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Laser bandwidth effects on squeezing in intracavity parametric oscillation

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Finite input laser bandwidths alter the output quantum statistics in an intracavity parametric oscillator. We show that in an above-threshold experiment, the obtainable squeezing can be greatly reduced by these laser fluctuations. The effect is not eliminated just by reducing the laser bandwidth to less than the cavity bandwidth. Instead, the cavity/laser bandwidth ratio must be improved to the order of the intracavity photon number. Thus, highly stabilized lasers are essential in above-threshold squeezing experiments.

The discovery of squeezing[1] in four-wave mixing[2] has led to a large increase in activity in this field. More recently, attention has focused on the optical parametric oscillator (OPO) because of its low-noise characteristics.[3] Work with OPO's has led to substantial noise reduction[4] below the vacuum level in one quadrature of the radiation field. This is compatible with Heisenberg's uncertainty principle, since the other quadrature has increased fluctuations—but it allows more precise measurements in the quadrature of interest.[5]

Spectroscopic or interferometric applications are the most obvious ones for this new form of radiation. However, the squeezed radiation produced so far has been restricted to the squeezed vacuum. This type of output is produced by an OPO below threshold. It would be useful to produce OPO radiation above threshold, which results in squeezed coherent radiation with reduced fluctuations in phase. This has direct applications in ultrasensitive interferometry. Radiation of this type has yet to be found in experiment.

In this Rapid Communication we find that the input laser bandwidth has a large and hitherto unexpected effect on the extent of squeezing obtainable in these above threshold types of experiment. The input-laser-bandwidth corrections are shown to scale with the intracavity photon number. These, therefore, have dramatically increased relative size compared to quantum fluctuation effects, when the OPO is operated above threshold. For this reason, laser stabilization is extremely significant in obtaining squeezed coherent radiation. We also compare the different regions where near perfect squeezing occurs. This can be either near threshold or well above threshold.

The starting point of the calculation is the Hamiltonian for the OPO[6] in a doubly resonant interferometer:

$$\begin{aligned} H = {} & \hbar\omega_1 a_1^\dagger a_1 + \hbar\omega_2 a_2^\dagger a_2 + i\hbar\left(\frac{g}{2}\right)(a_1^{\dagger 2} a_2 - a_1^2 a_2^\dagger) \\ & + i\hbar[E(t)a_2^\dagger - E^*(t)a_2] \\ & + \Gamma_1^\dagger a_1 + \Gamma_1 a_1^\dagger + \Gamma_2^\dagger a_2 + \Gamma_2 a_2^\dagger \,. \end{aligned} \tag{1}$$

This Hamiltonian describes a degenerate optical parametric oscillator, with an input field with positive frequency part proportional to $E(t)$, that is nearly resonant to a pump mode at ω_2. This is coupled through a nonlinearity coefficient g to a signal mode at half the frequency $(\omega_1 = \omega_2/2)$. Both modes are damped through the cavity losses, described by the reservoirs Γ_1 and Γ_2. We suppose the input laser is intensity stabilized, but experiences phase diffusion.[7] This results in a finite bandwidth around ω_2, so that

$$\begin{aligned} & E(t) = E e^{i[\omega_2 t + \phi(t)]} \,, \\ & \langle \dot\phi(t)\dot\phi(t')\rangle = \gamma\delta(t-t') \,. \end{aligned} \tag{2}$$

The input laser has a Lorentzian spectrum with a FWHM bandwidth of $(\gamma/2\pi)$ Hz. While lasers generally have a more complex spectrum than that described by Eq. (2), these spectral effects can also be included in the general formalism of Eq. (1). In general, the correlations in phase will have a finite relaxation time, resulting in a spectrum that cuts off more rapidly than Lorentzian at large relative frequencies. The case of a Lorentzian spectrum is, therefore, a worst-case model. It is reasonable to expect that lasers with reduced spectral components at large relative detunings would have reduced overall effects on squeezing.

Our calculation proceeds using a master equation followed by a Fokker-Planck equation in a generalized positive P representation.[8] This permits the introduction of a stochastic differential equation with Stratonovich delta-correlation noise terms $\zeta(t),\zeta^\dagger(t)$, and equivalent moments to the full operator moments. The equations are

$$\begin{aligned} & \dot\alpha_1 = g\alpha_1^\dagger \alpha_2 - \kappa_1\alpha_1 + (g\alpha_2)^{1/2}\zeta(t) + \frac{i}{2}\dot\phi(t)\alpha_1 \,, \\ & \dot\alpha_2 = E - \frac{g}{2}\alpha_1^2 - \kappa_2\alpha_2 + i\dot\phi(t)\alpha_2 \,, \end{aligned} \tag{3}$$

where $\langle\zeta(t)\zeta(t')\rangle = \delta(t-t')$.

