a surface wave has been shown[7] to be an allowed eigenstate at the interface between a linear and nonlinear Kerr medium for a two-dimensional (2D) bounded beam. This type of wave is a likely candidate for the path of the missing energy, as the experimental procedure used would not have permitted this energy to be observed due to a beam stop placed at the interface. The question thus arises: How does energy become coupled into the surface wave?

This paper describes calculations similar to those of Marcuse and Tomlinson *et al.* which, instead of possessing strictly Kerr nonlinearity, have a diffusive Kerr-like nonlinearity relating the intensity of the beam through a simple diffusion equation to a second quantity to which the local nonlinear index of refraction is proportional. This second quantity, called the nonlinear mechanism density, might be interpreted as the density of carriers, heat, or whatever is generated by the light intensity that results in the changing index of refraction. The product of this calculation, involving the diffusive Kerr-like nonlinearity, shows behavior that is significantly different from that obtained by Tomlinson *et al.* for the nondiffusive case, and suggests the nature of the coupling to the surface wave observed in the first experiment.

NUMERICAL TECHNIQUE

The geometry of the 2D interface problem is shown in Fig. 1. The z axis corresponds to the interface, with the positive x half-space having an index of n_0 and the negative x half-space having a nonlinear index of the form $n = n_0 - \Delta + n_{2p}p$, where Δ is a small positive constant, n_{2p} is positive, and p is the solution to Eq. (1). Problem boundaries were chosen at $x = \pm 60$ μm and at $z = -200$ μm and $z = +400$ μm in order to achieve maximum accuracy while minimizing the required computational time. The calculation technique employed to obtain the results presented here is similar to those of Marcuse and Tomlinson *et al.* and will be described in more detail in a later paper.[8] Briefly, the scalar, nonlinear wave equation was solved using the Adams-Moulton technique of Marcuse, substituting a fifth-order corrector, thus achieving higher accuracy without an increase in the required computer storage. Nondiffusive results were found to agree with those of Tomlinson *et al.*: No surface wave was evident for a strictly Kerr nonlinearity. For example, Fig. 2 shows a nondiffusive result with $n_2/\Delta = 1.0$. This is exactly the output obtained by Tomlinson *et al.* (see Fig. 9 of Ref. 5), for the same input parameters as in that paper.

The diffusion equation, of the form

$$D_0 \frac{\partial^2 p(x)}{\partial x^2} + G_0 I(x) - \frac{p(x)}{\tau} = 0 \tag{1}$$

was solved using a finite difference technique. Note that diffusion in the z direction ($\partial^2 p/\partial z^2$) was neglected in the spirit of the slowly varying envelope approximation employed in the calculation. This approximation is valid for small glancing angles ($\psi \leq 10°$). Two different sets of boundary conditions, both of which relate to particular physical systems, were employed. The first set used was $p(x = 60\ \mu\text{m}) = 0.0$ and

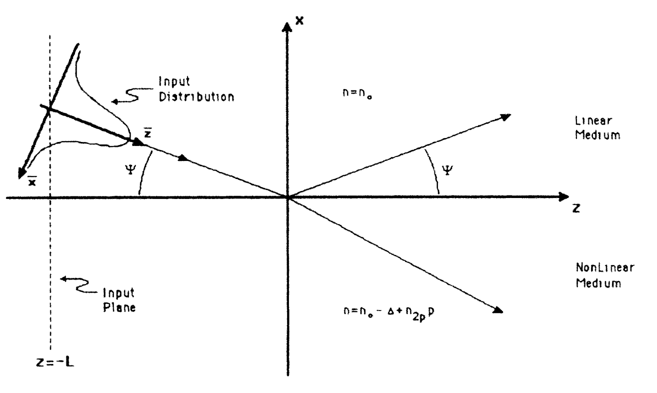

FIG. 1. Geometry of the simulation. $x > 0$ corresponds to the linear region and $x < 0$ to the nonlinear region. The interface is at $x = 0$. For all figures the Gaussian beam input is calculated to have a maximum amplitude of 1 in the absence of the interface at $x = z = 0$.

