Quantum theory of fourth-order interference

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The quantum theory of fourth-order interference of light is presented in a general format and compared with classical wave theory. The conditions under which nonclassical phenomena occur are discussed. In particular, the interference between the quantum field and classical field may give rise to a nonclassical effect. For some special states of light, the interference pattern does not disappear even though one field is much stronger than the other, for which no classical analog exists. Fourth-order effects in the interference between two independent fields are analyzed in detail. It is pointed out that the fourth-order interference between independent fields will not disappear when the integration time of detection is of the order of the reciprocal bandwidth of the two light fields as long as the spectra of the two fields are symmetric around the same center frequency, and for some correlated fields, the interference does not vanish even if the detection time is much larger than the reciprocal bandwidth of the fields. A new type of fourth-order interference experiment involving a beam splitter is proposed in which local realism of the Einstein-Podolsky-Rosen form is violated for quantum mechanics. This general argument is then applied to the interference between two photons generated in the parametric down-conversion process. The possibility of violations of Bell's inequalities in interference experiments is investigated.

I. INTRODUCTION

Interference phenomena as evidence for wavelike behavior of light were investigated a very long time ago. The classical wave theory of light established at that time successfully explained all the interference phenomena which involve quantities of the second order in the field amplitude. Fourth-order effects in interference were not noticed until Hanbury Brown and Twiss discovered intensity correlations.[1,2] Quantum features of light are best known in connection with such things as the photoelectric effect,[3] whereas the wavelike behavior of light in interference can usually be described without quantum theory.[4] However, recently there have been several discussions[5-8] focusing on the nonclassical feature in interference, especially in fourth-order form. These treatments have generally dealt with some specific systems such as resonance fluorescence from a single atom[5] and the parametric down-conversion process,[8] which exhibit strong nonclassical effects such as photon antibunching,[9] sub-Poissonian statistics[10,11] and squeezing.[12-15] The two interfering fields in these treatments are both very nonclassical. Therefore, it is not very surprising that they behave nonclassically in fourth-order interference. It was proved[5-7] that fourth-order interference between two classical fields with random phases has maximum relative modulation of 50%, whereas quantum fields may interfere to generate a relative modulation up to 100%. Therefore, it may be possible that fourth-order interference between nonclassical and classical fields has a relative modulation larger than the classical limit of 50%.

Interference phenomena produced by two independent light fields have been studied for many years.[16-19] Many of the experiments were limited to second-order interference, which is extremely phase sensitive. Therefore, special techniques[16,18,19] had to be used to reveal the interference pattern. Fourth-order interference, however, is not phase sensitive. It has been proved[5-8] that fourth-order interference is present for both independent and correlated fields, even though second-order interference may not exist. Therefore, fourth-order interference of two independent fields may be easier to observe than second order.

Recently, it was pointed out[5,6,8] that fourth-order interference phenomena in quantum mechanics provide an example of violation of local realism discussed by Einstein, Podolsky, and Rosen[20] (EPR) and give rise to another quantum-mechanical paradox. A hidden-variables theory is then needed to solve this paradox. However, this theory was proved by Bell[21] and others[22,23] to conflict with quantum mechanics in some systems involving polarization correlation measurements and several experiments[24,25] in these systems have been performed to test the theory. All the experiments were of the Bohm-type[26] EPR Gedanken experiment, in which polarization correlation measurements are performed. In fourth-order interference experiments, position correlations are measured and the variables are positions instead of polarization angles. So it is interesting to study the hidden-variables theory in interference experiments.

In the following sections, we first present a general quantum theory of fourth-order interference with emphasis on independent fields and compare this theory with classical wave theory. We then discuss the conditions for nonclassical effects to occur. In Sec. III, a new type of fourth-order interference experiment is given, in which the two detectors can be put as far apart as we wish. We then apply the theory discussed in Sec. III to the parametric down-conversion process, in which recent experiments[27,28] showed strong evidence for fourth-order interference. The possibility of violations of Bell's inequalities in interference experiments is studied in Sec. V.

II. GENERAL FORMALISM

We start off by considering two light fields from two sources. These two fields may or may not be independent of each other, but have independent random phases so that no second-order interference exists. We assume that these two fields have well-defined directions of propagation with a small angle $\delta\theta$ between them (see Fig. 1), and have the same polarization so that we can use a scalar treatment. The fields are assumed to be homogeneous and stationary. The positive frequency part of electric field operator of each field can be written as

$$\hat{E}_1^{(+)}(\mathbf{R},t)=\frac{1}{\sqrt{\mathcal{V}}}\sum_{\{\omega_1\}} l(\omega_1)\hat{a}(\omega_1)e^{i(\omega_1\boldsymbol{\kappa}_1\cdot\mathbf{R}/c-\omega_1 t)} ,$$
$$\hat{E}_2^{(+)}(\mathbf{R},t)=\frac{1}{\sqrt{\mathcal{V}}}\sum_{\{\omega_2\}} l(\omega_2)\hat{a}(\omega_2)e^{i(\omega_2\boldsymbol{\kappa}_2\cdot\mathbf{R}/c-\omega_2 t)} , \tag{1}$$

where $l(\omega)=i(\hbar\omega/2\epsilon_0)^{1/2}$, $\boldsymbol{\kappa}_1$ and $\boldsymbol{\kappa}_2$ are unit vectors of the directions of propagation for corresponding fields, $\mathcal{V}$ is the quantization volume, and $\{\omega_1\}$ and $\{\omega_2\}$ are the frequency sets with bandwidths $\Delta\omega_1$ and $\Delta\omega_2$ for the two fields, respectively. We then write for the total field in the interference plane as

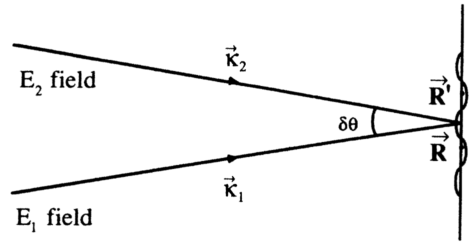

FIG. 1. Fourth-order interference between two fields.

$$\hat{E}^{(+)}(\mathbf{R},t)=\hat{E}_1^{(+)}(\mathbf{R},t)+\hat{E}_2^{(+)}(\mathbf{R},t) . \tag{2}$$

In the fourth-order interference experiments, we measure the joint probability of detecting one photon at position $\mathbf{R}$ at time t and another at $\mathbf{R}'$ at time $t+\tau$. This probability $P_{12}(\mathbf{R},t,\mathbf{R}',t+\tau)$ is expressed in the form[29]

$$\begin{aligned}P_{12}(\mathbf{R},t,\mathbf{R}',t+\tau)&=K\langle\mathcal{T}{:}\hat{I}(\mathbf{R},t)\hat{I}(\mathbf{R}',t+\tau){:}\rangle\\&=K\langle\mathcal{T}[\hat{E}^{(-)}(\mathbf{R},t)\hat{E}^{(-)}(\mathbf{R}',t+\tau)\hat{E}^{(+)}(\mathbf{R}',t+\tau)\hat{E}^{(+)}(\mathbf{R},t)]\rangle ,\end{aligned} \tag{3}$$

where K is a proportionality constant, $\mathcal{T}$ stands for time ordering and :: for normal ordering. Substituting Eq. (2) into Eq. (3), we can write P_{12} as

$$\begin{aligned}P_{12}(\mathbf{R},t,\mathbf{R}',t+\tau)=K\langle\mathcal{T}&[\hat{E}_1^{(-)}(\mathbf{R},t)+\hat{E}_2^{(-)}(\mathbf{R},t)][\hat{E}_1^{(-)}(\mathbf{R}',t+\tau)+\hat{E}_2^{(-)}(\mathbf{R}',t+\tau)]\\&\times[\hat{E}_1^{(+)}(\mathbf{R}',t+\tau)+\hat{E}_2^{(+)}(\mathbf{R}',t+\tau)][\hat{E}_1^{(+)}(\mathbf{R},t)+\hat{E}_2^{(+)}(\mathbf{R},t)]\rangle .\end{aligned} \tag{4}$$

We now denote $(\mathbf{R}',t+\tau)$ by a prime and $(\mathbf{R},t)$ without prime. Expanding Eq. (4), we find that the unpaired terms, say $\langle\mathcal{T}[\hat{E}_1^{(-)}\hat{E}_1^{(-)\prime}\hat{E}_2^{(+)\prime}\hat{E}_2^{(+)}]\rangle$, $\langle\mathcal{T}[\hat{E}_1^{(-)}\hat{E}_1^{(-)\prime}\hat{E}_1^{(+)\prime}\hat{E}_2^{(+)}]\rangle$, etc., vanish because of the independent random phases of the fields. Only six terms survive and Eq. (4) becomes

$$\begin{aligned}P_{12}(\mathbf{R},t,\mathbf{R}',t+\tau)=K[&\langle\mathcal{T}{:}\hat{I}_1\hat{I}_1'{:}\rangle+\langle\mathcal{T}{:}\hat{I}_2\hat{I}_2'{:}\rangle+\langle\mathcal{T}{:}\hat{I}_1\hat{I}_2'{:}\rangle+\langle\mathcal{T}{:}\hat{I}_2\hat{I}_1'{:}\rangle\\&+\langle\mathcal{T}(\hat{E}_2^{(-)}\hat{E}_1^{(-)\prime}\hat{E}_2^{(+)\prime}\hat{E}_1^{(+)})\rangle+\langle\mathcal{T}(\hat{E}_1^{(-)}\hat{E}_2^{(-)\prime}\hat{E}_1^{(+)\prime}\hat{E}_2^{(+)})\rangle] .\end{aligned} \tag{5}$$

