Atom-field interaction without the rotating-wave approximation: A path-integral approach

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We use the path-integral technique to solve the quantum-mechanical problem of interaction of a single two-level atom with a single-mode quantized field without the rotating-wave approximation (RWA). We calculate the full spin-Bose propagator in the form of a matrix as an expansion around a path corresponding to the RWA. The effect of energy-nonconserving terms on the atomic population inversion is studied, and we show that even under the conditions in which the RWA is considered to be valid there is a significant effect on the atomic inversion due to these terms.

I. INTRODUCTION

The problem of a single two-level atom interacting with a single-mode quantized field has drawn considerable interest both theoretically and experimentally.[1–9] Jaynes used the rotating-wave approximation (RWA) to reduce it to an exactly solvable problem. Jaynes and Cummings subsequently solved the model for a quantized radiation field.[1] Certain interesting features about the dynamical behavior of the atom such as collapses and revivals of the atomic population inversion were prediced by Eberly *et al.*, who later on gave analytic expressions for the revival time and amplitude of the envelope when the field mode is initially in a coherent state.[6] These predictions have been verified in a recent experiment by Rempe *et al.*[9] It is the exactly solvable model, i.e., in the RWA which is usually referred to as the Jaynes-Cummings model.

The rotating-wave approximation is used in many problems in quantum optics and it is acceptable to neglect the rapidly oscillating terms in resonance problems. Tavis and Cummings,[10] however, noted that the breakdown in the RWA occurs for extremely high intensity fields. Milonni, Ackerhalt, and Galbraith[11] have shown that the terms neglected in the RWA in the semiclassical Jaynes-Cummings model may lead to chaos when they are kept in the high intensity regime. Fox and Eidson[12] studied the onset of chaos in the level population expectation value as the atom-field coupling strength is increased. Strong field effect has also been studied by Munz and Marowsky,[13] without the RWA, using a semiclassical treatment which emphasizes the weak field and small detuning conditions under which the RWA is justified. The effect of energy-nonconserving terms on the photon statistics in a single-mode laser has been investigated by Vyas and Singh[14] using a perturbation technique.

The fully quantum-mechanical model without the RWA, however, is not solvable by usual techniques since the eigenstates of the Hamiltonian cannot be found in closed form. In this paper, we calculate the coherent state propagator for the Jaynes-Cummings model without the RWA.

The idea of a coherent-state representation for the path integral was first discussed by Klauder.[15] The concept was later introduced in quantum optics and coherent state propagators were calculated for various Hamiltonians involving the boson operators.[16] Multitime correlation functions have been calculated with the help of coherent-state propagators with operators in any order.[16,17] The importance and application of this technique to quantum optics lies in the perturbation methods and the approximations to which they lead.[18] Path integrals have also been employed to study spin-Bose systems.[19] In the RWA, the Jaynes-Cummings model is similar to the Lee model of nuclear interaction in which a heavy nucleon interacts with a light-particle field. Marshall and Pell calculated all possible transition probabilities for this model in an occupation-number representation.[20]

We use a path-integral technique in the coherent-state representation to obtain the propagator as a perturbation series, the perturbation parameter being the ratio of the Rabi frequency to the field frequency. The zeroth-order term in our expansion corresponds to the RWA results. In Sec. II, we define the coherent-state propagator for a system of a two-level atom interacting with a single-mode field without the RWA. In Sec. III, we obtain the full spin-Bose propagator by performing path integrations. In Sec. IV, we obtain an expression for the atomic population inversion and discuss its dynamical behavior.

