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Effect of detuning on a single-mode modulated laser

Binoy K. Goswami and Dhruba J. Biswas*
Laser Division, Bhabha Atomic Research Centre, Trombay, Bombay 400085, India
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We have compared the impact of loss, pump, and frequency modulations on a single-mode homogeneously broadened laser under varying detunings and modulation frequencies. In the case of loss and gain modulations the impact on intensity shows a minimal behavior at the line center for gain values more than twice the lasing threshold. The system relaxation time with respect to any transient perturbation depends on the magnitude of detuning and is minimal at the line center. In the case of frequency modulation, the impact on intensity vanishes at the line center irrespective of the gain values. These results are based on linear analysis.

Frequency, gain, and loss modulations have been found to be an effective way of destabilizing the emission of a single-mode laser for which one of the variables is adiabatically eliminated.[1-8] Emission at the modulation frequency, subharmonic bifurcation, quasiperiodic motion, and chaotic emission have been observed in many of these systems. Recently the critical dependence of the instability phenomenon on the cavity detuning for a gain-modulated single-mode cw CO_2 laser has been shown.[8] Here we present theoretical results, based on a perturbation technique, showing the dependence of detuning on the impact of such modulations on the system. In particular, we find that the modulation will have more impact on the detuned system than in the resonant case for gain more than double the threshold. We also have compared the effects of frequency, loss, and pump modulations under varying detunings and varying modulation frequencies. Similar studies for loss and gain have earlier been made for a resonantly tuned system with modulation at close to the relaxation oscillation frequency and for gain equal to twice the threshold.[9] These studies showed that to get instability by gain modulation similar to that obtained by loss modulation a very large ratio of the gain-to-loss modulation depth would be required. It is also important to note here that from linear analysis on intensity only, one may have a comparative analysis of instability behavior of a modulated single-mode cw laser.

Starting from the equation of motion of the density matrix and electromagnetic radiation inside a unidirectional ring-laser cavity, containing a collection of homogeneously broadened two-level atoms, we obtain the following equations (in dimensionless form) after adiabatic elimination of the polarization:

$$\dot{I}(t)=-k(t)I(t)+\omega D(t)I(t)L(\omega,\nu)\ , \tag{1a}$$

$$\dot{D}(t)=-[D(t)-D_u(t)]-I(t)D(t)L(\omega,\nu)\ , \tag{1b}$$

$$\nu_0=\nu_{p0}\{1+\tfrac{1}{2}D_s[(\omega-\nu_{p0})/\gamma_\perp]L(\omega,\nu_{p0})\}\ , \tag{1c}$$

where $I(t)$ is the slowly varying intensity inside the laser cavity normalized by the saturation intensity $p^2/\hbar^2\gamma_\parallel\gamma_\perp$, p the transition dipole-moment matrix element. $D(t)$ is the population inversion, $D_u(t)$ is the pump parameter, and D_s is the population inversion at steady state, all normalized by $p^2/\hbar\gamma_\perp\epsilon$, where ϵ is the dielectric constant of the medium. $[p^2/(\hbar\gamma_\perp\epsilon)]^{-1}$ is the population inversion required to have a fractional mode pulling equal to half of the Lorenzian line shape per unit-mode frequency per unit detuning. This definition comes from Eq. (1c). $L(\omega,\nu)$ is the Lorentzian line-shape function given by

$$1/\{1+[(\omega-\nu)/\gamma_\perp]^2\}\ ,$$

ω is the center of the transition, and ν is the slowly varying time-dependent active-mode frequency in the presence of the frequency modulation. ν_0 and ν_{p0} are the corresponding time-independent active- and passive-mode frequencies in the absence of any modulation. ω, ν, ν_0, ν_{p0}, $\gamma_\perp$, $\gamma_\parallel$, and the cavity decay rate $k(t)$ are all normalized with respect to $\gamma_\parallel$ while t is normalized with respect to $\gamma_\parallel^{-1}$.