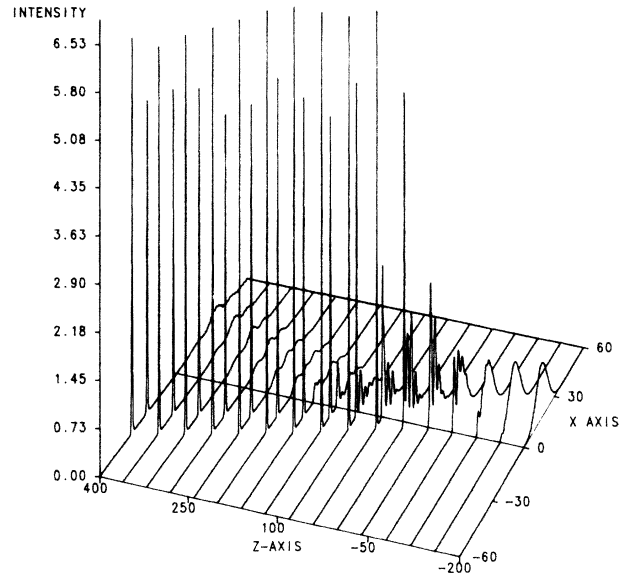

FIG. 2. Diffusionless simulation results for $n_2/\Delta = 1.0$. This figure corresponds exactly to Fig. 9 of Ref. 5 and results in two transmitted, self-focused channels. Each channel appears to emanate from an interference fringe that successfully crossed the interface.

$$\frac{\partial p(x = 0.0)}{\partial x} = 0.0 .$$

This set of boundary conditions models the case when the nonlinear mechanism p is not permitted to diffuse into the linear medium, as would be the case with a glass-liquid interface. The second set was $p(x = -60\ \mu\text{m}) = 0.0$ and $p(x = +60\ \mu\text{m}) = 0.0$. Here diffusion into the linear medium is permitted, as would be the case for a solid-solid interface where p represented, e.g., heat. Sourcing of p is not permitted for $x > 0.0$. Each of these sets of boundary conditions exhibited strikingly different behavior.

The simulation was run on a CRAY X/MP-48 computer and, for a mesh of 800 points in the x direction by 120 000 points in the z direction, took approximately 300 sec of CPU time.

RESULTS

The results of the simulations are shown in the series of Figs. 2–8. It should be noted that the x axis is expanded with respect to the z axis in order to show the detail. Figure 2 is a diffusionless case with the same input conditions as Fig. 9 of Ref. 5, illustrating agreement with the previous simulation. The other results are illustrated in the remaining figures. Parameters are given in terms of the glancing angle Ψ, the nonlinearity n_2, and the diffusion length L_D. The nonlinearity is given in units of n_2, where n_2 is the total change in the index of refraction that would occur at the interface in the absence of diffusion. The actual nonlinearity is calculated from the nonlinear mechanism density $p(x,z)$ as

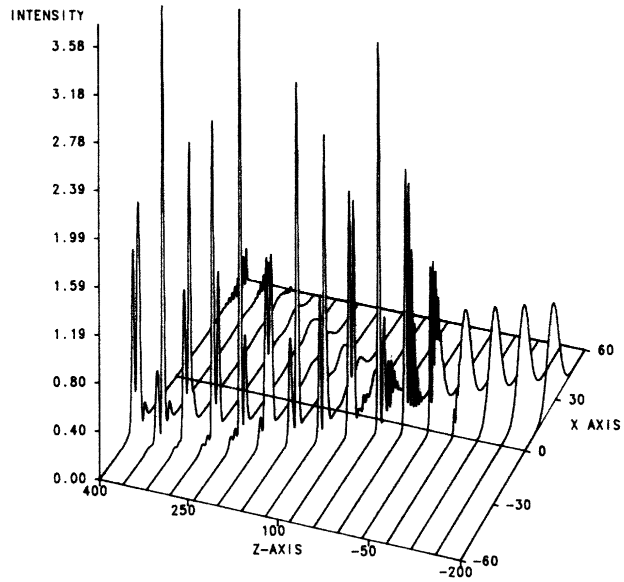

FIG. 3. Simulation results for $n_2/\Delta=5.6$, $\psi=8°$, and $L_D=10.0$. Diffusion is permitted across the interface and no surface wave is evident.