II. COHERENT-STATE PROPAGATOR

The Hamiltonian for a system of a two-level atom interacting with a single-mode quantized radiation field in the interaction picture is

$$H_I = -g(a\sigma_+ + a^\dagger\sigma_- + a\sigma_- e^{-2i\omega t} + a^\dagger\sigma_+ e^{2i\omega t}) , \tag{1}$$

where $a^\dagger$ and a are the creation and annihilation operators, respectively, for the field, g is the atom-field coupling constant, and σ_+ and σ_- are the atomic flipping operators

$$\sigma_+ = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad \sigma_- = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} . \tag{2}$$

For simplicity, we have taken the field to be resonant with the atomic transition frequency. We obtain the propagator as a matrix element[16]

$$K(\alpha_f,t_f;\alpha_i,t_i)=\langle \alpha_f \mid U(t_f,t_i) \mid \alpha_i \rangle \;, \tag{3}$$

where the coherent state $\mid \alpha_i \rangle$ is the eigenstate of the destruction operator a at time t_i and

$$U(t_f,t_i)=T\exp\left[-i\int_{t_i}^{t_f} dt' H(t')\right] \tag{4}$$

is the time evolution operator where T is Dyson's time-ordering operator and we have chosen units such that $\hbar=1$.

Since the Hamiltonian in Eq. (4) is a matrix in the space spanned by both atom and field states, the spin-Bose propagator given by Eq. (3) will be a 2×2 matrix. The relationship of the propagator with certain quantities of interest was determined in Refs. 16 and 17. Here we derive an expression for the atomic inversion in terms of the propagator.

The dynamical behavior of the atomic inversion for an atom initially ($t=0$) in the excited state can be obtained by noting that the wave function at a later time t is

$$\mid \psi(t) \rangle = U(t,0)\begin{bmatrix}1\\0\end{bmatrix}\otimes \mid \alpha_i \rangle \;, \tag{5}$$

where the vector product $\binom{1}{0}\otimes \mid \alpha_i \rangle$ represents the initial state of the atom-field system. The corresponding density matrix is given by

$$\begin{aligned}\rho(t)&=\mid \psi(t)\rangle\langle \psi(t) \mid \\ &=U(t,0)\begin{bmatrix}1\\0\end{bmatrix}\otimes \mid \alpha_i\rangle\langle \alpha_i \mid \otimes (1 \;\; 0)U^{\dagger}(t,0) \;.\end{aligned} \tag{6}$$

The probability that the atom is in the excited state at time t is

$$\begin{aligned}P_{11}(t)&=\mathrm{Tr}_F(1\;\;0)\rho(t)\begin{bmatrix}1\\0\end{bmatrix}\\ &=\frac{1}{\pi}\int d^2\alpha_f \langle \alpha_f \mid U(t,0) \mid \alpha_i\rangle_{00} \mid^2 \\ &=\frac{1}{\pi}\int d^2\alpha_f \mid K_{00}(\alpha_f,t_t;\alpha_i,0)\mid^2 \;.\end{aligned} \tag{7a}$$

Here we have taken the trace over field variables and the subscripts on $P_{11}(t)$ denote the initial and final states of the atom.

Similarly, the probability that the atom is in the ground state at time t is

$$P_{10}(t)=\frac{1}{\pi}\int d^2\alpha_f \mid K_{10}(\alpha_f,t_t;\alpha_i,0)\mid^2 \;. \tag{7b}$$

Starting with the initial ground state, upper and lower level probabilities at time t are

$$P_{01}(t)=\frac{1}{\pi}\int d^2\alpha_f \mid K_{01}(\alpha_f,t_t;\alpha_i,0)\mid^2 \;, \tag{7c}$$

$$P_{00}(t)=\frac{1}{\pi}\int d^2\alpha_f \mid K_{11}(\alpha_f,t_t;\alpha_i,0)\mid^2 \;. \tag{7d}$$

