PHYSICAL REVIEW A VOLUME 37, NUMBER 1 JANUARY 1, 1988

Ion-channel guiding in a steady-state free-electron laser

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(Received 16 March 1987)

Ion-channel guiding of an intense relativistic electron beam in a microwave (cm–mm) steady-state free-electron laser (FEL) is proposed. By averaging equations of motion over the high-frequency variation of the radio-frequency field, the expulsion mechanism of plasma electrons is examined and a criterion to allow development of an ion channel in a steady-state FEL is derived. Key issues on FEL operation induced by ion-channel guiding are discussed.

When an electron beam is injected into an ionized channel, the beam space charge ejects plasma electrons leaving an ion core, and the beam electrons are electrostatically attracted to the ion channel. Such ion-channel guiding has been used for the transport of the relativistic electron beam (REB); particularly, Martin *et al.*[1] reported first use of a uv-laser ionized channel for REB guiding, focusing, and damping in beam-transport experiments over distances of several meters. This technique has been used to guide a beam 95 m through the Advanced Test Accelerator[2] (ATA) without the use of magnetic guide fields. Recently this has been demonstrated to function even in the injector diode.[3] In this paper we propose an important extension of this technique in which an ionized channel is introduced into a "steady-state" free-electron laser[4] (FEL) motivated by its use in a two-beam accelerator.[5] This approach eliminates the need for conventional quadrupole and solenoid focusing magnets. Introduction of laser guiding into the steady-state FEL will make a capital and running cost less expensive because there is no power consumption and the induction unit without a solenoid employs relatively small scale ferrite cores to allow its easy assembling. The transverse instabilities of the driving beam, e.g., the beam breakup instability, in a steady-state FEL which consists of FEL portions and inductive acceleration units, is potentially troublesome. When normal quadrupole magnets are employed for focusing, the beam breakup instability can be estimated as follows: Once the beam breakup instability occurs, the transverse beam motion grows exponentially with a scale length $L_{\rm BBU}$ given by[6]

$$L_{\rm BBU}=\left[\frac{17\,{\rm kA}}{I_B}\right]\left[\frac{L_g}{\omega_0 Z_\perp}\right]2k_\beta\gamma\ ,$$

where $2\pi/k_\beta$ is the betatron wavelength, γ the beam energy in the electron-rest-mass unit. I_B beam current, L_g induction-gap spacing, ω_0 angular frequency of beam breakup mode of problem, and $Z_\perp$ transverse impedance of induction gap. For the typical parameters of $k_\beta=2\pi$ $\rm m^{-1}$, $L_g=2$ m, $\gamma=16$, and $\omega_0 Z_\perp=0.4\ {\rm cm}^{-1}$,[7] numerical evaluation yields $L_{\rm BBU}=85$ m. This value is crucial for a steady-state FEL. If ion-channel guiding is employed instead of quadrupole magnets, the beam breakup instability will be suppressed by Landau damping[8] resulting from the variation in betatron frequency introduced by the nonlinearities of the ion channel. In fact, the ATA experiments demonstrated that ion-channel guiding is very effective for suppression of the transverse beam breakup mode in an induction accelerator. From these situations ion-channel guiding of REB in a steady-state FEL is very attractive now.

Unlike an induction accelerator such as the ATA, a steady-state FEL is associated with considerably strong wiggler fields (of several kG) and quite strong radio-frequency (rf) fields (of several hundred $\rm MV\,m^{-1}$). Motion of plasma electrons in such strong external fields may be characterized by magnetic trapping and an increase in relativistic mass due to rapid acceleration. In addition, plasma electrons are remarkably affected by the space-charge forces of an electron beam. It is not clear whether or not in these circumstances plasma electrons can be ejected by an injected beam in the desired time period (nsec) which allows accumulation of a sufficient amount of ion charge for partial charge neutralization of the beam body. In this paper behaviors of plasma electrons in the FEL portion are carefully explored and a critical condition, which allows an ion channel to form in the above sense, is derived. Key issues on FEL operation, e.g., rf phase variation and rf breakdown, induced by ion-channel guiding, are discussed.

All of the present analyses were derived subject to the following assumptions.

