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Traveling waves in large-aspect-ratio thermosolutal convection

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Numerical simulations of two-dimensional thermosolutal convection in a long box of aspect ratio $d=16$, with free but impenetrable boundaries, reveal solutions in the form of traveling, standing, and modulated waves. These nonlinear solutions are closely related to the theoretical predictions for an unbounded layer. An unexpected feature is that the traveling waves reverse their direction of propagation in an episodic manner; this is not the case for the modulated waves. The calculations are related to recent experiments on convection in binary fluid mixtures.

Recent experiments[1-3] on convection in water-ethanol mixtures have demonstrated the existence of convection in the form of a horizontally translating pattern of rolls. More complex translating patterns have also been observed.[1] These states are not transient, and persist for the duration of the experiment. We refer to them as *traveling waves.* It is probable that such waves were also present in experiments[4] with a bulk mixture of normal ^{3}He-^{4}He, although visualization was not possible.

Two-dimensional traveling waves were discovered independently[5] in thermosolutal convection, and a theory developed to account for the numerically generated solutions of the partial differential equations in a plane layer with periodic boundary conditions in the horizontal.[6] The theory predicts [5-7] that, in both thermosolutal convection and convection in binary fluid mixtures, small-amplitude oscillatory convection in the form of standing waves is unstable to traveling waves. Convection in a binary fluid mixture and thermosolutal convection are closely related.[8] In the former a stabilizing concentration gradient develops in response to an applied destabilizing temperature gradient by means of the Soret effect; in the latter it is maintained at the boundaries.

In a previous paper,[9] we have described both theoretically and numerically the transitions between traveling waves (TW's), standing waves (SW's), and steady states (SS's) in two-dimensional thermosolutal convection with periodic boundary conditions in the horizontal, and obtained evidence for the presence of modulated waves (MW's) as well as chaotic waves (CW's). Motivated by the experiments, we present here numerical results on two-dimensional thermosolutal convection in boxes with a larger but finite aspect ratio and impenetrable insulating sidewalls. The boundaries are assumed to be stress-free, and the temperature and solute concentration fixed at the top and bottom. The system is described by the nondimensionalized equations[9]

$$\frac{1}{\sigma}[\nabla^2\Psi_t+J(\Psi,\nabla^2\Psi)]=R_T T_x-R_S S_x+\nabla^4\Psi \; , \tag{1a}$$

$$T_t+J(\Psi,T)=\nabla^2 T \; , \tag{1b}$$

$$S_t+J(\Psi,S)=\tau\nabla^2 S \; , \tag{1c}$$

with x and z the horizontal and vertical coordinates and t the time, subject to the boundary conditions

$$\Psi=\Psi_{zz}=0,\; T=S=1 \text{ on } z=0 \; , \tag{1d}$$

$$\Psi=\Psi_{zz}=0,\; T=S=0 \text{ on } z=1 \; , \tag{1e}$$

$$\Psi=\Psi_{zz}=T_x=S_x=0 \text{ on } x=0,d \; . \tag{1f}$$

Here Ψ, T, and S are the stream function, the temperature, and the solute concentration, respectively, and $J(f,g)\equiv f_x g_z-f_z g_x$. The system is specified by five constants: the thermal and solutal Rayleigh numbers R_T, R_S, the Prandtl and Lewis numbers σ, τ, and the aspect ratio d. All the calculations reported here use $\sigma=1$, $\tau=10^{-1/2}$, $d=16$, and the vertical thermal diffusion time as the unit of time.

Equations (1) were solved numerically by finite-difference techniques,[10] typically employing 256 mesh points in the horizontal and 16 in the vertical. Some of the solutions were obtained by applying small random perturbations to T and S in a quiescent state, and then evolving the growing disturbances to a mature state. Other simulations used nearby mature solutions as initial conditions.

When $R_T=0.95\times10^4$, $R_S=10^4$, the initial instability evolves into a pattern of rolls translating either to the left or to the right. The rolls originate in a nearly quiescent region at one end, and travel to the other where reflection from the wall creates a small region of SW. Although such TW resemble those observed in the binary fluid experiments,[1-3] in our simulations a TW may change its direction of propagation after a time interval corresponding to about four or five wave transit times across the box. A transition from left TW to right TW is shown in Fig. 1(a), revealing that the region on the left becomes a pronounced source of right TW which overwhelm the original incoming waves. Eventually a small region of SW is established on the right, as can be seen in the upper panels in Fig. 1(a). The whole cycle generally repeats many times during the course of such a simulation. During the changeover there are pronounced variations in the wavelengths of the rolls with x and t, with consequent variations in phase speeds that yield the fanlike patterns evident in Fig. 1(a).