While Eqs. (1a) and (1b) are standard Maxwell-Bloch equations, (1c) represents the amount of mode pulling.[10] Equations (1a) and (1b) alone are insufficient to give rise to chaotic motion; therefore, we introduce an extra degree of freedom through sinusoidal modulation of the loss, the pump, or the cavity-mode frequency, in the form

$$k(t)=k_0[1+l\sin(\omega_1 t)]\ , \tag{2a}$$

$$D_u(t)=D_{u0}[1+g\sin(\omega_1 t)]\ . \tag{2b}$$

l and g are loss- and pump-modulation coefficients, respectively. Similarly, from a sinusoidal modulation on top of passive-mode frequency, it is easy to show that the modulation of the detuning will have the following form:

$$1/L(w,\nu)=1+[\delta_0+f\sin(\omega_1 t)]^2\ , \tag{2c}$$

where $\delta_0=(\omega-\nu_0)/\gamma_\perp$ is the detuning parameter (taking mode pulling into account) and f is the modulation of the detuning.

In order to find the impact of these modulations on the system, we solve the stationary state of the coupled Maxwell-Bloch equation by the perturbation technique (up to first order) where the perturbing amplitude is very small (~ 0). First we consider modulation only of the cavity-mode frequency, i.e., $D_u(t)=D_{u0}$ and $k(t)=k_0$.

Assume $I(t)=\sum_{i=0}^{\infty}f^i I^{(i)}(t)$ and $D(t)=\sum_{i=0}^{\infty}f^i D^{(i)}(t)$, i being the order of perturbation. From zeroth-order perturbation of Eqs. (1a) and (1b) we get

$$\dot{I}^{(0)}(t)=-k_0 I^{(0)}(t)+\omega D^{(0)}(t)I^{(0)}(t)L(\omega,\nu_0) \quad (3a)$$

[henceforth, $L(\omega,\nu_0)$ will be denoted by L only], and

$$\dot{D}^{(0)}(t)=-[D^{(0)}(t)-D_{u0}]-I^{(0)}(t)D^{(0)}(t)L \ . \quad (3b)$$

Equations (3a) and (3b) represent the kinetics for constant gain and loss. Asymptotic solutions reduce to the steady state defined by

$$I^{(0)}=(D_{u0}/D^{(0)}-1)/L \ , \quad (4a)$$

$$D^{(0)}=k_0/\omega L \ , \quad (4b)$$

and lasing bandwidth $=2\sqrt{(\varepsilon-1)}$, where $\varepsilon=D_{u0}\omega/k_0$ is the excitation rate normalized with respect to threshold. It is noted that D_s in Eq. (1c) is the same as $k_0/\omega L$. From first order of perturbation, we find

$$\dot{I}^{(1)}(t)=-2k_0 I_0 L\delta_0 \sin(\omega_1 t)+\omega I^{(0)}LD^{(1)}(t) \ ,$$

$$\dot{D}^{(1)}(t)=-D^{(1)}(t)(1+LI^{(0)})-LD^{(0)}I^{(1)}(t)+2L^2\delta_0 I^{(0)}D^{(0)}\sin(\omega_1 t) \ .$$

Hence,

$$\ddot{I}^{(1)}(t)+(1+LI^{(0)})\dot{I}^{(1)}(t)+k_0 LI^{(0)}I^{(1)}(t)=-2k_0 LI^{(0)}\delta_0[\omega_1\cos(\omega_1 t)+\sin(\omega_1 t)] \ . \quad (5a)$$

Similarly, treating loss modulation alone [from (2a)], one can find a similar relation, viz.,

$$\ddot{I}^{(1)}(t)+[1+LI^{(0)}]\dot{I}^{(1)}(t)+k_0 LI^{(0)}I^{(1)}(t)=-k_0 I^{(0)}\omega_1\cos(\omega_1 t)-k_0[1+LI^{(0)}]I^{(0)}\sin(\omega_1 t), \quad (5b)$$

and, with gain modulation, it takes the following form:

$$\ddot{I}^{(1)}(t)+[1+LI^{(0)}]\dot{I}^{(1)}(t)+k_0 LI^{(0)}I^{(1)}(t)=\omega LD_{u0}I^{(0)}\sin(\omega_1 t) \ . \quad (5c)$$