(i) The preionized channel is produced by laser induced ionization.

(ii) The channel radius should be larger than the spacial size swept by a quivering electron beam including beam size itself.

(iii) The electron-beam density should be greater than the channel-ionization density to assume complete plasma-electron expulsion and to prevent a two-stream instability.[9]

(iv) Relativistic effects are taken into account to order v^2 in the exact kinetic equations and to order v in drift motion appearing after averaging over the high-frequency variation of the rf fields.

We choose a Cartesian coordinate system so that an electron beam is traveling in the z direction and the planar wiggler field is in the y direction. The variation of the y component of the wiggler field along the z direction can be written as $B_W = B_0 \cos(2\pi z/\lambda_W)$, where B_0 is the amplitude and λ_W is the wavelength of the wiggler. The accompanying radiation fields of the TE_{01} mode are written in the linearized forms

$$E_x = E_0 \cos(\omega t - k_g z) ,$$
$$B_y = \frac{E_0 k_g}{\omega} \cos(\omega t - k_g z) , \tag{1}$$
$$E_y = E_z = B_x = B_z = 0 ,$$

where E_0 is the amplitude of the x component of the radiation field, ω and k_g the angular frequency and wave number in the waveguide. Assuming a Gaussian distribution of the electron-beam charge, that is, the linearized space-charge fields, the radial electric field and azimuthal magnetic field are given by

$$E_r = 2 \left[\frac{I_B}{I_0} \right] \left[\frac{mc^2}{e} \right] \frac{r}{a^2}, \quad B_\theta = \frac{E_r}{c} , \tag{2}$$

where I_B is the beam current, $I_0 = 17$ kA, a the root-mean-square radius of the beam, r the distance from the beam center, m the electron mass, e the unit charge, and c the speed of light.

Then, a plasma electron ejected from the beam core is governed by the following relativistic equations to order v^2:

$$\dot u = g \left[\left[1 - \tfrac{3}{2}u^2 - \frac{v^2}{2} - \frac{w^2}{2} \right] [Gx + E_0 \cos(\omega t - k_g z)] - uvGy - wc \left[B_0 \cos(k_W z) + \frac{E_0 k_g}{\omega} \cos(\omega t - k_g z) + \frac{G}{c}x \right] \right] , \tag{3a}$$

$$\dot v = g \left[\left[1 - w - \frac{u^2}{2} - \frac{3}{2}v^2 - \frac{w^2}{2} \right] Gy - uvGx \right] , \tag{3b}$$

$$\dot w = g \left[-uv[Gx + E_0 \cos(\omega t - k_g z)] + v(1-w)Gy + uc \left[B_0 \cos(k_W z) + \frac{E_0 k_g}{\omega} \cos(\omega t - k_g z) + \frac{G}{c}x \right] \right] , \tag{3c}$$

where u,v,w are the velocity normalized by c in each direction, the dots denote differentiation with respect to t, $g = (-e/mc)$, $G = 2(I_B/I_0)(mc^2/e)/a^2$, and $k_W = 2\pi/\lambda_W$.

If the frequency with which the electromagnetic field varies is large compared with the Larmor frequency Ω_L $(= eB/2m\gamma)$ (≤ 5 GHz for $B = 3$ kG) and the reciprocal transit time $\sqrt{eG/m\gamma}$ (≤ 5 GHz for $I_B = 1$ kA and $a = 2$ cm), the equations of motion can be averaged over the high-frequency variation of the field. After tedious mathematical calculations following the method of averaging developed by Bogolyubov and Mitropolskii,[10] Eqs. (3) can be replaced by the average equation to first order of u,v,w,

$$\dot u = g \left[(1-w)Gx - wcB_0 \cos(k_W z) , + \frac{(gE_0)^2}{\omega^2} \left[(-\tfrac{3}{4} + \tfrac{3}{2}w)Gx + \frac{u}{2}Gy + \tfrac{1}{2}(1+w)cB_0 \cos(k_W z) \right] \right] , \tag{4a}$$

$$\dot v = g \left[(1-w)Gy + \frac{(gE_0)^2}{\omega^2}(-\tfrac{1}{4} + \tfrac{1}{2}w)Gy \right] , \tag{4b}$$

$$\dot w = g \left[uGx + vGy + ucB_0 \cos(k_W z) + \frac{(gE_0)^2}{\omega^2} \left[-\frac{v}{2}Gy \right] \right] . \tag{4c}$$

Here we assume that drift motion is not ultrarelativistic. Let us now consider the particular cases where essential properties of motion are understood without loss of generality.