It can be seen in Eq. (5) that the first four terms depend weakly on $\mathbf{R},\mathbf{R}'$, whereas the last two terms are mixed in the two fields at $\mathbf{R},\mathbf{R}'$; these are the interference terms. The first two terms in Eq. (5) depend only on the relative distance of $\mathbf{R}$ and $\mathbf{R}'$ because of the homogeneity property we assumed for the two fields. Since the two fields propagate along $\boldsymbol{\kappa}_1$ and $\boldsymbol{\kappa}_2$ which are almost same, the first two terms in Eq. (5) are functions only of the relative distance between $\mathbf{R}$ and $\mathbf{R}'$ along direction $\boldsymbol{\kappa}_1$ or $\boldsymbol{\kappa}_2$. If we furthermore assume that $\mathbf{R},\mathbf{R}'$ are on the interference plane that is nearly perpendicular to $\boldsymbol{\kappa}_1$ and $\boldsymbol{\kappa}_2$, these two terms will not depend on $\mathbf{R},\mathbf{R}'$ and are equal to autocorrelations for the corresponding fields and can be written as

$$\begin{aligned}\langle\mathcal{T}{:}\hat{I}_1\hat{I}_1'{:}\rangle&=\langle\mathcal{T}{:}\hat{I}_1(t+\tau)\hat{I}_1(t){:}\rangle\\&=\langle\hat{I}_1\rangle^2[1+\lambda_1(\tau)] ,\\\langle\mathcal{T}{:}\hat{I}_2\hat{I}_2'{:}\rangle&=\langle\mathcal{T}{:}\hat{I}_2(t+\tau)\hat{I}_2(t){:}\rangle\\&=\langle\hat{I}_2\rangle^2[1+\lambda_2(\tau)] ,\end{aligned} \tag{6}$$

where $\lambda_i(\tau)$ is the normalized intensity correlation for the corresponding field and is defined as

$$\begin{aligned}\lambda_i(\tau)&\equiv\frac{\langle\mathcal{T}{:}\hat{I}_i(t)\hat{I}_i(t+\tau){:}\rangle}{\langle\hat{I}_i\rangle^2}-1\\&=\frac{\langle\mathcal{T}{:}\Delta\hat{I}_i(t)\Delta\hat{I}_i(t+\tau){:}\rangle}{\langle\hat{I}_i\rangle^2}\quad(i=1,2) .\end{aligned} \tag{7}$$

$\lambda_i(0)$ is non-negative for classical fields and can approach -1 for nonclassical fields.[30] The middle two terms on the right-hand side of Eq. (5) are the cross correlations between the two fields at two points, and the last two terms give rise to interference. In order to calculate these terms, we need to know the state of the fields. However, we can draw some general conclusions just by comparing the terms.

It can be proved (see the Appendix) that, for classical fields, the first two terms and the middle two terms in Eq. (5) are both larger than the last two terms or the interference terms, that is

$$\langle \mathfrak{T}{:}\hat{I}_1(\mathbf{R},t)\hat{I}_2(\mathbf{R}',t+\tau){:}\rangle + \langle \mathfrak{T}{:}\hat{I}_2(\mathbf{R},t)\hat{I}_1(\mathbf{R}',t+\tau){:}\rangle \geq |\langle \mathfrak{T}[\hat{E}_1^{(-)}(\mathbf{R},t)\hat{E}_2^{(-)}(\mathbf{R}',t+\tau)\hat{E}_1^{(+)}(\mathbf{R}',t+\tau)\hat{E}_2^{(+)}(\mathbf{R},t)]\rangle + \text{c.c.}| , \quad (8)$$

$$\langle \mathfrak{T}{:}\hat{I}_1(\mathbf{R},t)\hat{I}_1(\mathbf{R}',t+\tau){:}\rangle + \langle \mathfrak{T}{:}\hat{I}_2(\mathbf{R},t)\hat{I}_2(\mathbf{R}',t+\tau){:}\rangle \geq |\langle \mathfrak{T}[\hat{E}_1^{(-)}(\mathbf{R},t)\hat{E}_2^{(-)}(\mathbf{R}',t+\tau)\hat{E}_1^{(+)}(\mathbf{R}',t+\tau)\hat{E}_2^{(+)}(\mathbf{R},t)]\rangle + \text{c.c.}| . \quad (9)$$

Therefore, the largest modulation of interference pattern is 50% for classical interference. For nonclassical fields, the first inequality can be proved (see the Appendix) to be satisfied, the second one, however, is violated for some states of fields. The fields with $\lambda_1(\tau)=-1$ ($i=1$ and 2), for example, will make the left-hand side of the inequality (9) zero, and the inequality (9) is violated as long as the right-hand side of the inequality is nonzero. From these two inequalities, we can see that one necessary condition for which nonclassical effect occurs in interference experiment is that the first two terms in Eq. (5) or the auto-correlation terms are less than the maximum of the absolute value of the interference terms, namely

$$\langle \mathfrak{T}{:}\hat{I}_1(\mathbf{R},t)\hat{I}_1(\mathbf{R}',t+\tau){:}\rangle + \langle \mathfrak{T}{:}\hat{I}_2(\mathbf{R},t)\hat{I}_2(\mathbf{R}',t+\tau){:}\rangle < |\langle \mathfrak{T}[\hat{E}_1^{(-)}(\mathbf{R},t)\hat{E}_2^{(-)}(\mathbf{R}',t+\tau)\hat{E}_1^{(+)}(\mathbf{R}',t+\tau)\hat{E}_2^{(+)}(\mathbf{R},t)]\rangle + \text{c.c.}|_M . \quad (10a)$$

By using inequality (8), which is true for all fields including quantum fields, we change condition (10a) to

$$\langle \mathfrak{T}{:}\hat{I}_1(\mathbf{R},t)\hat{I}_1(\mathbf{R}',t+\tau){:}\rangle + \langle \mathfrak{T}{:}\hat{I}_2(\mathbf{R},t)\hat{I}_2(\mathbf{R}',t+\tau){:}\rangle < \langle \mathfrak{T}{:}\hat{I}_1(\mathbf{R},t)\hat{I}_2(\mathbf{R}',t+\tau){:}\rangle + \langle \mathfrak{T}{:}\hat{I}_1(\mathbf{R},t)\hat{I}_2(\mathbf{R}',t+\tau){:}\rangle . \quad (10b)$$

Therefore, the modified necessary condition for nonclassical effect to occur in fourth-order interference is that the sum of the autocorrelations of the two interfering fields is less than the sum of the cross correlations between the two fields at two locations.

It is interesting to notice that if one field, say $\hat{E}_1$, is very nonclassical so that $\lambda_1(\tau)=-1$, the inequality (10a) will always be satisfied regardless of what kind of field the other source emits, whenever the nonclassical field is much stronger than the other one and the right-hand side of this inequality is significantly different from zero. Therefore the nonclassical effect may occur in the interference between nonclassical field and classical field. If the interference pattern exists, it will not disappear even though $\langle \hat{I}_1\rangle \gg \langle \hat{I}_2\rangle$ because, when $\lambda_1(\tau)=-1$ and $\langle \hat{I}_1\rangle \gg \langle \hat{I}_2\rangle$, the cross correlation terms and the interference terms in Eq. (5) will dominate and have the same order of $\langle \hat{I}_1\rangle\langle \hat{I}_2\rangle$. When both fields are very nonclassical and $\lambda_i(\tau)=-1$ for $i=1$ and 2, the inequality (10a) is satisfied whenever the interference terms are nonzero, and the interference pattern, if it exists, will not disappear for any ratio of $\langle \hat{I}_1\rangle$ and $\langle \hat{I}_2\rangle$ because the first two terms in Eq. (5) vanish and the interference terms have the same order as the cross correlation terms. In Sec. IV, we will see such a case. This kind of phenomenon is purely quantum mechanical and has no classical origin, because, for classical fields, the autocorrelation terms in Eq. (5) can never vanish and therefore if one field, say $\hat{E}_1$, is much stronger than the other one, the first term in Eq. (5) will dominate and the interference effect will be very small.

When the two fields are independent of each other, the cross correlation terms in Eq. (5) become extremely simple and can be written as

$$\begin{aligned}\langle \mathfrak{T}{:}\hat{I}_1\hat{I}'_2{:}\rangle &= \langle \hat{I}_1\rangle\langle \hat{I}'_2\rangle = \langle \hat{I}_1\rangle\langle \hat{I}_2\rangle ,\\ \langle \mathfrak{T}{:}\hat{I}_2\hat{I}'_1{:}\rangle &= \langle \hat{I}_2\rangle\langle \hat{I}'_1\rangle = \langle \hat{I}_1\rangle\langle \hat{I}_2\rangle ,\end{aligned} \quad (11)$$

where we have used the homogeneity and stationarity properties of the fields. The interference terms in Eq. (5) are also easy to calculate. With the help of Eq. (1), they reduce to

$$\begin{aligned}&\langle \mathfrak{T}[\hat{E}_1^{(-)}(\mathbf{R},t)\hat{E}_2^{(-)}(\mathbf{R}',t+\tau)\hat{E}_1^{(+)}(\mathbf{R}',t+\tau)\hat{E}_2^{(+)}(\mathbf{R},t)]\rangle + \text{c.c.}\\ &\quad = \langle \hat{E}_1^{(-)}(\mathbf{R},t)\hat{E}_2^{(-)}(\mathbf{R}',t+\tau)\hat{E}_1^{(+)}(\mathbf{R}',t+\tau)\hat{E}_2^{(+)}(\mathbf{R},t)\rangle + \text{c.c.} \quad (\tau\geq 0)\\ &\quad = \frac{1}{\mathcal{V}^2}\sum_{\omega',\omega''} |l(\omega_{01}+\omega')l(\omega_{02}+\omega'')|^2 n_1(\omega_{01}+\omega')n_2(\omega_{02}+\omega'')e^{i\omega'(\boldsymbol{\kappa}_1\cdot\delta\mathbf{R}/c-\tau)-i\omega''(\boldsymbol{\kappa}_2\cdot\delta\mathbf{R}/c-\tau)}\\ &\qquad \times e^{-i\tau(\omega_{01}-\omega_{02})}e^{i[(\boldsymbol{\kappa}_1\omega_{01}-\boldsymbol{\kappa}_2\omega_{02})\cdot\delta\mathbf{R}/c]} + \text{c.c.} \quad (\tau\geq 0)\end{aligned} \quad (12)$$