III. PATH-INTEGRAL REPRESENTATION FOR THE PROPAGATOR

As pointed out earlier, the coherent-state propagator corresponding to the Hamiltonian (1) is in the form of a 2×2 matrix which can be expressed in terms of a path-integral representation by the following expression (for details of the derivation, see Appendix A):

$$K(\alpha_f,t_f;\alpha_i,t_i)=\lim_{N\to\infty}\int\cdots\int P(\alpha)e^{S(\alpha)}\mathcal{D}^{N-1}\{\alpha\} \;, \tag{8}$$

where

$$\mathcal{D}^{N-1}\{\alpha\}\equiv\prod_{j=1}^{N-1}\frac{d^2\alpha(j)}{\pi} \;, \tag{9}$$

$$S(\alpha)=-\tfrac{1}{2}\sum_{j=1}^{N}[\,\mid\alpha(j)\mid^2+\mid\alpha(j-1)\mid^2 -2\alpha^*(j)\alpha(j-1)] \;, \tag{10}$$

$$P(\alpha)=\prod_{\substack{j=1\\ \leftarrow}}^{N}[I+i\epsilon\eta(j)\sigma_-+i\epsilon\eta'(j-1)\sigma_+] \;. \tag{11}$$

Here the arrow under the product symbol indicates the time ordering in the product, i.e.,

$$\prod_{\substack{j=1\\ \leftarrow}}^{N} f(j)=f(N)f(N-1)\cdots f(2)f(1) \;.$$

Also,

$$\eta(j)=g[\alpha^*(j)+\alpha(j-1)f(j-1)] \;, \tag{12a}$$

$$\eta'(j-1)=g[\alpha(j-1)+\alpha^*(j)f^*(j)] \;, \tag{12b}$$

$$f(j)=e^{-2i\omega j\epsilon} \;. \tag{12c}$$

Each of the matrix elements in Eq. (8) requires multifold integrations. We use a characteristic function technique to carry out these integrations. The propagator elements can then be obtained by appropriately differentiating the characteristic function and taking the limit $N\to\infty$. It is difficult to obtain a closed form for the propagator matrix elements. We define a perturbation parameter which is the ratio of Rabi frequency to the field frequency, i.e., $\mu\sqrt{\bar{n}}$ where $\bar{n}$ is the mean number of photons and

$$\mu=\frac{g}{2\omega} \;. \tag{13}$$

The propagator, then, is in the form of a perturbation series. (The detailed calculations are given in Appendix B. Here we simply give the results):

$$K=K^{(0)}+i\mu\sqrt{\bar{n}}K^{(1)}+\cdots \;. \tag{14}$$

The zeroth-order term in Eq. (14) corresponds to the solution with the RWA and $i\mu\sqrt{\bar{n}}K^{(1)}$ is the first-order correction due to the energy-nonconserving terms. We only retain terms up to first order in μ in Eq. (14). The various matrix elements of the coherent-state propagator under this approximation are given by

$$K_{00}(\alpha_f,t_f;\alpha_i,t_i)=\sum_{n=0}^{\infty}\frac{(\alpha_f^*)^n(\alpha_i)^n}{n!}e^{-(1/2)(|\alpha_f|^2+|\alpha_i|^2)}\times\left[\cos[g(t_f-t_i)\sqrt{n+1}]+i\mu\left[(\alpha_f^{*2}e^{2i\omega t_f}+\alpha_i^2e^{-2i\omega t_i})\frac{\sin[g(t_f-t_i)\sqrt{n+1}]}{\sqrt{n+1}}-\sqrt{n+1}\sin[g(t_f-t_i)\sqrt{n+1}]+g(t_f-t_i)\cos[g(t_f-t_i)\sqrt{n+1}]\right]\right], \tag{15a}$$

$$K_{01}(\alpha_f,t_f;\alpha_i,t_i)=-\sum_{n=0}^{\infty}\frac{(\alpha_f^*)^n(\alpha_i)^n}{n!}e^{-(1/2)(|\alpha_f|^2+|\alpha_i|^2)}\times\left[i\alpha_i\frac{\sin[g(t_f-t_i)\sqrt{n+1}]}{\sqrt{n+1}}+\mu\left[\alpha_f^*e^{2i\omega t_f}\cos[g(t_f-t_i)\sqrt{n}\,]-\alpha_f^*e^{2i\omega t_i}\cos[g(t_f-t_i)\sqrt{n+2}]-\alpha_i g(t_f-t_i)\frac{\sin[g(t_f-t_i)\sqrt{n+1}]}{\sqrt{n+1}}\right]\right], \tag{15b}$$