When $y = \dot y = 0$, in the small-amplitude region $z < \lambda_W$, Eqs. (4a) and (4c) reduce to

$$\dot u = g \left[Gx(1 - \tfrac{3}{4}\epsilon) + wGx(-1 + \tfrac{3}{2}\epsilon) + wcB_0 \left[-1 + \frac{\epsilon}{2} \right] + \frac{\epsilon}{2}cB_0 \right] , \tag{5a}$$

$$\dot w = g(uGx + ucB_0) , \tag{5b}$$

where $\epsilon = (gE_0/\omega)^2$. Substitution of the first integral of (5b) into (5a) yields

$$\ddot x = cg \left\{ \frac{gG^2}{2c}(-1 + \tfrac{3}{2}\epsilon)x^3 + gB_0G(-\tfrac{3}{2} + \tfrac{7}{4}\epsilon)x^2 + \left[G(1 - \tfrac{3}{4}\epsilon) + cgB_0^2 \left[-1 + \frac{\epsilon}{2} \right] \right] x + \frac{\epsilon}{2}cB_0 \right\} . \tag{6}$$

In the limit of weak radiation fields, motion of a particle is characterized in the schematic potential curve shown in Fig. 1; namely, plasma electrons initially placed in the beam region are confined there.

Meanwhile, for motion in the vertical direction Eqs. (4b) and (4c) become

$$\dot u = gG \left[\left[1 - \frac{\epsilon}{4} \right] y - \left[1 - \frac{\epsilon}{2} \right] wy \right] , \tag{7a}$$

$$\dot w = g \left[Gxu + B_0 \dot x + G \left[1 - \frac{\epsilon}{2} \right] yv \right] . \tag{7b}$$

Integrating the latter equation and substituting into Eq. (7a), we then have

$$\ddot{y}=cgG\left[\left[1-\frac{\epsilon}{4}\right]-\left[1-\frac{\epsilon}{2}\right]\left[gB_0(x-x_0)+\frac{gG}{2c}(x^2-x_0^2)\right]\right]y-\frac{(gG)^2}{2}\left[1-\frac{\epsilon}{2}\right]^2y^3\,, \quad (8)$$

where x is the initial position in the horizontal direction. The first term in the right-hand side corresponds to the expulsion force due to space-charge force ($\propto E_r$). The second term involving the horizontal excursion x and the third term ($\propto y^3$) originate from the $y_{\dot{z}}\times B_\theta$ force. Magnitude of each term degrades due to relativistic effects ϵ depending on rf field parameters. If $\epsilon=0$, Eq. (8) is completely in agreement with the nonrelativistic kinetic equation without rf fields. The solution for the orbit $y(t)$ from Eq. (8) is complicated, but we are mainly interested in the outer limit of the orbit and the characteristic expulsion time in the small-amplitude region. So we define the following instantaneous turning-point radius and expulsion time:

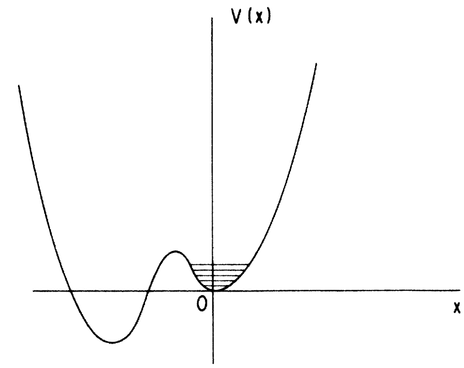

FIG. 1. Potential curve $V(x)$.