A time trace of the averaged vertical heat transport at midlayer, as measured by the Nusselt number N_T, is shown in Fig. 1(b). The N_T attains maxima during periods corresponding to nearly pure traveling waves, labeled by L and R to indicate their sense. Superimposed on the slow trends are many rapid fluctuations in N_T, with amplitudes of about 0.3 involving a broadband of frequencies ω centered at 20.1, corresponding to the appearance and disappearance of the rolls at the sidewalls and the associated standing wave there. The number of traveling rolls decreases slowly with time during each cycle as the quiescent region expands, with the principal minima in N_T realized when the smallest number of traveling rolls occupy the box. The reversal in the sense of the traveling waves is initiated at this stage, although it may not succeed, as indicated in Fig. 1(b). We have examined four TW solutions at nearby parameter values and have found all of them to be of this episodic character. The implication is that the layer at such parameters is unable to support simple traveling waves of a given direction.

During periods of reversal both waves occur simultaneously, and many persist for a considerable period in the remarkable chevron pattern shown in Fig. 2 for $R_T = R_S = 10^4$. Here rolls appear at both ends, travel toward the center at nearly constant speed, and annihilate there. The solution appears to be composed of two oppositely directed TW, both with $d = 8$. Such a pattern is eventually unstable to left or right TW filling the entire box.

Figure 3 shows a modulated traveling wave (MW) realized at $R_T = 0.85 \times 10^4$, $R_S = 10^4$. As the rolls travel to the right, their amplitudes show a distinct modulation and they translate more rapidly at small amplitudes. These waves are analogous to the modulated waves predicted theoretically[7,9] in the case of periodic boundary conditions in the horizontal.[11] We have studied six parameter values where MW are realized; in all these cases the MW continue to translate in the same direction for the duration of the lengthy simulations, whether to the right or to the left. Figure 3 also reveals that SW are present in the regions close to both sidewalls.

The temporal power spectrum of N_T for this MW solution is shown in Fig. 4(b), together with those of two solutions at adjacent values of R_T. The MW of Fig. 3 possesses two frequencies, $\omega_1 = 24.6$, corresponding to the translation of the rolls, and $\omega_2 = 49.2$, corresponding to their temporal modulation. The frequencies are locked in the ratio 1:2. The accompanying Figs. 4(a) and 4(c) for