$$K_{10}(\alpha_f,t_f;\alpha_i,t_i)=-\sum_{n=0}^{\infty}\frac{(\alpha_f^*)^n(\alpha_i)^n}{n!}e^{-(1/2)(|\alpha_f|^2+|\alpha_i|^2)}\times\left[i\alpha_f^*\frac{\sin[g(t_f-t_i)\sqrt{n+1}]}{\sqrt{n+1}}-\mu\left[\alpha_i e^{-2i\omega t_f}\cos[g(t_f-t_i)\sqrt{n+2}]-\alpha_i e^{-2i\omega t_i}\cos[g(t_f-t_i)\sqrt{n}\,]+\alpha_f^* g(t_f-t_i)\frac{\sin[g(t_f-t_i)\sqrt{n+1}]}{\sqrt{n+1}}\right]\right], \tag{15c}$$

$$K_{11}(\alpha_f,t_f;\alpha_i,t_i)=\sum_{n=0}^{\infty}\frac{(\alpha_f^*)^n(\alpha_i)^n}{n!}e^{-(1/2)(|\alpha_f|^2+|\alpha_i|^2)}\times\left[\cos[g(t_f-t_i)\sqrt{n}\,]-i\mu\left[(\alpha_f^{*2}e^{2i\omega t_i}+\alpha_i^2e^{-2i\omega t_f})\frac{\sin[g(t_f-t_i)\sqrt{n+2}]}{\sqrt{n+2}}-\sqrt{n}\sin[g(t_f-t_i)\sqrt{n}\,]-g(t_f-t_i)\cos[g(t_f-t_i)\sqrt{n}\,]\right]\right]. \tag{15d}$$

Equations (15a)–(15d) give the full spin-Bose propagator. Note that the zeroth-order terms correspond to the results given in Ref. 7.

IV. ATOMIC POPULATION INVERSION

Assuming that the atom is initially (at $t=0$) in the ground state, the population inversion at a later time t is

$$W(t)=P_{01}(t)-P_{00}(t)\ . \tag{16}$$

On substituting K_{01} and K_{11} from Eqs. (15c) and (15d) into Eqs. (7c) and (7d) and using some properties of coherent states we obtain

$$W(t)=-\sum_{n=0}^{\infty}|\langle n|\alpha\rangle|^2\left[\cos(2gt\sqrt{n}\,)+i\mu\left[(\alpha^2-\alpha^{*2})\frac{\sin[2gt\sqrt{n+2}]}{\sqrt{n+2}}+2(\alpha^{*2}e^{2i\omega t}-\alpha^2e^{-2i\omega t})\cos(gt\sqrt{n}\,)\frac{\sin(gt\sqrt{n+2})}{\sqrt{n+2}}\right]\right], \tag{17}$$

where we have taken $t_f = t$, $t_i = 0$, and $\alpha_i = \alpha$, and we have retained only the terms linear in μ. It is trivial to see that the probability amplitudes satisfy the unitarity condition, i.e.,

$$P_{01}(t) + P_{00}(t) = \sum_{n=0}^{\infty} |\langle n|\alpha\rangle|^2 = 1 . \tag{18}$$

In the limit $\bar{n} \gg 1$ and for an initial number state, Eq. (18) reduces to the semiclassical results obtained by Munz and Marowsky (Ref. 13). For

$$\alpha = \sqrt{\bar{n}}\, e^{i\phi}, \quad \tau \equiv 2gt ,$$

where ϕ is the phase of the field, Eq. (17) becomes

$$W(\tau) = -\sum_{n=0}^{\infty} \frac{\bar{n}^{\,n} e^{-\bar{n}}}{n!} \left\{ \cos(\tau\sqrt{n}\,) - \mu\sqrt{\bar{n}} \left[2\sqrt{\bar{n}}\, \sin(2\phi) \frac{\sin(\tau\sqrt{n+2})}{\sqrt{n+2}} + 4\sqrt{\bar{n}}\, \sin\left[\frac{\tau}{2\mu} - 2\phi\right] \cos\left[\frac{\tau}{2}\sqrt{n}\right] \times \frac{\sin[(\tau/2)\sqrt{n+2}]}{\sqrt{n+2}} \right] \right\} . \tag{19}$$