$$y_M=\frac{2}{(2-\epsilon)}\left\{\frac{c}{gG}\left[(8-2\epsilon)-(8-4\epsilon)\left[gB_0\bar{x}+\frac{gG}{2c}(x^2-x_0^2)\right]\right]\right\}^{1/2}\,, \quad (9)$$

$$\tau=\left\{cGg\left[\left[1-\frac{\epsilon}{4}\right]-\left[1-\frac{\epsilon}{2}\right]\left[gB_0\bar{x}+\frac{gG}{2c}(x^2-x_0^2)\right]\right]\right\}^{-1/2}\,. \quad (10)$$

Here we recall that for motions in the horizontal plane plasma electrons are confined around the initially placed position. Therefore, it seems reasonable to assume $|x|,|x_0|\le a$. Thus we can neglect the terms of $gG(x^2-x_0^2)/2c$ in Eqs. (9) and (10) since

$$(gG/2c)x^2\le\frac{gG}{2c}a^2=I_B/I_0\simeq 0.06$$

for $I_B=1$ kA and $gxB_0\le gaB_0\sim 2$ for $a=1$ cm, $B_0=3$ kG. If the steady-state FEL parameters satisfy the condition: $b/2<y_M$ and $\tau\sim$nsec for $|\bar{x}|=|x-x_0|\le a$, plasma electrons in the wiggler section are understood to escape along the y direction parallel to the wiggler field, as shown in Fig. 2.

Thus from Eqs. (9) and (10), after algebraic calculations the criterion which allows an ion channel to be realized is written in the form,

$$\frac{1}{2(c\tau_0)^2\left[\left[1-\frac{\epsilon}{4}\right]-\left[1-\frac{\epsilon}{2}\right]gB_0\bar{x}\right]}<\frac{I_B}{a^2I_0}<\frac{8[(4-\epsilon)-(4-2\epsilon)gB_0\bar{x}]}{b^2(2-\epsilon)^2}\,, \quad (11)$$

where $\epsilon=(\lambda/2\pi)^2(eE_0/mc^2)^2$, λ is the wavelength of radiation fields, and τ_0 the desired ion-channel forming time, typically measured in nsec as described above.

Now, the criterion (11) is represented in the parameter space (ϵ, $(I_B/I_0)/a^2$) as shown in Fig. 3; a hatched zone corresponds to the parameter regions where ion-channel guiding in a steady-state FEL is realizable. When the parameter set of a typical steady-state FEL under consideration,[4,11] $E_0=200$ MV/m, $\lambda=1.8-2.4$ cm, $I_B=1-3$ kA, waveguide height $b=\sqrt{8}$ cm, and $B_0=3$ kG is plotted on Fig. 3, it falls in the hatched zone. There we assumed $\tau_0=1$ nsec and $a=1$ cm. This indicates that an ion channel is able to form in the desired

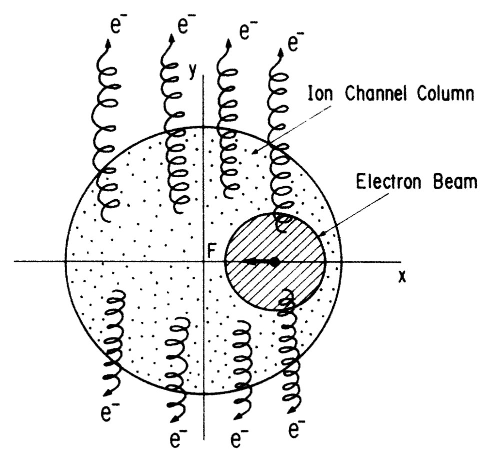

FIG. 2. Schematic of ion channel, electron beam, and ejected plasma electron in part of a FEL.