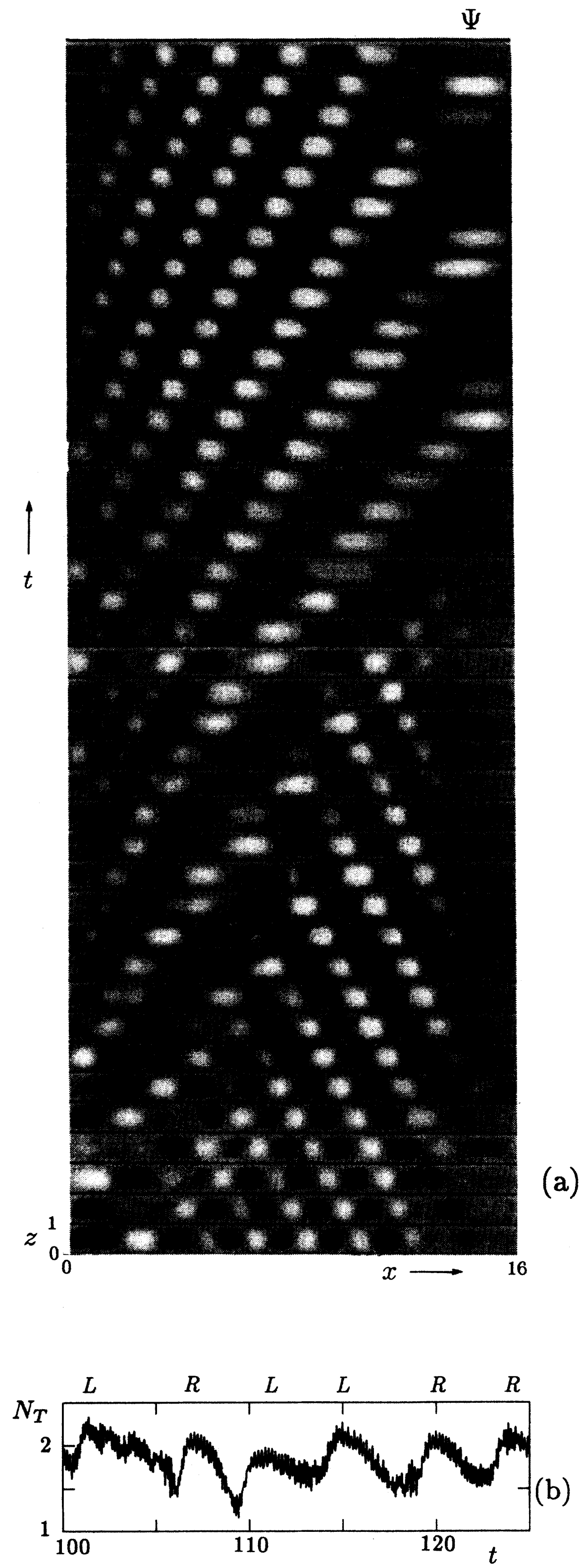

FIG. 1. (a) Transition from left TW to right TW shown in spatial contour plots of the stream function Ψ at 40 successive times, with t advancing upward, spanning $t = 104.57$–107.24 in increments of 0.068. The light areas indicate counterclockwise motion. (b) Time trace of associated N_T at midlayer depth.

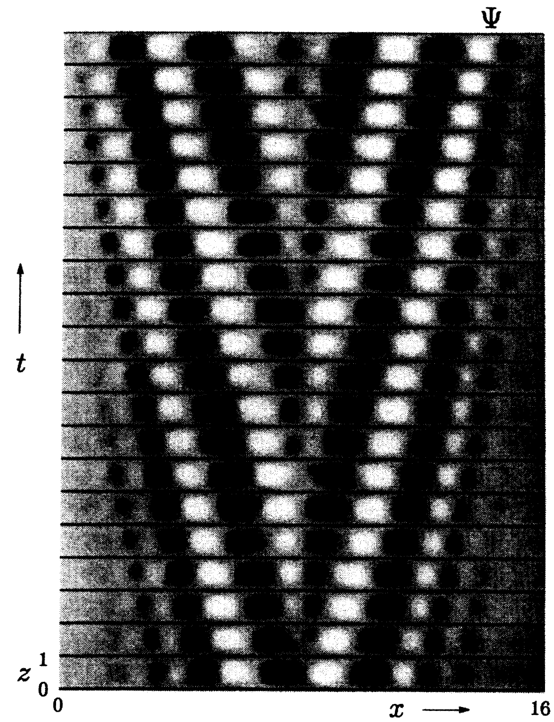

FIG. 2. Contour plots of Ψ at 20 successive times for solution displaying right and left TW which converge at the center, spanning $t = 16.70-17.33$ in increments of 0.033.

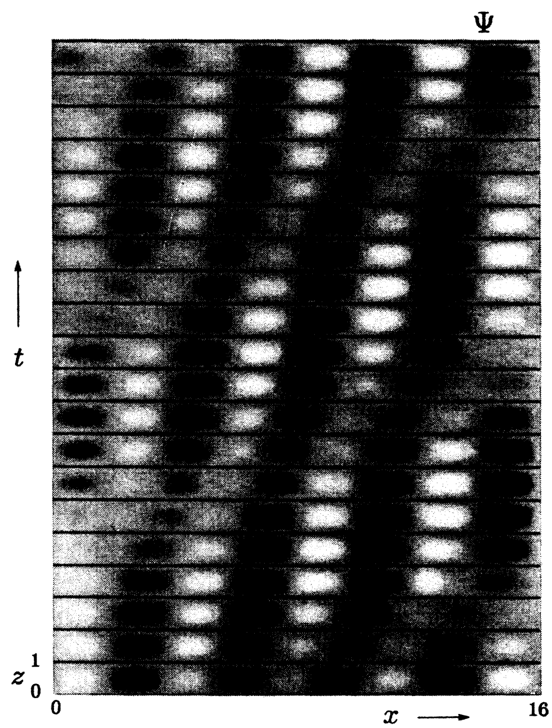

FIG. 3. Contour plots of Ψ for solution possessing modulated traveling waves (MW), spanning $t = 92.14-92.49$ in increments of 0.018.