The first term in Eq. (19) corresponds to the RWA results. The second term depends upon the phase of the initial field and the third term oscillates at the field frequency. The phase contributions in the second and third terms tend to cancel each other and the population inversion is essentially unaffected due to a change in the phase of the initial field. Indeed, it is the third term in Eq. (19) which makes the significant contribution. Comparison of Figs. 1 and 2 highlights the effect of energy-nonconserving terms on the population inversion. The envelope does not collapse altogether but there are rapid oscillations of the population inversion during the otherwise relaxed period. There are, also, small nutations on the Rabi precessions.

V. CONCLUSION

In conclusion, we have extended the path-integral technique to spin-Bose systems in quantum optics in the context of the Jaynes-Cummings model without the rotating-wave approximation.

Our results are valid for small values of the perturbation parameter which is the ratio of the Rabi frequency to the field frequency, since we only have retained terms up to the first order in the propagator. There is, however, no restriction on the excitation intensity. We have also shown that under the conditions in which the rotating-wave approximation is considered to be justified, i.e., exact resonance and weak field, there is a significant contribution to the dynamical behavior of the atom due to the energy-nonconserving terms.

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APPENDIX A: DERIVATION OF EQ. (8)

The derivation of the path-integral representation of the coherent-state propagator is along the same lines as in Ref. 16. On inserting N resolutions of identity in Eq. (3),

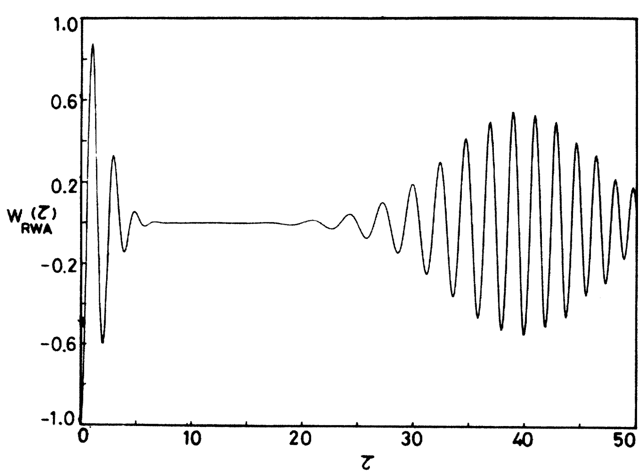

FIG. 1. Plot of $W_{\rm RWA}(\tau)$ against a dimensionless time $\tau = 2gt$ and for $\bar{n} = 10$. The Gaussian envelope collapses and revival occurs after a time $2\pi\sqrt{\bar{n}}$ as given in Ref. 6.

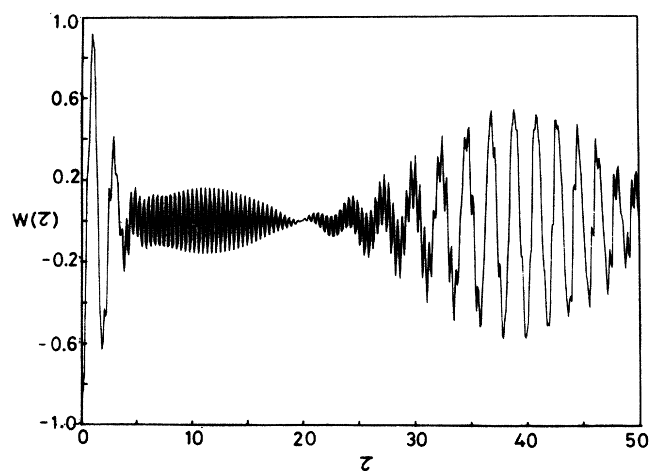

FIG. 2. Plot of $W(\tau)$ with perturbation parameter $\mu\sqrt{\bar{n}} = 0.1$, $\phi = 0$, and $\bar{n} = 10$. The contribution due to the energy-nonconserving terms is most significant during the relaxation period where the population inversion oscillates rapidly.