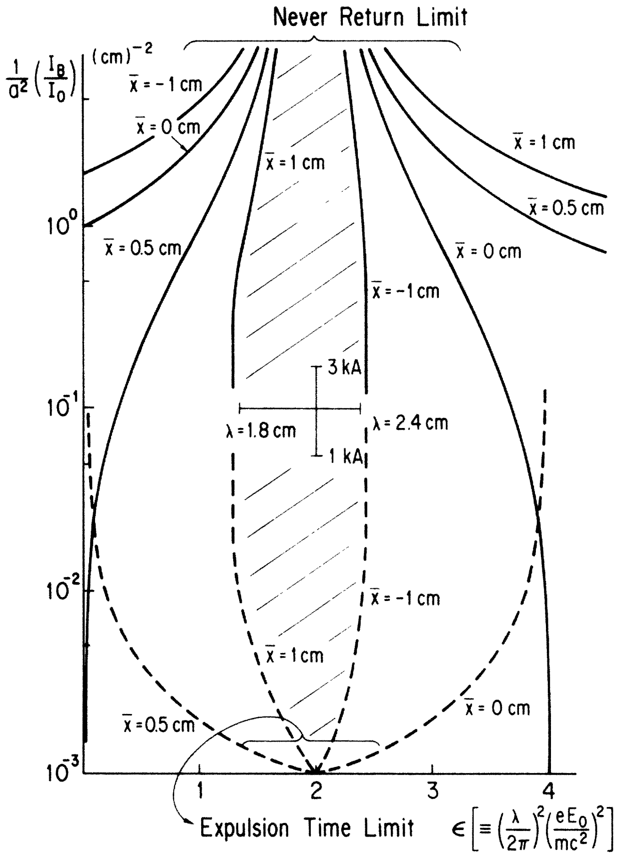

FIG. 3. Criterion represented in the parameter space $(\epsilon, (I_B/I_0)/a^2)$. The ordinate scale corresponds to the case of $b=\sqrt{8}$ cm and $\tau_0=1$ nsec.

time period at least in the microwave region of current interest.

There is a particular problem for an ion channel in a steady-state FEL. Since an ion channel produced by a laser trails the laser path, the electron beam performing a quiver motion is necessarily off centered from the ion channel as seen in Fig. 2. In this configuration the electron beam always feels a restoring force which is proportional to the distance from the channel center. This not only gives focusing about the equilibrium orbit but also results in distortion in the equilibrium orbit itself originated by the wiggler field. Let us estimate the size of distortion.

Making the assumption that the ion channel has an uniform cylinder shape, the equilibrium orbit is a periodic solution of the equation

$$\frac{d^2x_0}{dz^2}+\frac{en_i}{2m\gamma c\beta\epsilon_0}x_0=-\frac{eB_0}{m\gamma}\cos(k_W z)\ , \tag{12}$$

where n is the charge density of ion channel β, γ the usual relativistic factor, and ϵ_0 the electric permittivity in vacuum. It is written as

$$x_0(z)=\frac{-(eB_0/m\gamma)}{k_i^2-k_W^2}\cos(k_W z)\ , \tag{13}$$

where the abbreviation $k_i^2=en_i/(2m\gamma c\beta\epsilon_0)$ is used. Solution (13) tells us a very simple result, that if the betatron wavelength $\lambda_\beta=2\pi/k_i$ is sufficiently large compared to the wiggler wavelength, the magnitude of distortion will be negligibly small. Note that the betatron wavelength could be varied in wide ranges by control of gas pressure or ionizing laser fluence.

One may be concerned with any coupled instability between the electron beam and ion channel. Ions will oscillate in the strong radial electric field produced by the electron beam at a frequency

$$\omega_{\rm os}=[2(m/M)(I_B/I_0)]^{1/2}(c/a)\ ,$$

where M is the mass of the ion. With a 2-kA beam current and 1 cm beam size, the oscillation period τ $(=2\pi/\omega_{\rm os})$ is about 20 nsec for proton and 70 nsec for nitrogen. For the electron beam with relatively short pulse length (20–30 ns), therefore, we will not meet serious coupled instabilities.

Ion-channel guiding gives beam focusing on both planes in addition to intrinsic focusing in the wiggler-field direction: $k_x^2=k_i^2$, $k_y^2=k_0^2+k_i^2$, where k_0 and k_i represent the intrinsic focusing number and ion-channel focusing number, respectively. This necessarily duplicates the stop-band width in which the synchrotron-betatron resonance is excited. However, our theoretical and computational works have indicated the resonance can be easily avoided by choosing appropriate machine parameters.