where we have used the assumption that $\langle \hat{a}^\dagger(\omega)\hat{a}(\omega')\rangle = \langle \hat{a}^\dagger(\omega)\hat{a}(\omega)\rangle\delta_{\omega,\omega'} \equiv n(\omega)\delta_{\omega,\omega'}$ because of the stationarity of the fields, and have put $\omega_1=\omega_{01}+\omega'$ and $\omega_2=\omega_{02}+\omega''$ with middle frequencies ω_{01},ω_{02} and $\delta\mathbf{R}=\mathbf{R}_1-\mathbf{R}_2$. It is easy to see that, when $\tau<0$, the expression for interference terms is exactly same as the last equation of Eqs. (12). Therefore we will ignore the condition $\tau\geq 0$. Combining Eqs. (6), (11), and (12), we obtain

$$P_{12}(\mathbf{R},\mathbf{R}',\tau)=K\Bigg[\langle\hat{I}_1\rangle^2[1+\lambda_1(\tau)]+\langle\hat{I}_2\rangle^2[1+\lambda_2(\tau)]+2\langle\hat{I}_1\rangle\langle\hat{I}_2\rangle + \Bigg[\frac{1}{\mathcal{V}^2}\sum_{\omega',\omega''}|l(\omega_{01}+\omega')l(\omega_{02}+\omega'')|^2 n_1(\omega_{01}+\omega')n_2(\omega_{02}+\omega'')e^{i\omega'(\boldsymbol{\kappa}_1\cdot\delta\mathbf{R}/c-\tau)-i\omega''(\boldsymbol{\kappa}_2\cdot\delta\mathbf{R}/c-\tau)} \times e^{-i\tau(\omega_{01}-\omega_{02})}e^{i[(\boldsymbol{\kappa}_1\omega_{01}-\boldsymbol{\kappa}_2\omega_{02})\cdot\delta\mathbf{R}/c]}+\text{c.c.}\Bigg]\Bigg]. \tag{13}$$

We now integrate over the detection time T and the measured probability becomes

$$P_{12}(\mathbf{R},\mathbf{R}',T)=KT\Bigg[\langle\hat{I}_1\rangle^2\left[1+\frac{1}{T}\int_{-T/2}^{T/2}d\tau\,\lambda_1(\tau)\right]+\langle\hat{I}_2\rangle^2\left[1+\frac{1}{T}\int_{-T/2}^{T/2}d\tau\,\lambda_2(\tau)\right]+2\langle\hat{I}_1\rangle\langle\hat{I}_2\rangle + \Bigg[\frac{1}{\mathcal{V}^2}\sum_{\omega',\omega''}|l(\omega_{01}+\omega')l(\omega_{02}+\omega'')|^2 n_1(\omega_{01}+\omega')n_2(\omega_{02}+\omega'')e^{i\omega'(\boldsymbol{\kappa}_1\cdot\delta\mathbf{R}/c-\tau)-i\omega''(\boldsymbol{\kappa}_2\cdot\delta\mathbf{R}/c-\tau)} \times n_2(\omega_{02}+\omega'')\frac{1}{T}\int_{-T/2}^{T/2}d\tau\, e^{i\omega'(\boldsymbol{\kappa}_1\cdot\delta\mathbf{R}/c-\tau)-i\omega''(\boldsymbol{\kappa}_2\cdot\delta\mathbf{R}/c-\tau)} \times e^{-i\tau(\omega_{01}-\omega_{02})}e^{i[(\boldsymbol{\kappa}_1\omega_{01}-\boldsymbol{\kappa}_2\omega_{02})\cdot\delta\mathbf{R}/c]}+\text{c.c.}\Bigg]\Bigg]. \tag{14}$$

We can see from Eq. (14) that, if $|T(\omega_{01}-\omega_{02})|\sim 1$, the interference terms may integrate to zero. So we need to make ω_{01} and ω_{02} close enough so that $|T(\omega_{01}-\omega_{02})|\ll 1$ and $\omega_{01}\approx\omega_{02}\equiv\omega_0$. We furthermore assume that $n_1(\omega_{01}+\omega'')$ and $n_2(\omega_{02}+\omega'')$ are symmetric around ω_{01} and ω_{02}. Then Eq. (14) becomes

$$P_{12}(\mathbf{R},\mathbf{R}',T)=KT\Bigg[\langle\hat{I}_1\rangle^2\left[1+\frac{1}{T}\int_{-T/2}^{T/2}d\tau\,\lambda_1(\tau)\right]+\langle\hat{I}_2\rangle^2\left[1+\frac{1}{T}\int_{-T/2}^{T/2}d\tau\,\lambda_2(\tau)\right]+2\langle\hat{I}_1\rangle\langle\hat{I}_2\rangle + 2\cos[(\boldsymbol{\kappa}_1-\boldsymbol{\kappa}_2)\cdot\delta\mathbf{R}\omega_0/c]\frac{1}{T}\int_{-T/2}^{T/2}d\tau\,\mathcal{L}_1(\boldsymbol{\kappa}_1\cdot\delta\mathbf{R}/c-\tau)\mathcal{L}_2(\boldsymbol{\kappa}_2\cdot\delta\mathbf{R}/c-\tau)\Bigg], \tag{15}$$

where we have put

$$\mathcal{L}_i(\tau)=C\int d\omega'\,|l(\omega_{0i}+\omega')|^2 n_i(\omega_{0i}+\omega')e^{i\omega'\tau}\quad(i=1,2),$$

which is real and non-negative when $n_i(\omega_{0i}+\omega_i')$ is symmetric around $\omega_i'=0$ and $l(\omega_{0i}+\omega_i')$ varies slowly, and has width of order $1/\Delta\omega_i$, and is approximately equal to $\langle\hat{I}_i\rangle$ when $\Delta\omega_i\tau\ll 1$. C is a scaling constant when we change from a sum to an integral with respect to ω_i'. If we introduce the visibility v of the interference, which is

$$v\equiv\frac{\frac{1}{T}\int_{-T/2}^{T/2}d\tau\,2\mathcal{L}_1(\boldsymbol{\kappa}_1\cdot\delta\mathbf{R}/c-\tau)\mathcal{L}_2(\boldsymbol{\kappa}_2\cdot\delta\mathbf{R}/c-\tau)}{\langle\hat{I}_1\rangle^2\left[1+\frac{1}{T}\int_{-T/2}^{T/2}d\tau\,\lambda_1(\tau)\right]+\langle\hat{I}_2\rangle^2\left[1+\frac{1}{T}\int_{-T/2}^{T/2}d\tau\,\lambda_2(\tau)\right]+2\langle\hat{I}_1\rangle\langle\hat{I}_2\rangle}, \tag{16}$$

Eq. (15) can be further reduced to

$$P_{12}(x,x',T)\propto 1+v\cos[2\pi(x-x')/L], \tag{17}$$

where $L=2\pi c/(\omega_0\delta\theta)=\lambda_0\delta\theta$ is the fringe spacing, x and x' are positions of detectors along the direction of $\boldsymbol{\kappa}_1-\boldsymbol{\kappa}_2$, and $\delta\theta$ is the angle between $\boldsymbol{\kappa}_1$ and $\boldsymbol{\kappa}_2$. By inspection of Eq. (16), we see that if $\Delta\omega_i(\boldsymbol{\kappa}_i\cdot\delta\mathbf{R})/c\ll 1$, v has its largest value when $T\Delta\omega_i\ll 1$ and is finite when $T\Delta\omega_i\sim 1$ although v goes to zero when $T\to\infty$. Therefore, the fringes will not go away even if $T\Delta\omega_i\sim 1$. P_{12} is also phase insensitive and the interference fringes do not disappear if the two fields have independent random phase fluctuations. That is because fourth-order photon detection does not depend on the phases of fields. The frequency is of course related to the phase of the field, so even if the phase changes almost by 2π when $T\Delta\omega_i\sim 1$, the fringes do not average to zero. However, if T is so long that the detectors can tell which photon comes from which source via its center frequency, then no interference occurs. That is why we need $T(\omega_{01}-\omega_{02})\ll 1$. All the properties described above are typical for fourth-order interference. Under the same circumstances, second-order interference will disappear. Although v in fourth-order interference also goes to zero when $T\to\infty$ like second-order interference, the mechanism is not same for these two cases. In second-order interference, the phase linearly increases when $T\to\infty$ and therefore the interference fringes average to zero, whereas in fourth-order interference, as $T\to\infty$, the cross correlation terms go to ∞ because of the independence of the fields while the interference terms stay finite. Later in Sec. V, we will see that, for some correlated fields, the cross correlation terms are finite as $T \to \infty$, and therefore v depends weakly on detection time T so that v does not vanish even if $T \to \infty$.

We can also see in Eq. (16) that, if the two detectors are so far away from each other that $\Delta\omega_i(\boldsymbol{\kappa}_i \cdot \delta\mathbf{R})/c \sim 1$, the visibility v will be very small. This suggests a way to estimate how many fringes we can observe.[8]

In the following, we shall assume that $T\Delta\omega_i \ll 1$ and $\Delta\omega_i(\boldsymbol{\kappa}_i \cdot \delta\mathbf{R})/c \ll 1$ in order to achieve maximum visibility, and under these conditions $\lambda_i(\tau)$ will be approximately $\lambda_i(0)$ because $|\tau| \ll 1/\Delta\omega_i$, which is of the order of the correlation time T_{ci} of each field. Equation (16) then becomes

$$v \equiv \frac{2\langle \hat{I}_1 \rangle \langle \hat{I}_2 \rangle}{\langle \hat{I}_1 \rangle^2[1+\lambda_1(0)]+\langle \hat{I}_2 \rangle^2[1+\lambda_2(0)]+2\langle \hat{I}_1 \rangle \langle \hat{I}_2 \rangle} . \tag{18}$$

For classical fields, $\lambda_i(0) \geq 0$ and we have

$$v_{\text{classical}} \leq \frac{2\langle \hat{I}_1 \rangle \langle \hat{I}_2 \rangle}{\langle \hat{I}_1 \rangle^2+\langle \hat{I}_2 \rangle^2+2\langle \hat{I}_1 \rangle \langle \hat{I}_2 \rangle} . \tag{19}$$

The right-hand side of this inequality reaches its maximum value of $\frac{1}{2}$ when $\langle \hat{I}_1 \rangle = \langle \hat{I}_2 \rangle$. Therefore the largest modulation of the interference pattern is 50% for classical fields. For nonclassical fields, $\lambda_i(0)$ may be less than zero and therefore v can go beyond 50% and reach to 100% when $\lambda_i(0)$ $(i=1,2)$ takes its minimum value of -1.