Equations (5) describe conventional forced damped oscillatory motion of different forcing modulations. The system will relax to steady state with respect to any transient perturbation with frequency

$$\Omega=(k_0 LI^{(0)})^{1/2}=[k_0(\varepsilon L-1)]^{1/2}$$

and with a damping rate $L\varepsilon$.

The relaxation oscillation frequency and the damping rate are symmetrically dependent on the detuning parameter δ_0 and their values are maximum at line center ($\nu=\omega$). Thus, the system takes a longer time to attain the steady state with respect to any transient perturbation the higher the detuning is. The asymptotic amplitude of

$$I^{(1)}(t)\equiv I_0^{(1)}=2\Omega^2\delta_0\{(\omega_1^2+1)/[(\Omega^2-\omega_1^2)^2+L^2\varepsilon^2\omega_1^2]\}^{1/2} \text{ (for frequency modulation)} \ , \quad (6a)$$

$$=(\Omega^2/L)\{(\omega_1^2+\varepsilon^2L^2)/[(\Omega^2-\omega_1^2)^2+L^2\varepsilon^2\omega_1^2]\}^{1/2} \text{ (for loss modulation)} \ , \quad (6b)$$

$$=(\Omega^2\varepsilon)/[(\Omega^2-\omega_1)^2+L^2\varepsilon^2\omega_1^2]^{1/2} \text{ (for gain modulation)} \ . \quad (6c)$$

The dependence of $I_0^{(1)}$ on detuning as well as frequency of modulation (for $\varepsilon=2$) is illustrated three dimensionally in Fig. 1 for the case of frequency modulation. Maximum values of $I_0^{(1)}$ occur at modulation frequencies which are equal to the respective relaxation oscillation frequencies for different detunings. Qualitatively, this behavior is similar for all three kinds of modulation. In Fig. 2 we have plotted this maximum value of $I_0^{(1)}$ versus detuning for all types of modulation for $\varepsilon=2$. From this figure a relative measure of the impact of all three kinds of modulation can be made. It will be observed

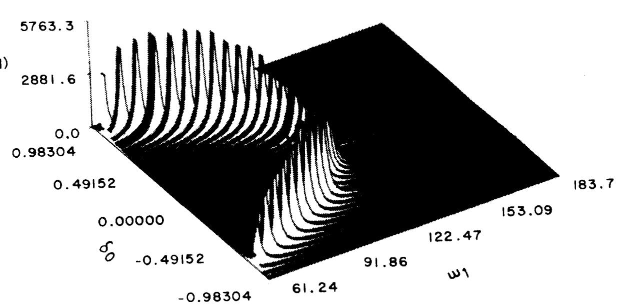

FIG. 1. Dependence of $I_0^{(1)}$ on detuning δ_0 and ω_1 in case of frequency modulation. Excitation ration $\varepsilon=2$ and $k_0=1.5\times10^4$.

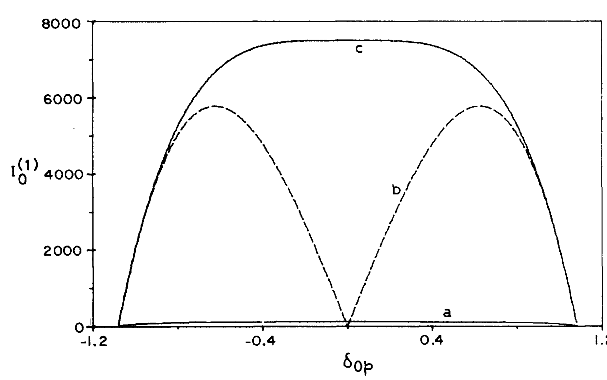

FIG. 2. Dependence of $I_0^{(1)}$ on detuning (passive cavity) δ_{0p} ($\omega_1=\Omega$) for all three modulations. Excitation ratio $\varepsilon=2$ and $k_0=1.5\times10^4$. (a) gain, (b) frequency, and (c) loss modulations, respectively.