The head and tail portions of beam pulse feel different focusing strength due to the longitudinal nonuniformity in the ion channel. The variation in focusing strength leads to the error variation in the amplitude and phase of the microwave. It may be roughly estimated by

$$\delta\phi/L \propto k_1\left[-\frac{k_g\bar{\epsilon}_0 k_i(T)}{8k_W\gamma_r}\right]\left[\frac{k_i(H)}{k_i(T)}-1\right]$$

or

$$\propto k_1\left[-\frac{k_g\bar{\epsilon}_0 k_i(T)}{8k_W\gamma_r}\right]\left[\left[\frac{n_i(H)}{n_i(T)}\right]^{1/2}-1\right]\ , \tag{14}$$

with

$$k_1=\frac{Z_0}{\sqrt{2}}\left[\frac{I_B}{ab}\right]\frac{1}{E_0}\left[\frac{eB_0}{\sqrt{2}mck_W}\right]\frac{1}{\gamma_r}\ ,$$

where the right-hand side is obtained by averaging the FEL rf phase shift equation[12] over the betatron and synchrotron wavelength, and the emittance distribution of particles, L the length along the steady-state FEL, Z_0(377 Ω) impedance in vacuum, $\bar{\epsilon}$ rms normalized emittance, ab cavity size, γ_r resonant energy, and n_i ion density. With $\bar{\epsilon}_0=10^{-3}$ mrad, $I_B=2$ kA, $ab=2\times5$ cm^2, $E_0=100$ MV/m, $B_0=2.5$ kG, $\lambda_W=27$ cm, $\gamma_r=16$, $\lambda=1$ cm (Ref. 13), $k_i(T)=2\pi$ m^{-1}, $n_i(H)/n_i(T)=\frac{1}{2}$, and $L=300$ m, numerical evaluation shows $|\delta\phi|=42°$. This value still seems to be tolerable because the superposition of the electric fields is made through the summation of cosine terms.

Generally, rf breakdown in the waveguide is classified into ionization breakdown and electron multipactoring breakdown. The ionization mean-free path at a given electron energy is described by $\lambda=(n\sigma)^{-1}$, where n is the neutral gas density and σ the cross section. For nitrogen with the maximum cross section ($\sim 2.5\times 10^{-16}$ cm^{-2}), the minimum ionization mean-free path is $\lambda \simeq [0.11/P(\text{torr})]$ cm. In the pressure range of current interest (10^{-3}–10^{-4} torr), $\lambda=1-10$ m, which is much larger than the cavity size, ~ 10 cm. Thus we will not meet ionization breakdown. However, one may worry that bombardment of runaway plasma electrons (of order $10^{10}/cm^2$) on the waveguide wall ignites serious electron multipactoring and leads to rf breakdown, since the rf field with a very high frequency employed in a steady-state FEL violates the multipactoring-suppression condition;[14] $E_0 > m\omega^2 d/2e$, where d is the waveguide size. However, in the wiggler section, one-dimensional motion on secondary electrons produced by first bombardment is always perpendicular to the field direction of TE mode, therefore multipactoring effects will not be enhanced. Even in the induction section, remarkably strong static space-charge forces of electron beam (of tens of MV/m) will prevent secondary electrons from approaching the waveguide center; thus multipactoring cannot occur.

Limitation of the ion-channel length is set by the diffraction and absorption of the laser pulse. It may be possible to place laser amplifier sections directly in the beam line to regenerate the laser pulse in a manner that maintains the timing. At that time, the driving electron beam may also be refreshed.

Beam front erosion in a FEL waveguide with rectangular shape has not been discussed here because beam front erosion dynamics is in principle the same as that in beam propagation through a cylindrical aperture of relatively large size which has been considered in much literature. Most of the discussions in literature[15] can be applied for the present case. Nonlinearities in space-charge forces and radiation fields of TE mode, which have not been included in the present derivation, may slightly modify the boundary regions for criterion (11), but do not affect the main discussions of our paper.

In conclusion, it turned out that a steady-state FEL, especially in the microwave (cm–mm) range of interest in the high-energy accelerators society, has a possibility of ion-channel guiding of its driving REB. So a "proof in principle" experiment is particularly expected now.

The authors would like to thank members of the FEL group for useful conversations about the present topic.

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