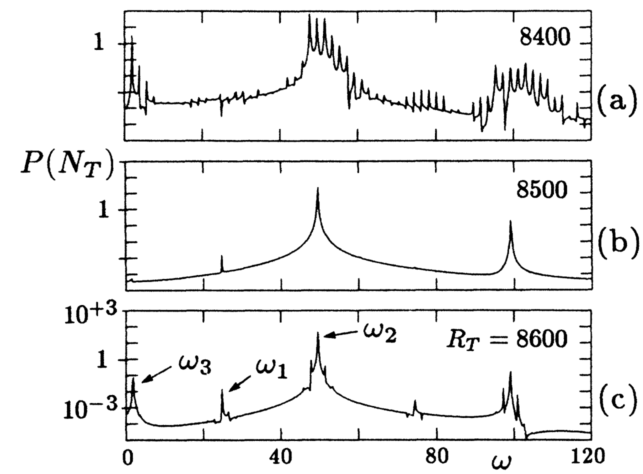

FIG. 4. Temporal power spectra of N_T for MW solutions at three adjacent values of R_T as indicated.

MW at adjacent values of R_T show the presence of a third, much lower frequency ω_3 which is locked to ω_1 in the ratio 1:13. All other spectral features are combinations of these three basic frequencies.

Finally, for $R_T = 1.2 \times 10^4$, $R_S = 1.46 \times 10^4$, we have located standing waves that occupy the full width of the box. For this solution, the rolls reverse their sense periodically in time and do not propagate. The rolls possess uniform wavelengths across the box. With periodic boundary conditions and $d = 3$ this solution is unstable to traveling waves.[9]

The results described above show that when the aspect ratio d is sufficiently large the system exhibits behavior characteristic of an unbounded system. Specifically, the three classes of solutions predicted with periodic boundary conditions, TW, SW, and MW, as well as SS, have clear counterparts in the finite system with impenetrable sidewalls. When d is too small, however, only SW and SS are found.[12] The sidewalls introduce relatively little spatial modulation, although they can stabilize SW with respect to TW disturbances. The one unexpected feature of the calculations is the episodic reversal of the direction of propagation of the traveling waves. This evolution is clearly aperiodic in detail, and the reversals occur over intervals considerably shorter than the horizontal diffusion time that might be expected. Such behavior has not been reported in the binary fluid experiments, nor has it been predicted theoretically.[13] A more complete account of our results will be given elsewhere.