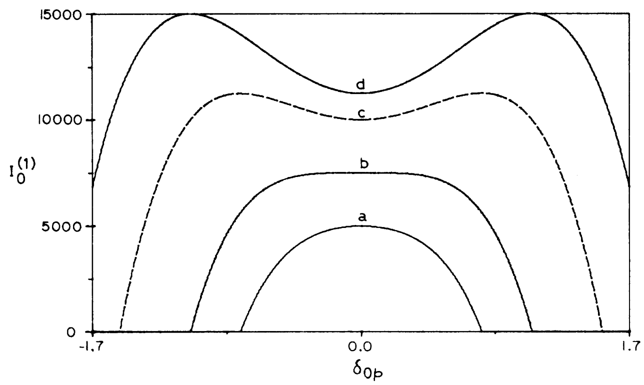

FIG. 3. Dependence of $I_0^{(1)}$ on detuning δ_{0p} ($\omega_1 = \Omega$) for various values of ε in case of loss modulation. $k_0 = 1.5 \times 10^4$ and (a) $\varepsilon = 1.5$, (b) $\varepsilon = 2.0$, (c) $\varepsilon = 3.0$, and (d) $\varepsilon = 4.0$, respectively.

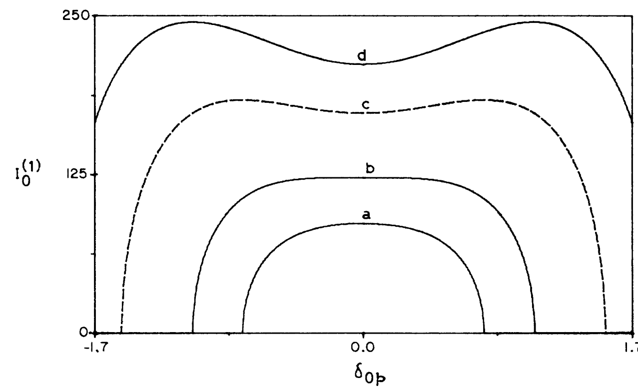

FIG. 4. Dependence of $I_0^{(1)}$ on detuning δ_{0p} ($\omega_1 = \Omega$) for various values of ε in case of gain modulation. $k_0 = 1.5 \times 10^4$ and (a) $\varepsilon = 1.5$, (b) $\varepsilon = 2.0$, (c) $\varepsilon = 3.0$, and (d) $\varepsilon = 4.0$, respectively.

that the impact of the loss modulation is always more than that of gain modulation. From Fig. 2 it will also be seen that frequency modulation $\{\delta_0 \gtrsim [(\varepsilon - 1)/3]^{1/2}\}$ will have an impact of the same order as that of loss. We note here that instabilities and chaos have been observed in CO_2 lasers with frequency[2,3] and loss[4,5] modulations at close to the relaxation oscillation frequency. The larger impact on intensity for loss and frequency modulations than for gain modulation could be well understood since the Maxwell-Bloch equations, from which Eqs. (6) were derived, reveal the fact that the laser intensity is susceptible to a change in the pump parameter (D_u) in the second order while also susceptible to a similar change in loss in the first order. On the other hand, population inversion is susceptible to a change in the pump parameter in the first order, while open to a similar change in loss in the second order. Frequency modulation will have first-order impact on both intensity and population inversion. Thus, as long as intensity is considered, effect of frequency or loss modulation will be dominant in comparison to gain modulation, and for the same reason, frequency or loss modulation will have similar impact on the intensity of the system. From Eqs. (6) one obtains in the limit $\omega_1 = \Omega$,