Let us now find the condition for $v > \frac{1}{2}$. In Eq. (18), v reaches its maximum value of

$$v_M = \frac{1}{1+\{[1+\lambda_1(0)][1+\lambda_2(0)]\}^{1/2}} , \tag{20}$$

when $\langle \hat{I}_1 \rangle^2[1+\lambda_1(0)] = \langle \hat{I}_2 \rangle^2[1+\lambda_2(0)]$. Therefore, if

$$[1+\lambda_1(0)][1+\lambda_2(0)] < 1 , \tag{21}$$

then $v_M > \frac{1}{2}$ and the classical limit of v is exceeded. We can see that when condition (21) is satisfied, conditions (10) are also satisfied for the intensities given above. To satisfy condition (21), it is not necessary for both interfering fields to be nonclassical. One of them may be classical.

When we examine Eq. (18), we see once again that, if one of the fields, say $\hat{E}_1$, is very nonclassical so that $\lambda(0) = -1$, then, no matter what kind of field the other source emits, the visibility v is nearly one as long as the nonclassical field $\hat{E}_1$ is much stronger than the other field; when both fields are very nonclassical so that $\lambda_i(0) = -1$ for $i=1$ and 2, v is one for any combination of strengths of the two fields. This is obviously nonclassical behavior of light and cannot be explained by classical wave theory. Actually, we can see from Eqs. (18) or (19) that the classical visibility v of the interference pattern tends to zero as the ratio of intensities of the two fields becomes either very large or very small.

III. FOURTH-ORDER INTERFERENCE EXPERIMENT WITH A BEAM SPLITTER

Usually we observe the interference pattern in some interference plane. The observations are limited to a small area because of the finite size of the beams and the finite number of observable fringes. This does not cause much difficulty for second-order interference because only one detector is involved. However, two detectors are used in fourth-order interference and it may be difficult to collect the photons when the observation area is small. The fourth-order interference phenomenon is also known[5,6,8] to provide evidence for locality violation of quantum mechanics. However, the detectors may have to be very close due to the finite size of the observable area. The possibility exists therefore that the two detection processes might interact with each other in such a short distance. In the following, a new type of fourth-order interference experiment is described in which the two detectors can be far apart.

Suppose now that two beams of light are incident from different sides of a beam splitter which has transmissivity $\mathcal{T}$ and reflectivity $\mathcal{R}$ with $\mathcal{T}+\mathcal{R}=1$. After the beam splitter, they come together to interfere. Usually, we observe the fringes in the plane on which either $\mathbf{R}_1$ or $\mathbf{R}_2$ is located (Fig. 2). Now we put two detectors in different sides of the beam splitter, one at $\mathbf{R}_1$ and another at $\mathbf{R}_2$. Let the origin of the system on the beam splitter plane and $\mathbf{R}_1', \mathbf{R}_2', \boldsymbol{\kappa}_1'\boldsymbol{\kappa}_2'$ be the mirror images of $\mathbf{R}_1, \mathbf{R}_2, \boldsymbol{\kappa}_1, \boldsymbol{\kappa}_2$ relative to the beam splitter (Fig. 2). We further assume that $\mathbf{R}_1, \mathbf{R}_2'$ are in the interference plane that is vertical to the directions of incoming fields (Fig. 2). The fields at $\mathbf{R}_1, \mathbf{R}_2$ are given as follows:

$$\hat{E}^{(+)}(\mathbf{R}_1,t) = \sqrt{\mathcal{T}}\hat{E}_1^{(+)}(\mathbf{R}_1,t) + i\sqrt{\mathcal{R}}\hat{E}_2^{(+)\prime}(\mathbf{R}_1,t) ,$$

$$\hat{E}^{(+)}(\mathbf{R}_2,t) = \sqrt{\mathcal{T}}\hat{E}_2^{(+)}(\mathbf{R}_2,t) - i\sqrt{\mathcal{R}}\hat{E}_1^{(+)\prime}(\mathbf{R}_2,t) , \tag{22}$$

where the prime means that $\boldsymbol{\kappa}_i$ is replaced by $\boldsymbol{\kappa}_i'$ in Eq. (1). Substituting Eq. (22) into Eq. (3), we get, with the same assumption used in deriving Eq. (5),

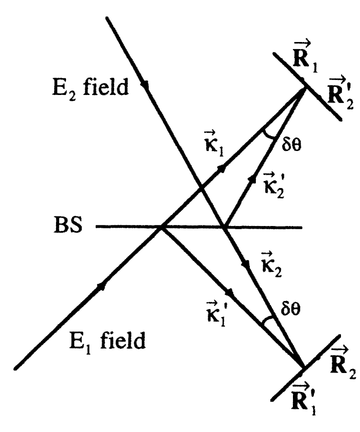

FIG. 2. Fourth-order interference with a beam splitter.

$$P_{12}(\mathbf{R}_1,t,\mathbf{R}_2,t+\tau)=K\{\mathcal{T}\mathcal{R}[\langle \mathfrak{T}:\hat{I}_1(\mathbf{R}_1,t)\hat{I}'_1(\mathbf{R}_2,t+\tau):\rangle+\langle \mathfrak{T}:\hat{I}'_2(\mathbf{R}_1,t)\hat{I}_2(\mathbf{R}_2,t+\tau):\rangle]$$
$$+\mathcal{T}^2\langle \mathfrak{T}:\hat{I}_1(\mathbf{R}_1,t)\hat{I}_2(\mathbf{R}_2,t+\tau):\rangle+\mathcal{R}^2\langle \mathfrak{T}:\hat{I}'_2(\mathbf{R}_1,t)\hat{I}'_1(\mathbf{R}_2,t+\tau):\rangle$$
$$-\mathcal{T}\mathcal{R}[\langle \mathfrak{T}\hat{E}_1^{(-)}(\mathbf{R}_1,t)\hat{E}_2^{(-)}(\mathbf{R}_2,t+\tau)\hat{E}_1^{(+)\prime}(\mathbf{R}_2,t+\tau)\hat{E}_2^{(+)\prime}(\mathbf{R}_1,t)\rangle+\text{c.c.}]\} \ . \tag{23}$$

As in Eq. (5), the first two terms are equal to autocorrelations for corresponding fields if the fields are homogeneous and stationary, and can be expressed as

$$\begin{aligned}\langle \mathfrak{T}:\hat{I}_1(\mathbf{R}_1,t)\hat{I}'_1(\mathbf{R}_2,t+\tau):\rangle&=\langle \hat{I}_1\rangle^2[1+\lambda_1(\tau)] \ ,\\ \langle \mathfrak{T}:\hat{I}'_2(\mathbf{R}_1,t)\hat{I}_2(\mathbf{R}_2,t+\tau):\rangle&=\langle \hat{I}_2\rangle^2[1+\lambda_2(\tau)] \ .\end{aligned} \tag{24}$$

The middle two terms are the cross correlations between the two fields and the last terms are the interference terms. If the two fields are also independent, these terms become

$$\begin{aligned}&\langle \mathfrak{T}:\hat{I}_1(\mathbf{R}_1,t)\hat{I}_2(\mathbf{R}_2,t+\tau):\rangle=\langle \hat{I}_1\rangle\langle \hat{I}_2\rangle \ ,\\ &\langle \mathfrak{T}:\hat{I}'_2(\mathbf{R}_1,t)\hat{I}'_1(\mathbf{R}_2,t+\tau):\rangle=\langle \hat{I}_1\rangle\langle \hat{I}_2\rangle \ ,\end{aligned} \tag{25}$$

$$\begin{aligned}&\langle \mathfrak{T}[\hat{E}_1^{(-)}(\mathbf{R}_1,t)\hat{E}_2^{(-)}(\mathbf{R}_2,t+\tau)\hat{E}_1^{(+)\prime}(\mathbf{R}_2,t+\tau)\hat{E}_2^{(+)\prime}(\mathbf{R}_1,t)]\rangle+\text{c.c.}\\ &=\langle \hat{E}_1^{(-)}(\mathbf{R}_1,t)\hat{E}_2^{(-)}(\mathbf{R}_2,t+\tau)\hat{E}_1^{(+)\prime}(\mathbf{R}_2,t+\tau)\hat{E}_2^{(+)\prime}(\mathbf{R}_1,t)\rangle+\text{c.c.}\quad(\tau\geq 0)\\ &=\frac{1}{\mathcal{V}^2}\sum_{\omega',\omega''}|l(\omega_{01}+\omega')l(\omega_{02}+\omega'')|^2 n_1(\omega_{01}+\omega')n_2(\omega_{02}+\omega'')e^{i\omega'[(\boldsymbol{\kappa}_1\cdot\mathbf{R}_1-\boldsymbol{\kappa}'_1\cdot\mathbf{R}_2)/c-\tau]-i\omega''[(\boldsymbol{\kappa}'_2\cdot\mathbf{R}_1-\boldsymbol{\kappa}_2\cdot\mathbf{R}_2)/c-\tau]}\\ &\qquad\times e^{-i\tau(\omega_{01}-\omega_{02})}e^{i[(\boldsymbol{\kappa}_1\omega_{01}-\boldsymbol{\kappa}'_2\omega_{02})\cdot\mathbf{R}_1-(\boldsymbol{\kappa}'_1\omega_{02}-\boldsymbol{\kappa}_2\omega_{02})\cdot\mathbf{R}_2]/c}+\text{c.c.}\quad(\tau\geq 0)\\ &=\frac{1}{\mathcal{V}^2}\sum_{\omega',\omega''}|l(\omega_{01}+\omega')l(\omega_{02}+\omega'')|^2 n_1(\omega_{01}+\omega')n_2(\omega_{02}+\omega'')e^{i\omega'[\boldsymbol{\kappa}_1\cdot(\mathbf{R}_1-\mathbf{R}'_2)/c-\tau]-i\omega''[\boldsymbol{\kappa}_2\cdot(\mathbf{R}'_1-\mathbf{R}_2)/c-\tau]}\\ &\qquad\times e^{-i\tau(\omega_{01}-\omega_{02})}e^{i[(\boldsymbol{\kappa}_1\omega_{01}-\boldsymbol{\kappa}'_2\omega_{02})\cdot\mathbf{R}_1-(\boldsymbol{\kappa}'_1\omega_{02}-\boldsymbol{\kappa}_2\omega_{02})\cdot\mathbf{R}_2]/c}+\text{c.c.}\quad(\tau\geq 0)\end{aligned} \tag{26}$$