$$K(\alpha_f,t_f;\alpha_i,t_i)=\lim_{N\to\infty}\int\cdots\int\langle\alpha(N)|e^{-i\epsilon\hat{H}}|\alpha(N-1)\rangle\langle\alpha(N-1)|\cdots|\alpha(2)\rangle \times\langle\alpha(2)|e^{-i\epsilon\hat{H}}|\alpha(1)\rangle\langle\alpha(1)|e^{-i\epsilon\hat{H}}|\alpha(0)\rangle\prod_{j=1}^{N-1}\frac{d^2\alpha(j)}{\pi}\ , \tag{A1}$$

where

$$\epsilon=(t_f-t_i)/N,\quad \alpha(N)=\alpha_f,\quad \alpha(0)=\alpha_i\ .$$

It follows on expanding the exponential to first order in ϵ that

$$\langle\alpha(j)|e^{-i\epsilon\hat{H}}|\alpha(j-1)\rangle\cong[1-i\epsilon H(\alpha_j^*,\alpha_{j-1};t_{j-1})]\times\exp[-\tfrac{1}{2}(|\alpha_j|^2+|\alpha_{j-1}|^2)+\alpha_j^*\alpha_{j-1}]\ , \tag{A2}$$

where for a normally ordered Hamiltonian, the function H is given by

$$H(\alpha_j^*,\alpha_{j-1};t_{j-1})=\frac{\langle\alpha(j)|H(a^\dagger,a;t)|\alpha(j-1)\rangle}{\langle\alpha(j)|\alpha(j-1)\rangle}\ . \tag{A3}$$

On substituting Eqs. (A2) and (A3) into Eq. (A1), we obtain the expression for the coherent-state propagator in the path-integral representation, which is given in the text as Eq. (8).

APPENDIX B: PATH INTEGRATIONS BY A CHARACTERISTIC-FUNCTION TECHNIQUE

Here we shall carry out the multifold integrations in Eq. (8) using the characteristic function technique. The time-ordered product in Eq. (11) can be written as[20]

$$\begin{aligned}P(\alpha)=&\prod_{\substack{j=1\\ \leftarrow}}^{N}(I+i\epsilon\eta(j)\sigma_-+i\epsilon\eta'(j-1)\sigma_+]\\ =&\sigma_1\sum_{k=0}^{[N/2]}\sum_{j_1<j_{2k}}^{N}{}'\prod_{n=1}^{k}[i\epsilon\eta(j_{2n})][i\epsilon\eta'(j_{2n-1}-1)]+\sigma_0\sum_{k=0}^{[N/2]}\sum_{j_1<j_{2k}}^{N}{}'\prod_{n=1}^{k}[i\epsilon\eta(j_{2n-1})][i\epsilon\eta'(j_{2n}-1)]\\ &+\sigma_+\sum_{k=0}^{[(N-1)/2]}\sum_{j_1<j_{2k}}^{N}{}'\sum_{n=1}^{k}[i\epsilon\eta(j_{2n})]\prod_{m=1}^{k+1}[i\epsilon\eta'(j_{2m-1}-1)]\\ &+\sigma_-\sum_{k=0}^{[(N-1)/2]}\sum_{j_1<j_{2k}}^{N}{}'\prod_{n=1}^{k}[i\epsilon\eta'(j_{2n}-1)]\prod_{m=1}^{k+1}[i\epsilon\eta(j_{2m-1})]\ ,\end{aligned} \tag{B1}$$

where

$$\sigma_1\equiv\begin{vmatrix}1&0\\0&0\end{vmatrix},\quad \sigma_0\equiv\begin{vmatrix}0&0\\0&1\end{vmatrix}\ , \tag{B2}$$

$$\sum_{j_1<j_{2k}}^{N}{}'\equiv\sum_{j_1=1}^{N}\sum_{j_1=1}^{j_1-1}\cdots\sum_{j_{2k}=1}^{j_{2k-1}-1}\ . \tag{B3}$$

The square brackets in the summation limits in Eq. (B1) indicate that the greatest integer less than or equal to the enclosed quantity is taken as the summation limit.