$$I_0^{(1)} = 2\Omega^2 \delta_0 / L\varepsilon \quad \text{(for frequency modulation)} , \tag{7a}$$

$$= \frac{\Omega^2}{L} (1/\Omega^2 + 1/L^2\varepsilon^2)^{1/2} \quad \text{(for loss modulation)} , \tag{7b}$$

$$= \Omega / L \quad \text{(for gain modulation)} . \tag{7c}$$

From Eqs. (7) we find that for gain up to twice the lasing threshold ($\varepsilon \leq 2$) the amplitude $I_0^{(1)}$ is maximum only at line center. But for gain more than twice the threshold ($\varepsilon > 2$) it will be seen that amplitude will have a dip at line center with a maximum symmetrically spaced on either side.[11] To illustrate this point graphically we have plotted $I_0^{(1)}$ vs δ_0 for $\varepsilon = 1.5$, 2, 3, and 4, and $\omega_1 = \Omega$ in Figs. 3 and 4 for loss and gain modulations, respectively. In the case of gain modulation and loss modulation the absolute value of δ_0 at which maximum will occur is $(\varepsilon/2 - 1)^{1/2}$. The ratio of dip height to $I_0^{(1)}$ at line center is

$$\frac{(\varepsilon^2/4) - \varepsilon + 1}{\varepsilon - 1} \quad \text{(for loss modulation)} ,$$

$$\left[\frac{[(\varepsilon^2/4) - \varepsilon + 1]}{\varepsilon - 1} \right]^{1/2} \quad \text{(for gain modulation)} ,$$

both of which increase as ε increases. The corresponding values of δ_0 in the case of frequency modulation are $[(\varepsilon - 1)/3]^{1/2}$, which is $1/\sqrt{3}$ times the lasing bandwidth (half-width). It is notable here that $I_0^{(1)}$ vanishes at line center. Indeed, instabilities and chaos have been observed in CO_2 lasers with frequency modulation in the detuned case.[2,3] It will be interesting to investigate the effect of gain and loss modulation with the use of detuning as a control parameter when gain is more than double the lasing threshold.

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*Also at Multidisciplinary Research Section, Bhabha Atomic Research Center, Bombay 400085, India.

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[10]From the basic equations, it will be seen that laser frequency ν is dependent on $D(t)$. For simplification, $D(t)$ is replaced by D_s. This is justified if population inversion already has reached a steady state, or the temporal fluctuation of inversion is much smaller compared to its average value. In both the cases, it is well known that D_sL is determined by passive cavity decay and active medium constants. For higher gain, this simplification may have to be reconsidered.

[11]Although effective gain is $L\varepsilon$, the dependence of $I_0^{(1)}$ on ε is not symmetric with that on L. Besides, dip arises along δ_0. Hence, a similar nature of $I_0^{(1)}$ with respect to ε at constant δ_0 cannot be expected.

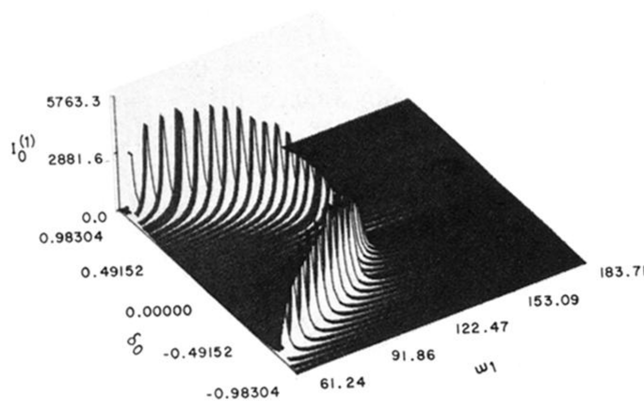

FIG. 1. Dependence of $I_0^{(1)}$ on detuning δ_0 and ω_1 in case of frequency modulation. Excitation ration $\varepsilon=2$ and $k_0=1.5\times10^4$.