where we used the properties that $\mathbf{R}'_1,\mathbf{R}'_2,\boldsymbol{\kappa}'_1,\boldsymbol{\kappa}'_2$ are the mirror images of $\mathbf{R}_1,\mathbf{R}_2,\boldsymbol{\kappa}_1,\boldsymbol{\kappa}_2$. As before, we will ignore the condition $\tau\geq 0$. With the assumption of $|T(\omega_{01}-\omega_{02})|\ll 1$, the measured probability is

$$\begin{aligned}P_{12}(x_1,x_2,T)=K\Bigg[&\mathcal{T}\mathcal{R}\langle \hat{I}_1\rangle^2\left[1+\frac{1}{T}\int_{-T/2}^{T/2}d\tau\,\lambda_1(\tau)\right]+\mathcal{T}\mathcal{R}\langle \hat{I}_2\rangle^2\left[1+\frac{1}{T}\int_{-T/2}^{T/2}d\tau\,\lambda_2(\tau)\right]\\ &+2(\mathcal{T}^2+\mathcal{R}^2)\langle \hat{I}_1\rangle\langle \hat{I}_2\rangle-2\mathcal{T}\mathcal{R}\cos[2\pi(x_1-x_2)/L]\\ &\times\frac{1}{T}\int_{-T/2}^{T/2}d\tau\,\mathcal{L}_1(\boldsymbol{\kappa}_1\cdot\delta\mathbf{R}_1/c-\tau)\mathcal{L}_2(\boldsymbol{\kappa}_2\cdot\delta\mathbf{R}_2/c-\tau)\Bigg] \ ,\end{aligned} \tag{27}$$

where $\delta\mathbf{R}_1=\mathbf{R}_1-\mathbf{R}'_2$ and $\delta\mathbf{R}_2=\mathbf{R}'_1-\mathbf{R}_2$, x_1 and x_2 are the coordinates of $\mathbf{R}_1$ and $\mathbf{R}_2$ along directions of $\boldsymbol{\kappa}'_1-\boldsymbol{\kappa}_2$ and $\boldsymbol{\kappa}_1-\boldsymbol{\kappa}'_2$, respectively, and all the other quantities have the same meaning as before.

Comparing Eq. (27) with Eq. (15), we can see that they have the same form. If $\mathcal{T}=\mathcal{R}=\frac{1}{2}$, Eq. (27) is exactly the same as Eq. (15), except for a minus sign in front of the interference term and a factor of $\frac{1}{4}$. The two detector in this case are far apart from each other and the possibility of interaction between the two detection processes can be ruled out if fast switching is done.[25]

It is not necessary for the two fields to come in from different sides of the beam splitter. If the two fields are superimposed before they strike the beam splitter, the factor i in Eq. (22) will be missing but otherwise the treatment is the same. The result, however, is a little different: in Eq. (27), the minus sign in front of the interference term is changed to a plus sign. The difference exists because reflection at the beam splitter and the special way the observation is made introduce an extra phase shift.

IV. FOURTH-ORDER INTERFERENCE OF CORRELATED FIELDS

We have given the treatment for independent fields. For correlated fields, however, we need to know the detailed relation between the two interfering fields so that we can calculate the cross correlation terms and the interference terms in Eq. (5) or (23). In the following, we will apply the argument presented in Sec. III to the two fields generated in parametric down-conversion processes. (For interference without beam splitter, the treatment is similar. Also see Ref. 8.)

The process of parametric down-conversion is known[31–35] to generate two highly correlated photons. The two photons have a wide bandwidth $\Delta\omega$ and the same polarization, and travel in well-defined directions when $\Delta\omega\ll\omega_0$.[33] They can be described by a two-photon state[8,36]

$$|\Psi\rangle = \sum_{\omega_1,\omega_2} \Phi(\omega_1,\omega_2)\exp[-i(\boldsymbol{\kappa}_1\cdot\mathbf{r}_1\omega_1+\boldsymbol{\kappa}_2\cdot\mathbf{r}_2\omega_2)/c+i(\omega_1+\omega_2)t]\,|\omega_1,\omega_2\rangle\ , \tag{28}$$

where the weight function $\Phi(\omega_1,\omega_2)$ has the form of $\psi(\omega_1-\omega_0)\delta(2\omega_0-\omega_1-\omega_2)$ and is symmetric with respect to ω_1,ω_2 due to energy conservation and the phase matching condition, and therefore $\psi(\omega_1-\omega_0)$ is symmetric around ω_0. This state represents a two-photon packet which is peaked at $\mathbf{r}_1,\mathbf{r}_2$ at time t.[8,37] For convenience, we consider the time t at which photon 1 arrives at the detectors, i.e., $\mathbf{r}_1$ is on the detection plane (Fig. 3) while photon 2 has not yet arrived at the detection plane due to a path delay. If the beam splitter is removed, then, at time t, photon 1 is located at $\mathbf{r}_1$ and photon 2 at $\mathbf{r}_2$ (Fig. 3). With the beam splitter in, $\mathbf{r}_1',\mathbf{r}_2'$ are the mirror images of $\mathbf{r}_1,\mathbf{r}_2$. Therefore, $\delta r\equiv(\mathbf{r}_1'-\mathbf{r}_2)\cdot\boldsymbol{\kappa}_2=(\mathbf{r}_1-\mathbf{r}_2')\cdot\boldsymbol{\kappa}_2'$ describes the path difference of these two photons.

The autocorrelation of each field is zero for all τ because there is only one photon in each field. So the first two terms in Eq. (23) are equal to zero. We easily find this by substituting Eq. (28) into these two terms. The cross correlation terms and the interference terms in Eq. (23) are now easy to calculate. Substituting Eq. (28) into these terms, we obtain, after some simplification,

$$\langle \mathfrak{T}{:}\hat{I}_1(\mathbf{R}_1,t)\hat{I}_2(\mathbf{R}_2,t+\tau){:}\rangle = |f(\tau-\tau_1)|^2 = f^2(\tau-\tau_1)\ , \tag{29}$$

$$\langle \mathfrak{T}{:}\hat{I}_1'(\mathbf{R}_2,t)\hat{I}_2'(\mathbf{R}_1,t+\tau){:}\rangle = |f(\tau-\tau_2)|^2 = f^2(\tau-\tau_2)\ , \tag{30}$$

with

$$\tau_1\equiv\delta r/c-[\boldsymbol{\kappa}_1\cdot(\mathbf{R}_1-\mathbf{r}_1)-\boldsymbol{\kappa}_2\cdot(\mathbf{R}_2-\mathbf{r}_1')]/c\ ,$$

$$\tau_2\equiv-\delta r/c-[\boldsymbol{\kappa}_2'\cdot(\mathbf{R}_1-\mathbf{r}_1)-\boldsymbol{\kappa}_1'\cdot(\mathbf{R}_2-\mathbf{r}_1')]/c\ ,$$

and

$$f(\tau)=C\int d\omega'\,|\,l(\omega_0+\omega')l(\omega_0-\omega')\psi(\omega')e^{i\omega'\tau}\ ,$$

which is real because of the symmetric property of $\psi(\omega')$ around $\omega'=0$ and has a width of $\delta t\sim 1/\Delta\omega$; and

$$\begin{aligned}\langle \mathfrak{T}[\hat{E}_1^{(-)}(\mathbf{R}_1,t)\hat{E}_2^{(-)\prime}(\mathbf{R}_2,t+\tau)\hat{E}_1^{(+)\prime}(\mathbf{R}_2,t+\tau)\hat{E}_2^{(+)\prime\prime}(\mathbf{R}_1,t)]\rangle+\text{c.c.}\\ =2f^*(\tau-\tau_1)f(\tau-\tau_2)e^{-i\omega_0(\boldsymbol{\kappa}_1-\boldsymbol{\kappa}_2')\cdot(\mathbf{R}_1-\mathbf{r}_1)/c+i\omega_0(\boldsymbol{\kappa}_1'-\boldsymbol{\kappa}_2)\cdot(\mathbf{R}_2-\mathbf{r}_1')/c}+\text{c.c.}\\ =2f(\tau-\tau_1)f(\tau-\tau_2)\cos[2\pi(x_1-x_2)/L]\ ,\end{aligned} \tag{31}$$

where $x_1\equiv(\boldsymbol{\kappa}_1-\boldsymbol{\kappa}_2')\cdot(\mathbf{R}_1-\mathbf{r}_1)/\delta\theta$ is the coordinate of detector 1 along the direction $\boldsymbol{\kappa}_1-\boldsymbol{\kappa}_2'$ and $x_2\equiv(\boldsymbol{\kappa}_1'-\boldsymbol{\kappa}_2)\cdot(\mathbf{R}_2-\mathbf{r}_1')/\delta\theta$ is the coordinate of detector 2 along the direction $\boldsymbol{\kappa}_1'-\boldsymbol{\kappa}_2$, and $L\equiv 2\pi c/(\omega_0\delta\theta)$. In deriving Eqs. (29), (30), and (31), we have used the properties that $\mathbf{R}_1',\mathbf{R}_2',\boldsymbol{\kappa}_1',\boldsymbol{\kappa}_2',\mathbf{r}_1',\mathbf{r}_2'$ are the mirror images of $\mathbf{R}_1,\mathbf{R}_2,\boldsymbol{\kappa}_1,\boldsymbol{\kappa}_2,\mathbf{r}_1,\mathbf{r}_2$, respectively, and the origin was chosen on the BS plane. Combining Eqs. (29), (30), and (31) and integrating τ over the detection time T, we obtain, with $\mathcal{T}=\mathcal{R}=\frac{1}{2}$ in Eq. (23),

$$\begin{aligned}P_{12}(x_1,x_2,T)&=K\left[\int_{-T/2}^{T/2}d\tau f^2(\tau-\tau_1)+\int_{-T/2}^{T/2}d\tau f^2(\tau-\tau_2)-2\cos[2\pi(x_1-x_2)/L]\int_{-T/2}^{T/2}d\tau f(\tau-\tau_1)f(\tau-\tau_2)\right]\\ &=K'\{1-v\cos[2\pi(x_1-x_2)/L]\}\ ,\end{aligned} \tag{32}$$

where the visibility v is

$$v\equiv\frac{2\int_{-T/2}^{T/2}d\tau f(\tau-\tau_1)f(\tau-\tau_2)}{\int_{-T/2}^{T/2}d\tau f^2(\tau-\tau_1)+\int_{-T/2}^{T/2}d\tau f^2(\tau-\tau_2)}\ . \tag{33}$$