We have used a rotating frame so that the output field-operator expectation values near the signal frequency are related to α_1 through the relations[9]

$$\langle\hat\Phi_{(\text{out})}(t)\rangle = (2\kappa_1)^{1/2}\langle\alpha_1(t)\exp[-i\omega_1 t - i\phi(t)/2]\rangle \,. \tag{4}$$

Here $\hat\Phi$ is the positive-frequency photon-flux operator at the output mirror of the cavity. Note that Eq. (4) will be modified in cases of substantial intracavity absorption or other losses in non-detector-coupled ports. We will first treat the case of a rapidly decaying pump so that $\kappa_2 \gg \kappa_1$. In this limit, α_2 can be adiabatically eliminated, and to lowest order in (κ_1/κ_2)

$$\dot{\alpha}_1 = \frac{g\alpha_1^\dagger}{\kappa_2}\left[E - \frac{g}{2}\alpha_1^2\right] - \kappa_1\alpha_1 + \left[\frac{g}{\kappa_2}\left[E - \frac{g}{2}\alpha_1^2\right]\right]^{1/2}\zeta(t) + \frac{i}{2}\dot{\phi}(t)\alpha_1 \ . \tag{5}$$

In addition to Eq. (5), there is a corresponding equation for $\alpha_1^\dagger$. This is obtained on replacing i by $-i$, and $\alpha_1(t),\zeta(t)$ by $\alpha_1^\dagger(t),\zeta^+(t)$ everywhere in Eq. (5). The quantum-noise term $\zeta^+(t)$ is uncorrelated with $\zeta(t)$, so that $\alpha_1(t)$ and $\alpha_1^\dagger(t)$ are not complex conjugate except in the mean.

While Eq. (5) permits an exact calculation of all the quantum statistics, it is useful to transform to generalized phase and intensity variables ϕ_j, I_j where

$$I_j = (\alpha_j\alpha_j^\dagger) \ ,$$
$$\phi_j = \frac{1}{2i}\ln\left[\frac{\alpha_j^\dagger}{\alpha_j}\right] \ . \tag{6}$$

The linearized equations above threshold $(E > E_T)$ are

$$\delta\dot{I}_1 = -\kappa_I\delta I_1 + (EI_1^0)^{1/2}[\zeta(t) + \zeta^+(t)] \ ,$$
$$\delta\dot{\phi}_1 = -\kappa_\phi\phi_1 + \frac{i}{2}\left[\frac{E}{I_1^0}\right]^{1/2}[\zeta(t) - \zeta^+(t)] - \tfrac{1}{2}\dot{\phi}(t) \ , \tag{7}$$

where

$$I_1 = I_1^0 + \delta I_1, \ \ E_T = \kappa_1\kappa_2/g,$$
$$\delta\phi_1 = \phi_1, \ \ \kappa_I = 2\kappa_1(E/E_T - 1),$$
$$I_1^0 = 2(E - E_T)/g, \ \ \kappa_\phi = 2\kappa_1(E/E_T).$$

Here the intensity fluctuations show critical slowing down near the threshold input amplitude of $E = E_T$. The above equations are only valid sufficiently far above the critical region that the intensity fluctuations δI_1 are less than the coherent intensity I_1^0. On solving Eq. (7) for its time evolution, we obtain

$$\delta I_1(t) = (\kappa_1 I_1^0)^{1/2}\int_{-\infty}^{t}\exp[-\kappa_I(t-t')]\zeta_I(t')dt' \ ,$$
$$\delta\phi_1(t) = \tfrac{1}{2}(\kappa_1/I_1^0)^{1/2}\int_{-\infty}^{t}\exp[-\kappa_\phi(t-t')]\zeta_\phi(t')dt' \ , \tag{8}$$

where

$$\zeta_I(t) = \zeta(t) + \zeta^+(t),$$
$$\zeta_\phi(t) = i[\zeta(t) - \zeta^+(t)] - [I_1^0/\kappa_1]^{1/2}\dot{\phi}(t).$$

The fluctuations of $\delta\phi_1$, δI_1 relative to the coherent state level give information on the degree of squeezing. In this normally ordered representation, fluctuations below the vacuum level correspond to a negative variance in $\delta\phi$ or δI where

$$\langle\delta I_1(t)\delta I_1(t+\tau)\rangle = (\kappa_1 I_1^0)\exp(-\kappa_I|\tau|)/\kappa_I \ ,$$
$$\langle\delta\phi_1(t)\delta\phi_1(t+\tau)\rangle = \frac{(\gamma/2 - \kappa_1/I_1^0)\exp(-\kappa_\phi|\tau|)}{4\kappa_\phi} \ . \tag{9}$$

We see that the fluctuations in the intensity are enhanced above the vacuum level, while the phase fluctuations are decreased below the vacuum level. Squeezing is clearly obtained only if $I_1^0\gamma < 2\kappa_1$, as the input bandwidth γ always tends to increase the fluctuations toward or above the vacuum level.