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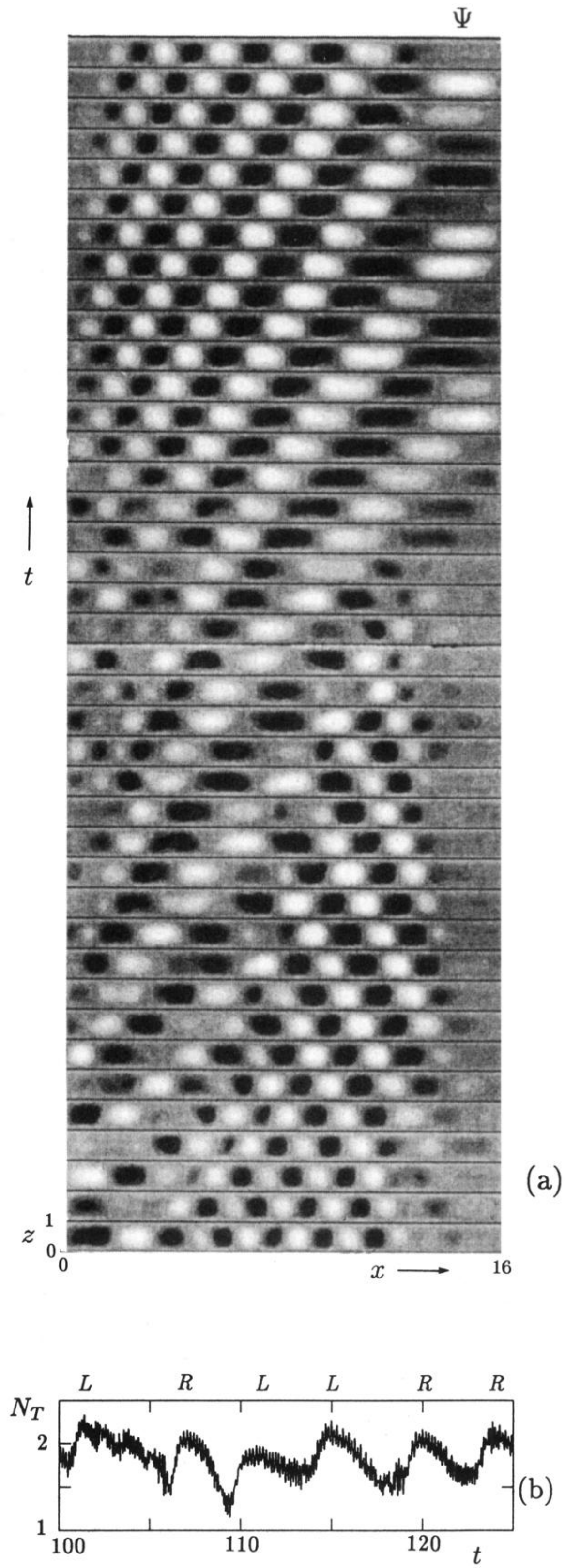

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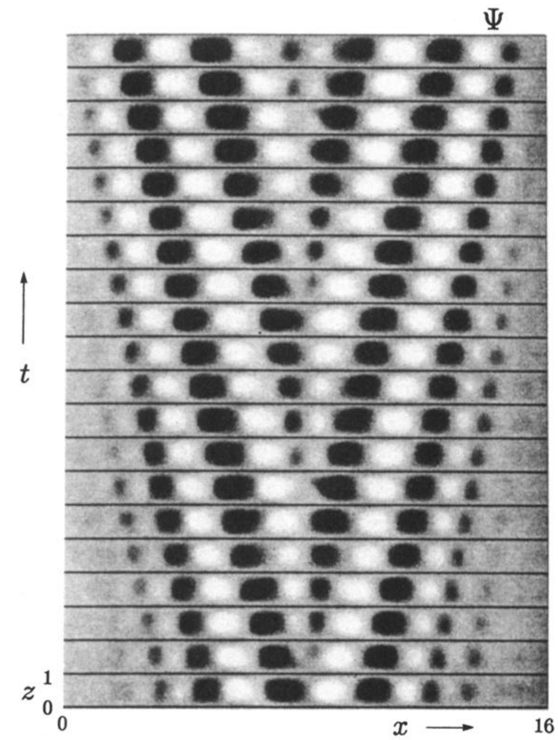

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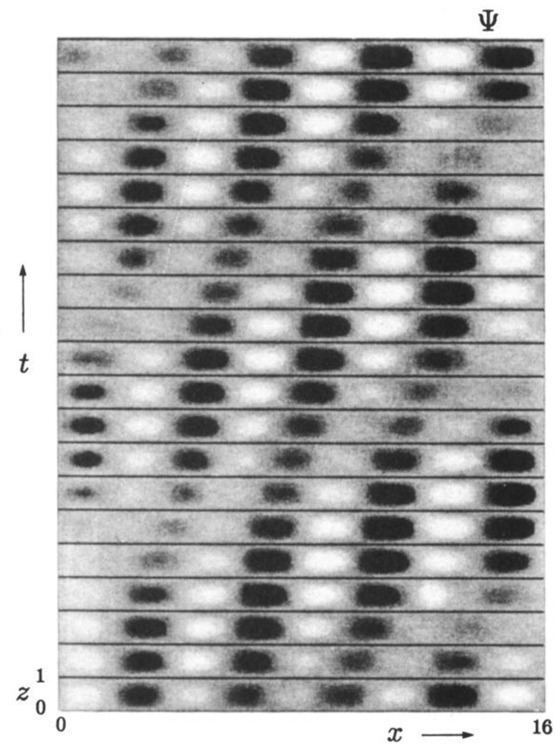

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