From Eq. (33), we find that whether v is close to unity depends mainly on the difference $\tau_1-\tau_2=2\delta r/c-2\pi(x_1+x_2)/(\omega_0 L)$, but not very much on the detection time T. So the interference does not disappear even if $T\gg 1/\Delta\omega$. This property is peculiar to fourth-order interference with correlated fields, and is not encountered for second-order interference. For independent fields, as $T\to\infty$, the cross correlation terms go to ∞ whereas interference terms stay finite and therefore v tends to 0. For correlated fields, however, the cross correlation terms have the same order as the interference terms for

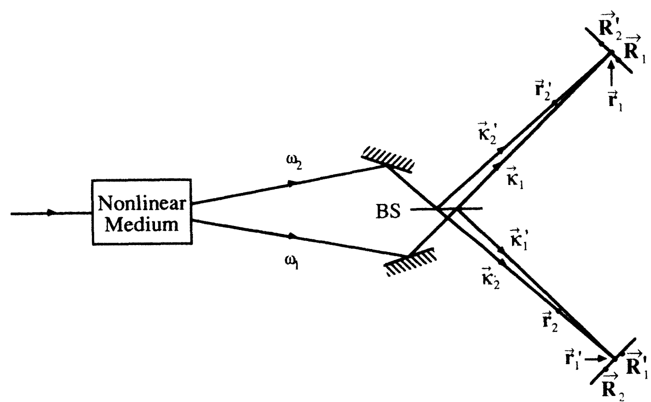

FIG. 3. Interference via a beam splitter in the parametric down-conversion process and the localization of the two down-converted photons.

all T as long as $\omega_{10}=\omega_{20}$ and the path is balanced, and they all stay finite as $T\rightarrow\infty$. That is why v does not go to zero when $T\rightarrow\infty$. We shall assume $T\gg 1/\Delta\omega$ and Eq. (33) becomes

$$v=\frac{\int_{-\infty}^{\infty}d\tau f(\tau-\tau_1)f(\tau-\tau_2)}{\int_{-\infty}^{\infty}d\tau f^2(\tau)} . \tag{34}$$

v is therefore proportional to the autoconvolution of $f(\tau)$. If $|\tau_1-\tau_2|\gg\delta t\sim 1/\Delta\omega$, then $v\approx 0$, and the fringes disappear; if $|\tau_1-\tau_2|\ll\delta t\sim 1/\Delta\omega$, then $v\approx 1$, and we get the sharpest fringes. So δt is the effective width of v. By measuring v as we change δr, we should be able to determine δt, which is a measure of the correlation time T_c of the two photons generated in parametric down-conversion processes.[35] This has been confirmed by a recent experiment[28] in which $\delta\theta$ was chosen to be close to zero. In that experiment, however, the visibility v did not quite reach 1 when $\tau_1-\tau_2=0$. This is probably due to the finite size of the detectors, for, if the fringe spacing is not much larger than the size of the detectors, we expected to measure an average modulation with decreased visibility.[27] The measurement of T_c by autocorrelation was also used by a French group.[32] In their experiment, the signal was very strong and nonlinear spectroscopy was used to detect the signal. This general method has been used for many years[38] to measure ultrashort pulse length. In the interference method, on the other hand, the signal can be very weak because photon counting is used.

Note in Eq. (34) that, for a fixed path difference δr, if x_1,x_2 are too large, i.e., if the detectors are placed too far from the peaks of the wave packet, then $|\tau_1-\tau_2|$ will be larger than δt and v will be close to 0. So this provides a way of estimating the number of interference fringes.[8]

As we mentioned in Sec. II, if $\lambda_i(\tau)=-1$ for $i=1$ and 2, the visibility v does not depend on the ratio of intensities of the two interfering fields, which is a conclusion that has no classical analog. Here, we consider such an example.

Suppose that one of the down-converted fields, say field 2, is reduced by a filter, so that the other field contains some unpaired single photon states. The effect of filter can be modeled as a beam splitter with transmissivity ζ. It can be shown from general arguments[39] that the density matrix of the system has the form

$$\rho=(1-\zeta)\sum_{\omega_1}\varphi(\omega_1)\,|\omega_1,0_2\rangle\langle\omega_1,0_2|+\zeta\rho_2 , \tag{35}$$

where $\varphi(\omega_1)$ is a normalized weight function, $\rho_2\equiv|\Psi\rangle\langle\Psi|$, with $|\Psi\rangle$ defined by Eq. (28). We can easily calculate the intensities of the two fields, which are

$$\begin{aligned}\langle\hat{I}_1\rangle&=\langle\hat{E}_1^{(-)}(\mathbf{R}_1,t)\hat{E}_1^{(+)}(\mathbf{R}_1,t)\rangle\\&=(1-\zeta)\frac{1}{\mathcal{V}}\sum_{\omega_1}|l(\omega_1)|^2\varphi(\omega_1)\\&\quad+\zeta\frac{1}{\mathcal{V}}\sum_{\omega'}|l(\omega_0+\omega')\psi(\omega')|^2 ,\end{aligned}\tag{36}$$

$$\begin{aligned}\langle\hat{I}_2\rangle&=\langle\hat{E}_2^{(-)}(\mathbf{R}_2,t)\hat{E}_2^{(+)}(\mathbf{R}_2,t)\rangle\\&=\zeta\frac{1}{\mathcal{V}}\sum_{\omega'}|l(\omega_0+\omega')\psi(\omega')|^2 .\end{aligned}\tag{37}$$

Therefore, the intensities of the two fields vary as ζ changes. If $\zeta\ll 1$, then $\langle\hat{I}_1\rangle\gg\langle\hat{I}_2\rangle$. On substituting Eq. (35) into Eq. (23), we immediately see that the first term of Eq. (35) will not contribute to P_{12} and the result is the same as Eq. (32), except for the factor ζ. So the visibility v has nothing to do with the constant ζ which determines the ratio of intensities of the two fields. This property of the two highly correlated photons is due to the characteristic of the measurement process. In the fourth-order photon detection processes, only photon pairs are registered as coincidence and highly correlated photons come in pairs. Therefore the unpaired photons will not be counted as coincidence. Here we are ignoring accidental coincidences which can be held to a low rate.

V. HIDDEN-VARIABLES THEORY IN INTERFERENCE EXPERIMENT

Fourth-order interference phenomena are an example of the nonlocal behavior of quantum mechanics.[5,6,8] In Eq. (32) with $v=1$, we can see that P_{12} vanishes if $x_1-x_2=NL$ and is maximum if $x_1-x_2=(N+\frac{1}{2})L$ where N is an integer and L is the fringe spacing. When two highly correlated photons are generated simultaneously and then travel far apart, one might think that what happens to one photon will not disturb the other as long as there is no action-at-a-distance. This is the kind of locality discussed by EPR.[20] However, as we have just seen, whether we can detect a photon at x_1 strongly depends on where the other photon is detected. This phenomenon implies that quantum mechanics violates the locality of EPR form and a hidden-variables theory, as EPR suggested,[20] is needed to restore the locality.

Hidden-variables theory has been well-studied for Bohm-type[26] EPR Gedanken experiments by Bell[21] and other workers,[22,23] who proved that this theory cannot give the same results as quantum mechanics for some cases and is violated in several experiments.[24,25] Polarization correlations experiments are examples of Bohn-type EPR Gedanken experiments. In fourth-order interference experiments, the quantity we consider is the position correlation. In the following, we will follow the procedure that Clauser *et al.*[22,23] used to derive Bell's inequalities, in order to obtain inequalities for position variables, and we show that these inequalities are violated by the predictions of quantum mechanics.

Consider the interference experiment with a beam splitter in Figs. 2 or 3. Let $p_1(x_1,\lambda)$ denote the probability of detecting one photon at position x_1 in a time period T by detector 1, given some other hidden variables λ, and $p_2(x_2,\lambda)$ is a similar quantity for detector 2. The locality property requires that the detection of one photon at x_1 is independent of the detection of another photon at x_2. Thus we write the joint probability of detecting two photons as

$$p_{12}(x_1,x_2,\lambda)=p_1(x_1,\lambda)p_2(x_2,\lambda) . \quad (38)$$

Since λ are hidden variables, the measured quantities are ensemble average over all λ with some weight function $g(\lambda)$, that is

$$p_i(x)=\int d\lambda\, g(\lambda)p_i(x,\lambda) \quad (i=1,2)$$
$$p_{12}(x_1,x_2)=\int d\lambda\, g(\lambda)p_1(x_1,\lambda)p_2(x_2,\lambda) . \quad (39)$$

We now use the following inequalities:[23]

$$-XY\leq xy-xy'+x'y+x'y'-x'Y-yX\leq 0 , \quad (40)$$

in which $0\leq x,x'\leq X$ and $0\leq y,y'\leq Y$. By putting $X=Y=1$, $p_1(x_1,\lambda)=x$, $p_1(x'_1,\lambda)=x'$, $p_2(x_2,\lambda)=y$, and $p_2(x_2,\lambda)=y$, and $p_2(x_,,\lambda)=y'$, multiplying $g(\lambda)$ and integrating over λ in inequalities (40), we obtain

$$-1\leq p_{12}(x_1,x_2)-p_{12}(x_1,x'_2)+p_{12}(x'_1,x_2)+p_{12}(x'_1,x'_2)-p_1(x'_1)-p_2(x_2)\leq 0 . \quad (41)$$

These are Bell's inequalities with respect to position variables.