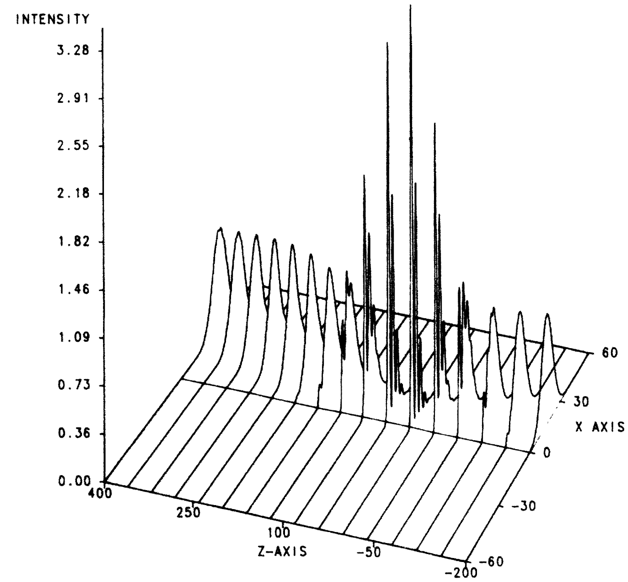

FIG. 5. Simulation results for $n_2/\Delta=8.0$, $\psi=5°$, and $L_D=10.0$. Diffusion is permitted across the interface and no surface wave is evident. Instead, total internal reflection occurs.

$n=n_0-\Delta+n_{2p}p(x,z)$, where $p(x,z)$ is related to the intensity through Eq. (1). The index offset at zero intensity is $\Delta=0.02$ for all the simulations shown. Figures 3 and 4 show simulation results for the following input parameters: $\Psi=8.0°$, $L_D=10.0$ μm, and $n_2=0.112$. Figure 3 does not exhibit evidence of any energy from the incident beam being coupled to the interface. Instead of multiple self-focused channels, however, a single beam (with continuously evolving sidelobes) appears to have formed from the interference fringes that have broken through the interface. This is a result of the lens formed by the diffusive nonlinear mechanism. The boundary conditions used here, $p(x=-60\ \mu\text{m})=p(x=+60\ \mu\text{m})=0$, correspond to the case of diffusion be-

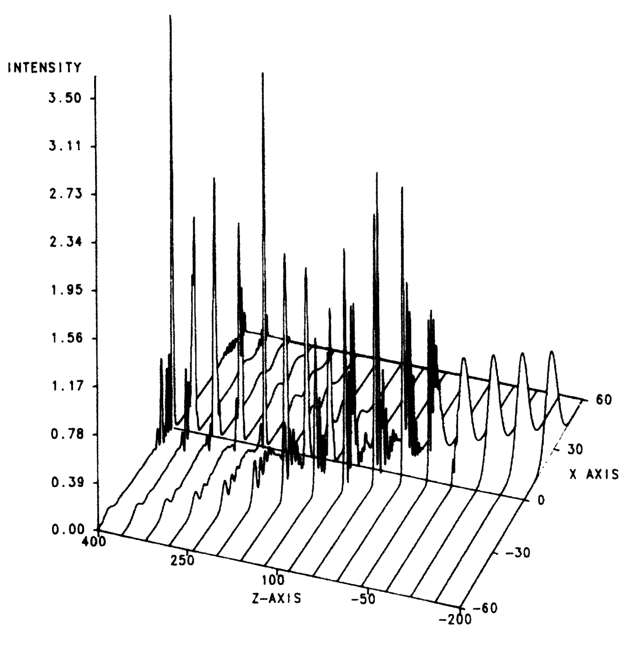

FIG. 4. Simulation results for the same input parameters as Fig. 3; however, diffusion is not permitted across the interface. In this case, a surface wave is clearly evident.

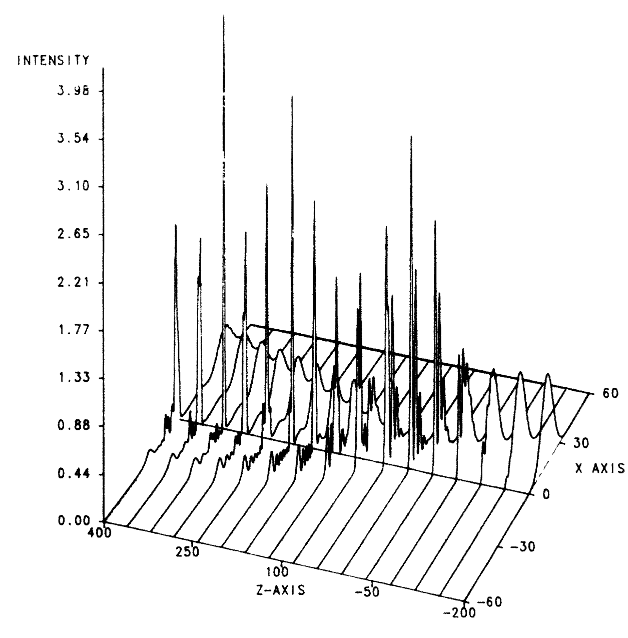

FIG. 6. Simulation results for the same input parameters as Fig. 5; however, diffusion is not permitted across the interface. In this case a surface wave is clearly evident and the transmitted energy appears to be coupling back to the interface as well.