To detect the squeezing, a local oscillator is used with frequency ω_1 and phase $[\theta + \phi(t)/2]$. This cancels the coherent rotating-frame effects. The intensity correlation spectrum, including shot-noise or vacuum fluctuation effects observed in local oscillator measurements,[2,4] is

$$V(\theta,\omega) = 1 + 4\kappa_1^2\cos^2\theta/(\kappa_I^2 + \omega^2) + 2\kappa_1\sin^2\theta(\gamma I_1^0 - 2\kappa_1)/(\kappa_\phi^2 + \omega^2) \ , \tag{10}$$

where

$$V(\theta,\omega) \equiv 1 + 2\kappa_1\int[\cos^2\theta\langle\delta I_1(t)\delta I_1(0)\rangle/I_1^0 + 4I_1^0\sin^2\theta\langle\delta\phi_1(t)\delta\phi_1(0)\rangle]e^{i\omega t}dt \ .$$

The observation of frequency-domain squeezing requires a phase angle of $\theta = \pi/2$, together with sufficiently low phase noise in the input laser. In this case of $\kappa_2 \gg \kappa_1$, perfect squeezing $[V(\theta,\omega) \to 0]$ is only obtainable near the critical region $(E \approx E_T)$. This corresponds to a region of anomalously large intensity fluctuations.

It would be more desirable to obtain squeezing outside the critical region. Therefore, we return to the more general case of arbitrary $r = \kappa_2/\kappa_1$. The result for the intensity correlation spectrum is then

$$V(\theta,\omega) = 1 + 4S_I(\omega)\cos^2\theta + 4S_\phi(\omega)\sin^2\theta \ ,$$

where

$$S_I(\omega) = \frac{r^2 + \bar{\omega}^2}{[2r(P-1) - \bar{\omega}^2]^2 + r^2\bar{\omega}^2} \ ,$$
$$S_\phi(\omega) = \frac{-(r^2 + \bar{\omega}^2) + (I_1^0\gamma/2\kappa_1)[(2+r)^2 + \bar{\omega}^2]}{(2rP - \bar{\omega}^2)^2 + \bar{\omega}^2(2+r)^2} \ , \tag{11}$$
$$\bar{\omega} = \omega/\kappa_1, \ \ P = E/E_T \ .$$

This gives near perfect squeezing $[V(\theta,\omega) \to 0]$ of phase fluctuations (i.e., $\theta = \pi/2$) in two different limits.[10] The first limit, as before, is near threshold $(E \gtrsim E_T)$ and at low frequencies $\bar{\omega} \sim 0$, where

$$V(\pi/2,0) = I_1^0 \gamma [1/(2\kappa_1) + 2/\kappa_2 + 2\kappa_1/\kappa_2^2] \ .$$

Thus, we require $\gamma I_1^0 \ll 2\kappa_1$, $\kappa_2/2$, and $\kappa_2^2/2\kappa_1$ for good squeezing. The second limit of excellent squeezing is for $r \ll 1$ and $\bar{\omega} \sim \pm\sqrt{2rP}$, where

$$V(\pi/2,\omega) = I_1^0 \gamma [1/(2\kappa_1) + 1/(\kappa_2 P)] \ .$$

Thus, we require $\gamma I_1^0 \ll 2\kappa_1$, $\kappa_2/2$, and $\kappa_2^2/2\kappa_1$ for good squeezing. The second limit of excellent squeezing is for $r \ll 1$ and $\bar{\omega} \sim \pm\sqrt{2rP}$, where

In general, squeezing near frequency $\bar{\omega}$ is only achievable if

$$I_1^0 \gamma < 2\kappa_1 \left(\frac{r^2 + \bar{\omega}^2}{(2+r)^2 + \bar{\omega}^2} \right) \ . \tag{12}$$

As I_1^0 corresponds to the photon number in the signal mode, this implies that the input linewidth must be less than the cavity linewidth by an increasingly large factor, as the intensity I_1^0 is increased above threshold.

The squeezing defined here is measured relative to a local oscillator whose phase is identical to the input field. Another situation of experimental interest is the intensity spectrum. In this case, the field operators must be transformed back to a nonrotating laboratory frame, which reintroduces the original laser linewidth into the observed spectrum.[11] The resulting spectrum will be given elsewhere. We note that, in general, local-oscillator measurements are both more sensitive to small fluctuations and less likely to introduce external phase noise.

Given the robustness towards phase noise normally expected in local-oscillator measurements, the calculation given here has unexpected results. In fact, there are excellent physical explanations for these effects. The interferometer itself is a storage device which remembers the phase history of the input laser. Thus, the averaged phase in the interferometer field is different from the input laser, whose current phase drives the local oscillator. This small difference is then effectively amplified by the number of photons in the signal field. The resulting additional noise in the detector is relatively large when compared to the small quantum fluctuations that are being reduced by the nonlinear squeezing interactions.

In summary, laser bandwidth reduction is of central importance in obtaining above threshold squeezing. Given current laser stabilization techniques which permit frequency stabilization of up to 10^{-15} of the input frequency,[12] this is an achievable requirement. With laser stabilization of this type, an intense coherent signal with squeezed phase is possible. This can be achieved either at low frequencies near threshold or at a sideband frequency well-above threshold for an interferometer having low pump losses. The resulting output field has potential applications in a variety of interferometer and low noise detection experiments.

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