The quantum mechanical predictions for these quantities are given by[29]

$$p_1(x_1)=\eta\int_T \langle \hat{I}(x_1,t)\rangle dt , \quad (42)$$

$$p_2(x_2)=\eta\int_T \langle \hat{I}(x_2,t)\rangle dt , \quad (43)$$

$$p_{12}(x_1,x_2)=\eta^2\int\int_T \langle \mathfrak{T}{:}\hat{I}(x_1,t_1)\hat{I}(x_2,t_2){:}\rangle dt_1 dt_2 , \quad (44)$$

where η is a constant which will be given later and it has been assumed that $p_i(x_i)$ $(i=1,2)$ and $p_{12}(x_1,x_2)$ do not change significantly in the detection areas and the two detectors are identical. Let us consider the experimental situation treated in Sec. IV. With the help of Eqs. (23) and (29)–(31), we have

$$p_{12}(x_1,x_2)=(T\eta^2/4)\left[\int_T^T d\tau f^2(\tau-\tau_1)+\int_{-T}^T d\tau f^2(\tau-\tau_2)-2\cos[2\pi(x_1-x_2)/L]\int_{-T}^T d\tau f(\tau-\tau_1)f(\tau-\tau_2)\right] , \quad (45)$$

where we have put $\mathcal{T}=\mathcal{R}=\frac{1}{2}$. Using Eqs. (22), (28), (42), and (43), we obtain

$$p_1(x_1)=p_2(x_2)=\eta T\frac{1}{\mathcal{V}}\sum_{\omega'} |l(\omega_0+\omega')\psi(\omega'|^2 . \quad (46)$$

As the fields propagate in well-defined directions, we can use a one-dimensional treatment and change summation to integration by using the correspondence[40]

$$\frac{1}{\mathcal{V}}\sum_{\omega'}\to\frac{1}{AL}\sum_{\omega'}\to\frac{1}{2\pi c A}\int d\omega'$$

where A is the detection area. Hence, Eq. (46) becomes

$$\begin{aligned}p_1(x_1)=p_2(x_2)&=\eta T\frac{1}{2\pi c A}\int d\omega' |l(\omega_0+\omega')\psi(\omega')|^2\\ &\approx\eta T\frac{1}{2\pi c A\,|l^2(\omega_0)|}\int d\omega' |l(\omega_0+\omega')l(\omega_0-\omega')\psi(\omega')|^2\\ &=\eta T\frac{cA}{|l^2(\omega_0)|}\int_{-\infty}^{\infty} d\tau f^2(\tau) ,\end{aligned} \quad (47)$$

where we have assumed that the bandwidth $\Delta\omega$ of $\psi(\omega')$ is much smaller than ω_0. In one dimensional treatment, η is given by[40]

$$\eta=\alpha_0 cA/|l(\omega_0)|^2 , \quad (48)$$

where α_0 is the quantum efficiency of the detector. Then Eq. (47) becomes

$$p_1(x_1)=p_2(x_2)=\left[\eta^2 T\int_{-\infty}^{\infty} d\tau f^2(\tau)\right]\Big/\alpha_0\equiv\Gamma/\alpha_0 . \quad (49)$$

In Eq. (45), if $T\gg|\tau_1|$, $|\tau_2|$, and δt, which is the width of $f(\tau)$, we obtain

$$p_{12}(x_1,x_2)=\frac{\Gamma}{2}\{1-v\cos[2\pi(x_1-x_2)/L]\} , \quad (50)$$

where Γ is the integral defined in Eq. (49) and v is defined in Eq. (34). Equations (49) and (50) are the quantum-mechanical predictions. Substituting Eq. (49) and (50) into the right inequality of inequalities (41), we obtain, with $x_1-x_2=3L/8$, $x_1-x'_2=L/8$, $x'_1x_2=5L/8$, $x'_1-x'_2=3L/8$, and $v=1$,

$$\Gamma\sqrt{2}+\Gamma-2\Gamma/\alpha_0\leq 0 , \quad (51a)$$

or

$$\sqrt{2}+1-2/\alpha_0\leq 0 . \quad (51b)$$

The inequality is violated as long as $\alpha_0 > 0.83$.

In a real experiment, it is difficult to obtain a photon detector with quantum efficiency larger than 0.83, therefore this result is not practical for a test of hidden-variables theory. In the following, by introducing two auxiliary assumptions, we will derive another Bell's inequality with respect to position correlations, which can be tested by the set-up described in Sec. IV.

We assume that the two interfering fields in Fig. 2 have orthogonal polarizations so that they do not ordinarily interfere. In order to obtain interference, we place a polarizer in front of each detector (see Fig. 4). Let $p_1(x_1,\theta_1,\lambda)$ be the probability of detecting one photon at position x_1 at detector 1, given the orientation angle θ_1 of polarizer 1, with some other hidden parameters λ needed to describe the system completely in the hidden-variables theory, and let $p_2(x_2,\theta_2,\lambda)$ be the corresponding probability for detector 2. $p_1(x_1,\infty,\lambda)$ and $p_2(x_2,\infty,\lambda)$ are similar quantities with the corresponding polarizers removed. As in Eqs. (38) and (39), we write the joint probability of detecting two photons at detectors 1 and 2 as

$$p_{12}(x_1,x_2,\theta_1,\theta_2,\lambda)=p_1(x_1,\theta_1,\lambda)p_2(x_2,\theta_2,\lambda) , \quad (52)$$

according to the locality assumption, and we take the measured quantity to be given by

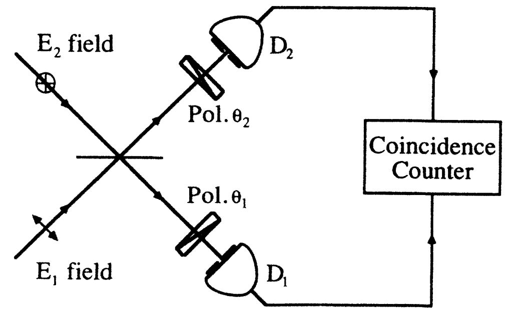

FIG. 4. Interference experiment for testing Bell's inequalities with respect to position correlations.

$$p_{12}(x_1,x_2,\theta_1,\theta_2) = \int d\lambda\, g(\lambda)p_1(x_1,\theta_1,\lambda)p_2(x_2,\theta_2,\lambda) , \quad (53)$$

From inequalities (40), by putting $p_1(x_1,\theta_1,\lambda)=x$, $p_1(x'_1,\theta_1,\lambda)=x'$, $p_2(x_2,\theta_2,\lambda)=y$, $p_2(x'_2,\theta_2,\lambda)=y'$, and $p_1(x_1,\infty,\lambda)=X$, $p_2(x_2,\infty,\lambda)=Y$, and with the assumptions (i) $p_i(x_i,\theta_i,\lambda)\le p_i(x_i,\infty,\lambda)$ (no enhancement assumption), (ii) $p_i(x_i,\infty,\lambda)=p_i(x'_i,\infty,\lambda)$ (homogeneity assumption), we obtain, after multiplying $g(\lambda)$ and integrating over λ,

$$p_{12}(x_1,x_2,\infty,\infty)\le p_{12}(x_1,x_2,\theta_1,\theta_2)-p_{12}(x_1,x'_2,\theta_1,\theta_2)+p_{12}(x'_1,x_2,\theta_1,\theta_2)+p_{12}(x'_1,x'_2,\theta_1,\theta_2)$$
$$-p_{12}(x_1,x_2,\infty,\theta_2)-p_{12}(x'_1,x_2,\theta_1,\infty)\le 0 , \quad (54a)$$

or

$$-1\le S\le 0 , \quad (54b)$$

where

$$S\equiv[p_{12}(x_1,x_2,\theta_1,\theta_2)-p_{12}(x_1,x'_2,\theta_1,\theta_2)+p_{12}(x'_1,x_2,\theta_1,\theta_2)+p_{12}(x'_1,x'_2,\theta_1,\theta_2)$$
$$-p_{12}(x_1,x_2,\infty,\theta_2)-p_{12}(x'_1,x_2,\theta_1,\infty)]/p_{12}(x_1,x_2,\infty,\infty) . \quad (54c)$$

Of the two assumptions above, the first one is fairly natural, but the second one needs a bit more discussion. If there was only one field, this assumption would be reasonable, as long as x_i,x'_i are not too far apart. When there are two fields, they might interfere with each other to generate a modulation, but this kind of interference will not occur here because the polarizations of the two fields are orthogonal. Therefore the second assumption is also reasonable. Here we only consider those hidden-variables theories in which the homogeneity property (ii) is satisfied for one field; in principle $p_1(x_1,\theta_1,\lambda)$ could change rapidly with x_1 even for one field in some unusual hidden-variables theories. We also exclude those hidden-variables theories in which $g(\lambda)$ depends on position x. Therefore inequalities (54) are somewhat less general than inequalities (41). But as we will see, they are violated by the predictions of quantum mechanics and can be tested in a real experiment like parametric down-conversion, even with detectors of small quantum efficiency.