ing permitted across the interface. Thus, the only effect of the diffusion is to smear out the nonlinear lens which is formed. Figure 4 was generated with the same input parameters as Fig. 3. The boundary conditions imposed upon this case are $p(x=-60\ \mu\text{m})=0$ and $\partial p(x=0)/\partial x=0$, corresponding to no diffusion of p across the interface. As can be readily seen, in this case the incident energy splits into three distinct channels—a transmitted beam, a reflected beam, and energy remaining coupled to the interface. The third output component is due to the lens formed by the nonlinear mechanism p trapped at the interface by the boundary condition at $x=0$. Figures 5 and 6 correspond to the following inputs: $\Psi=5$, $L_D=10.0$, and $n_2=0.16$. For this set of inputs Fig. 5 shows TIR which occurs when diffusion is permitted across the interface. Note that the self-focusing of the reflected beam due to the nonlinear Goos-Hanchen effect discussed in Ref. 5 is not nearly as extreme in this simulation as in the diffusionless case, primarily due to the smearing of the nonlinear lens by the diffusion of p. Figure 6 shows evidence of some energy coupling to the interface, reducing the reflection coefficient below $R=1.0$. In addition, the energy that is transmitted appears to couple back to the interface—again due to the lens formed as a result of the boundary condition.

Finally, although the simulation is only two dimensional and no quantitative analogies between experiment and simulation can be made, an attempt was made to use parameters nearly identical to the experimental input parameters of Ref. 2 in an attempt to draw more specific conclusions regarding the experiment. All parameters of the experiment except wavelength were scaled to a beamwidth of 10 μm and a low intensity index of refraction offset of 0.02. The wavelength was not scaled in order to maintain less than λ distance between mesh points. The glancing angle was thus $\Psi=6.625°$. The diffusion length was approximated in the CS_2 liquid as $L_D=(D\tau)^{1/2}$, where D is the self-diffusion coefficient of water and τ is the response time of the CS_2 nonlinearity. The scaled diffusion length obtained by this simple approximation was $L_D=0.51\ \mu$m. Results shown in Figs. 7 and 8 were particularly interesting in light of the comment in Ref. 2 that "... the transmitted beam appeared quite diffuse and a lens was used to collect the light over a range of $\sim 20X$ the incident beam diffraction angle around the expected output beam position." Both simulations do show an extreme divergence of the transmitted beam, apparently initiated by the inability of the various transmitted interference fringes to form a single beam because of the small diffusion length. It is expected that as the diffusion length becomes shorter than the spacing between interference fringes, the results would return to the strictly Kerr result known previously. This is because the fringes first in the nonlinear medium could not "assist" those following to pass into the nonlinear region by matching the indices of the two regions through the diffusive nonlinearity. As was shown in previous figures, however, the case in which diffusion is permitted across the interface does not result in a surface wave, whereas the case in which diffusion is not permitted across the interface does exhibit such a phenomenon.

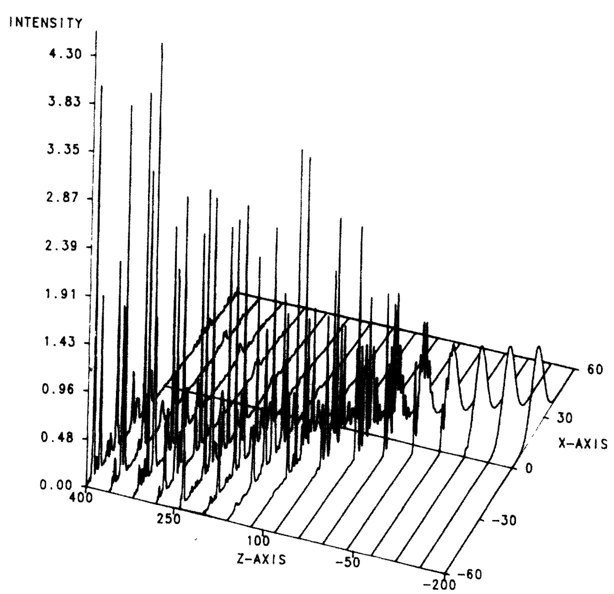

FIG. 7. Simulation results for $n_2/\Delta=6.3$, $\psi=6.625°$, and $L_D=0.51$. Diffusion is permitted across the interface and no surface wave is evident. Note, however, the extremely divergent nature of the transmitted beam.