Each of the matrix elements of the propagator can be obtained by multiple differentiations of the following characteristic function with respect to θ and θ' parameters:

$$K^C(\alpha_f,t_f;\alpha_i,t_i)=\int e^{S_\theta(\alpha)}\mathcal{D}^{N-1}\{\alpha\}\ , \tag{B4}$$

where

$$S_\theta(\alpha)=S(\alpha)+\sum_{j=1}^{N}[i\epsilon\eta(j)\theta_j+i\epsilon\eta'(j-1)\theta_j']\ . \tag{B5}$$

Using the results for the most general quadratic Hamiltonian given in Ref. 16, Eq. (B4) becomes

$$K^C(\alpha_f,t_f;\alpha_i,t_i)=\exp\left[-\tfrac{1}{2}(|\alpha_f|^2+|\alpha_i|^2)+\alpha_f^*\alpha_i-i\epsilon g\sum_{j=1}^{N}\{\alpha_f^*[\theta_j+f^*(j)\theta_j']+\alpha_i[\theta_j'+f(j-1)\theta_j]\}\right.\\ \left.-\epsilon^2g^2\sum_{j=1}^{N-1}\sum_{k=j+1}^{N}[\theta_j+f^*(j)\theta_j'][\theta_k'+f(k-1)\theta_k]\right]\ . \tag{B6}$$

We can now carry out the multiple differentiations of the characteristic function given in Eq. (B6) as follows:

$$K'_{ii}(\alpha_f,t_f;\alpha_i,t_i)=\sum_{k=0}^{[N/2]}\sum_{j_1<j_{2k}}^{N}{}' \prod_{n=1}^{k}\frac{d}{d\theta^p_{j_{2n}}}\frac{d}{d\theta^q_{j_{2n-1}}}K^C(\alpha_f,t_f;\alpha_i,t_i),\quad i=0,1 \tag{B7a}$$

$$K'_{ij}(\alpha_f,t_f;\alpha_i,t_i)=\sum_{k=0}^{[(N-1)/2]}\sum_{j_1<j_{2k}}^{N}{}' \prod_{n=1}^{k}\frac{d}{d\theta^p_{j_{2n}}}\prod_{m=1}^{k+1}\frac{d}{d\theta^q_{j_{2m-1}}}K^C(\alpha_f,t_f;\alpha_i,t_i),\quad i\neq j=0,1\ . \tag{B7b}$$

In the above equations $\theta^p=\theta$, $\theta^q=\theta'$ for $i=0$ and $\theta^p=\theta'$, $\theta^q=\theta$ for $i=1$.

The differentiations in Eq. (B7) are lengthy but straightforward. The propagator elements K_{ij} are obtained by setting the θ and θ' parameters in Eq. (B7) equal to zero and taking the limit $N\rightarrow\infty$; $\epsilon\rightarrow 0$. Under these limits, the summations are replaced by integrals, i.e.,

$$\epsilon^n\sum_{j_1=1}^{N}\sum_{j_2=1}^{j_1-1}\cdots\sum_{j_n=1}^{j_{n-1}-1}\rightarrow\int_{t_i}^{t_f}dt_1\int_{t_i}^{t_1}dt_2\cdots\int_{t_i}^{t_{n-1}}dt_n\ . \tag{B8}$$

So far we have not made any approximation. On performing the time integrations, we obtain the spin-Bose propagator as a perturbation series given in the text as Eqs. (14) and (15).

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