Now let us examine the quantum-mechanical predictions. We will consider the process of parametric down-conversion. In order to make the polarizations of two interfering fields orthogonal to each other, we need to rotate 90° the polarization of one of the two photons, say photon 2. Then, Eq. (28) is replaced by

$$|\Psi\rangle = \sum_{\omega_1,\omega_2}\Phi(\omega_1,\omega_2)e^{-i(\kappa_1\cdot r_1\omega_1+\kappa_2\cdot r_2\omega_2)/c+i(\omega_1+\omega_2)t}|\omega_1 x,\omega_2 y\rangle . \quad (55)$$

Because of the polarizers, the E fields at $\mathbf{R}_1,\mathbf{R}_2$ are changed to

$$\hat{E}^{(+)}(\mathbf{R}_1,t)=\sqrt{\mathcal{T}}\hat{E}^{(+)}_{1x}(\mathbf{R}_1,t)\cos\theta_1+i\sqrt{\mathcal{R}}\hat{E}^{(+)\prime\prime}_{2y}(\mathbf{R}_1,t)\sin\theta_1 ,$$
$$\hat{E}^{(+)}(\mathbf{R}_2,t)=\sqrt{\mathcal{T}}\hat{E}^{(+)}_{2y}(\mathbf{R}_2,t)\sin\theta_2-i\sqrt{\mathcal{R}}\hat{E}^{(+)\prime\prime}_{1x}(\mathbf{R}_2,t)\cos\theta_2 , \quad (56)$$

where θ_1,θ_2 are the orientations of polarizers 1, 2, respectively. Following the same procedure as before, we obtain

$$p_{12}(x_1,x_2,\theta_1,\theta_2)=K\{\mathcal{T}^2\cos^2\theta_1\sin^2\theta_2+\mathcal{R}^2\cos^2\theta_2\sin^2\theta_1+2\mathcal{T}\mathcal{R}v\cos\theta_1\sin\theta_1\cos\theta_2\sin\theta_2\cos[2\pi(x_1-x_2)/L]\} , \quad (57)$$

where K is a constant and v is defined in Eq. (34). If we choose $\theta_1=\pi/4$, $\theta_2=-\pi/4$ and $\mathcal{T}=\mathcal{R}=\frac{1}{2}$, we have

$$p_{12}(x_1,x_2,\pi/4,\pi/4)=\frac{K}{8}[1-v\cos 2\pi(x_1-x_2)/L] \ . \quad (58)$$

By using the relation $p_{12}(\infty)=p_{12}(\theta)+p_{12}(\theta+\pi/2)$, we easily obtain

$$p_{12}(x_1,x_2,\infty,\pi/4)=p_{12}(x_1,x_2,\pi/4,\infty)=\frac{K}{4} \ , \quad p_{12}(x_1,x_2,\infty,\infty)=\frac{K}{2} \ . \quad (59)$$

In particular, if we choose (i) $x_1-x_2=3L/8$, $x_1-x_2'=L/8$, $x_1'-x_2=5L/8$, $x_1'-x_2'=3L/8$; or (ii) $x_1-x_2=L/8$, $x_1-x_2'=3L/8$, $x_1'-x_2=L/8$, $x_1'-x_2'=L/8$, we find in Eq. (54c) that

$$S=\begin{cases}(\sqrt{2}v-1)/2 & \text{for set (i)}\\ -(\sqrt{2}v+1)/2 & \text{for set (ii)} \ ,\end{cases} \quad (60)$$

which violates the inequalities (54) as long as $v>0.707$. Therefore the hidden-variables theory is in conflict with quantum mechanics with respect to position correlation, and the demonstration of violations does not require a large quantum efficiency for the detector.

VI. SUMMARY

The theory of fourth-order interference has been studied in a general format. We have generally proved that, for classical fields with independent random phases, the visibility of the interference cannot exceed $\frac{1}{2}$. The conditions under which this classical limit is exceeded have been investigated. The necessary condition is that the sum of the autocorrelations of the two interfering fields is less than the sum of the cross correlations between the two fields at two points. Another nonclassical effect is that for some states of light, the visibility of the interference does not go to zero even though the ratio of the intensities of the two interfering fields is much larger than 1, and in some special cases, the visibility does not even depend on the ratio. It was pointed out that quantum field and classical field may interfere to generate nonclassical effects. Interference between independent fields was studied in detail, and it was shown for this case that the interference pattern does not go away when the detection time T is of the order of the reciprocal bandwidth of the fields, as long as their spectra are symmetric around the same center frequencies. For some correlated fields, the visibility ν does not depend very much on the detection time T and the interference pattern is present even when $T\rightarrow\infty$. A new type of fourth-order interference experiment with a beam splitter was proposed, in which the two detectors are far apart from each other and EPR-type locality is violated by quantum mechanics. The interference of two photons generated in the parametric down-conversion process was analyzed as an example of interference between correlated fields. A method of measuring the correlation time T_c of the two down-converted photons was described. Finally, the hidden-variables theory was applied to two different interference experiments, and corresponding to each experiment, Bell's inequalities with respect to position variables were derived and shown to be violated by the predictions of quantum mechanics.

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APPENDIX: PROOF OF INEQUALITIES (8) AND (9) FOR CLASSICAL FIELDS AND INEQUALITY (8) FOR QUANTUM FIELDS

We first write each term in the inequalities in Glauber-Sudarshan $\mathcal{P}$-representation[29,41] as follows:

$$\langle \mathfrak{T}{:}\hat{I}_1(\mathbf{R}_1,t)\hat{I}_1(\mathbf{R}_2,t+\tau){:}\rangle=\int d\{\alpha\}\mathcal{P}(\{\alpha\})E_1^*(\mathbf{R}_1,t)E_1^*(\mathbf{R}_2,t+\tau)E_1(\mathbf{R}_2,t+\tau)E_1(\mathbf{R}_1t) \ , \quad (A1)$$

$$\langle \mathfrak{T}{:}\hat{I}_2(\mathbf{R}_1,t)\hat{I}_2(\mathbf{R}_2,t+\tau){:}\rangle=\int d\{\alpha\}\mathcal{P}(\{\alpha\})E_2^*(\mathbf{R}_1,t)E_2^*(\mathbf{R}_2,t+\tau)E_2(\mathbf{R}_2,t+\tau)E_2(\mathbf{R}_1,t) \ , \quad (A2)$$

$$\langle \mathfrak{T}{:}\hat{I}_1(\mathbf{R}_1,t)\hat{I}_2(\mathbf{R}_2,t+\tau){:}\rangle=\int d\{\alpha\}\mathcal{P}(\{\alpha\})E_1^*(\mathbf{R}_1,t)E_2^*(\mathbf{R}_2,t+\tau)E_2(\mathbf{R}_2,t+\tau)E_1(\mathbf{R}_1,t) \ , \quad (A3)$$

$$\langle \mathfrak{T}{:}\hat{I}_2(\mathbf{R}_1,t)\hat{I}_1(\mathbf{R}_2,t+\tau){:}\rangle=\int d\{\alpha\}\mathcal{P}(\{\alpha\})E_2^*(\mathbf{R}_1,t)E_1^*(\mathbf{R}_2,t+\tau)E_1(\mathbf{R}_2,t+\tau)E_2(\mathbf{R}_1,t) \ , \quad (A4)$$

$$\langle \mathfrak{T}[\hat{E}_1^{(-)}(\mathbf{R}_1,t)\hat{E}_2^{(-)}(\mathbf{R}_2,t+\tau)\hat{E}_1^{(+)}(\mathbf{R}_2,t+\tau)\hat{E}_2^{(+)}(\mathbf{R}_1,t)]\rangle+\text{c.c.}$$
$$=\int d\{\alpha\}\mathcal{P}(\{\alpha\})E_1^*(\mathbf{R}_1,t)E_2^*(\mathbf{R}_2,t+\tau)E_1(\mathbf{R}_2,t+\tau)E_2(\mathbf{R}_1,t)+\text{c.c.} \ , \quad (A5)$$

where E_1,E_1^*, etc. in the rhs of these equations are c numbers and defined by $\hat{E}_i^{(+)}|\{\alpha\}\rangle=E_i|\{\alpha\}\rangle$, E_i^* is the complex conjugate of E_i. $\mathcal{P}(\{\alpha\})$ is non-negative for classical fields.

Consider the following two inequalities:

$$|E_1(\mathbf{R}_1,t)E_2(\mathbf{R}_2,t+\tau)\pm E_2(\mathbf{R}_1,t)E_1(\mathbf{R}_2,t+\tau)|^2\geq 0 \ , \quad (A6)$$

$$|E_1(\mathbf{R}_1,t)E_1^*(\mathbf{R}_2,t+\tau)\pm E_2(\mathbf{R}_1,t)E_2^*(\mathbf{R}_2,t+\tau)|^2\geq 0 \ . \quad (A7)$$

Expanding the lhs of each inequality above and multiplying by the non-negative number $\mathcal{P}(\{\alpha\})$ and integrating over $\{\alpha\}$, we immediately obtain the inequalities (8) and (9).

All the arguments above are for classical fields. For nonclassical fields, $\mathcal{P}(\{\alpha\})$ may be negative, therefore we cannot use the method above. Let us consider the operator

$$\hat{O} \equiv \hat{E}_2^{(+)}(\mathbf{R}_2, t+\tau)\hat{E}_1^{(+)}(\mathbf{R}_1, t) \pm \hat{E}_1^{(+)}(\mathbf{R}_2, t+\tau)\hat{E}_2^{(+)}(\mathbf{R}_1, t) , \tag{A8}$$

with $\tau \geq 0$. We then construct the following inequality:

$$\langle \hat{O}^\dagger \hat{O} \rangle \geq 0 . \tag{A9}$$

Substituting Eq. (A8) in and expanding the lhs of this inequality, we obtain

$$\langle \hat{E}_1^{(-)}(\mathbf{R}_1,t)\hat{E}_2^{(-)}(\mathbf{R}_2,t+\tau)\hat{E}_2^{(+)}(\mathbf{R}_2,t+\tau)\hat{E}_1^{(+)}(\mathbf{R}_1,t)\rangle + \langle \hat{E}_2^{(-)}(\mathbf{R}_1,t)\hat{E}_1^{(-)}(\mathbf{R}_2,t+\tau)\hat{E}_1^{(+)}(\mathbf{R}_2,t+\tau)\hat{E}_2^{(+)}(\mathbf{R}_1,t)\rangle$$
$$\pm[\langle \hat{E}_1^{(-)}(\mathbf{R}_1,t)\hat{E}_2^{(-)}(\mathbf{R}_2,t+\tau)\hat{E}_1^{(+)}(\mathbf{R}_2,t+\tau)\hat{E}_2^{(+)}(\mathbf{R}_1,t)\rangle + \text{c.c.}] \geq 0 \quad (\tau \geq 0) \tag{A10}$$

which is inequality (8) for $\tau \geq 0$. By interchanging $(\mathbf{R}_2, t+\tau)$ and $(\mathbf{R}_1, t)$ in Eq. (A8), and following the same procedure, we can easily prove that inequality (8) is also true for $\tau < 0$. Inequality (9) is violated for some nonclassical states. Therefore, only inequality (8) is generally satisfied by nonclassical fields.

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