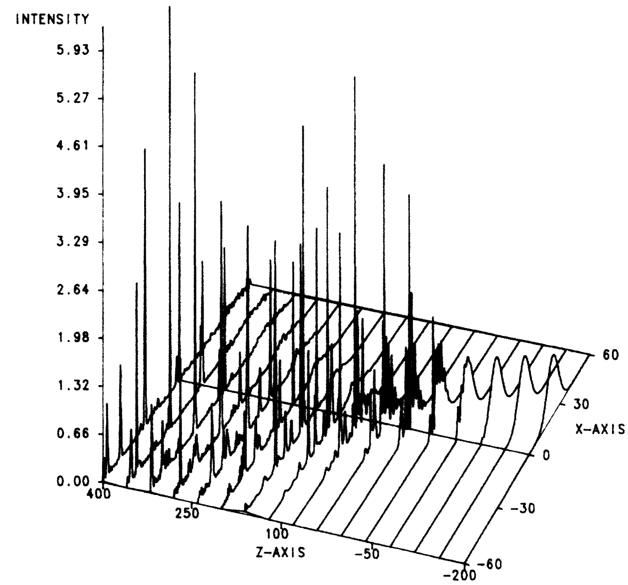

FIG. 8. Simulation results for the same input parameters as Fig. 7; however, diffusion is not permitted across the interface. In this case a surface wave is clearly evident. Also, the transmitted energy is very divergent, as in Fig. 7.

DISCUSSION

At an interface between a linear medium and a strictly Kerr nonlinear medium with a positive Kerr coefficient, no coupling can occur between a beam of light incident at small glancing angle and a surface wave which has been shown to be an eigenstate of such a system. Previous numerical simulations of the interface indicate that the transmitted beam of light breaks up into one or more self-focused channels, the exact number depending upon the magnitude of the nonlinearity. However, experimental results performed with a Schott glass-CS_2 interface exhibit the characteristic of missing energy in the partially transmissive state. As was suggested in the experimental paper, the probable path of this energy is a surface wave bound to the interface. However, no simulation or theory up to this point in time has suggested a possible mechanism for how coupling takes place.

This paper reports simulations exhibiting such coupling where the coupling comes about because of a certain set of boundary conditions imposed upon the diffusion equation governing the distribution of the nonlinear mechanism density. The boundary conditions which result in the coupling to the surface wave in the simulation are $p(x=-60\ \mu\text{m})=0.0$ and $\partial p(x=0)/\partial x=0.0$. Physically, this implies that none of the nonlinear mechanism density p is permitted to diffuse into the linear region. Simulations performed where p is permitted to diffuse across the interface exhibit no such surface wave; instead only transmission and reflection are observed.

A solid-liquid interface where the liquid is the nonlinear region is exactly the type of system where the boundary conditions supporting a surface wave would apply. Liquid cannot diffuse into the solid region, and so experiments with systems of this type should result in the type of surface wave which was observed in the simulation. The path of the experimental missing energy thus is attributed to this surface wave. The simulations also suggest the type of interface which would not exhibit such a surface wave. For example, an interface between two solid materials where the nonlinearity of the second region was thermal in nature would not give rise to this type of behavior (assuming adequate thermal coupling between the two regions) because the heat would tend to diffuse into the linear region. Although it would not change the linear region's index of refraction (because the region is assumed linear), the diffusing heat would prevent the lens effect at the interface, which promotes coupling from the incident beam to the surface wave, from occurring.

Finally, the simulations indicate that regardless of which boundary conditions are imposed on the diffusion equation, if the diffusion length is small with respect to the beam waist, the result is a very broad transmitted beam. This is attributed to the fact that the diffusion length is long enough to enhance coupling into the nonlinear region by the various interference fringes which form from the incident beam, but it is too short to aid in coupling of the various channels into a single beam through a diffuse, nonlinear lens. This result of the simulation also concurs with experimental results.

SUMMARY

Numerical simulations of an incident optical beam striking an interface between a linear medium and a diffusive Kerr-like nonlinear medium have been performed. These simulations indicate that significant differences exist between the diffusive and strictly Kerr cases. Although in the strictly Kerr case no coupling to a surface wave at the interface between the two media can occur, it is shown that in the diffusive case where no diffusion is permitted across the interface, such coupling can occur. The coupling is attributed to the nonlinear lens which develops at the interface due to the diffusion equation boundary conditions. Further investigation, both numerically and experimentally, into the evolution of the surface wave along the interface needs to be undertaken in order to understand the steady-state nature of this wave.

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