Kicked quantized cavity mode: An open-systems-theory approach

G. J. Milburn
Department of Physics and Theoretical Physics, Faculty of Science, The Australian National University, Canberra A.C.T. 2601 Australia
(Received 27 October 1986)

Techniques from the quantum theory of open systems (operations) are applied to the model of a kicked quantized cavity mode recently presented by Filipowicz, Javanainen, and Meystre [J. Opt. Soc. Am. B **3**, 906 (1986)]. These techniques permit a straightforward discussion of atomic velocity profile in the model.

I. INTRODUCTION

In a recent interesting paper[1] Filipowicz *et al.* have investigated the classical and quantum dynamics of a single-mode field interacting coherently with a stream of two-level atoms under the assumptions: (i) that only one atom at a time is coupled to the field, (ii) the interaction times are all equal, and (iii) the atoms arrive at definite known times. As they point out, this is the quantum optics analog of a coherently kicked harmonic oscillator. In this paper I present an analysis of the model using techniques from the quantum theory of open systems (as developed, for example, in the book of the same title by Davies[2]). These techniques are also of considerable use in quantum measurement theory.[3,4] In fact, some aspects of the model discussed here are examples of general results in quantum measurement theory (e.g., pointer basis evolution[5]).

In order to apply these techniques I assume that the atoms arrive in the cavity at Poisson distributed times. The assumption that only one atom at a time interacts with the cavity mode is retained in that for "sufficiently small" time intervals Δt the probability of more than one atom being present in the cavity is assumed to be zero. The precise meaning of "sufficiently small" will be explained in Sec. II. The advantages of the approach adopted here are (i) one can obtain an evolution equation for the field density operator, (ii) one can describe the probability distributions for postinteraction atomic measurements, (iii) there is a simple method to take account of the atomic velocity profile, and (iv) all essential quantum predictions of the discrete map dynamics are retained.

The results obtained confirm and extend the predictions of Filipowicz *et al.* The interesting prediction that one may drive the field into a number state by injecting initially inverted atoms of the right velocity class is upheld. Unfortunately, when the atomic velocity distribution is taken into account this does not occur. However, for a sufficiently small spread in atomic velocities the rate at which the photon number escapes the "trapping" eigenstate is quite small. Thus this unusual quantum feature may have observable effects.

The ability to describe the results of postinteraction atomic measurements enables the predictions of the model to be expressed in experimentally verifiable form. In a realization of the experiment as Rydberg atoms interacting with a microwave cavity, one can directly measure the final states of the atoms using ionization techniques.

The approach of this paper is generally applicable to a wide range of kicked dynamical models. In fact, some of the predictions of the model are special examples of general features of kicked quantum systems. These general features will be discussed in a forthcoming paper.

II. DYNAMICS OF KICKED QUANTUM SYSTEMS

In order to motivate the introduction of operations and effects, it is helpful to consider some ideas from quantum measurement theory. In fact, there is a close relationship between the dynamics of kicked systems and the effect of measurements; from the point of view of the measured system, a measurement is just a particular kind of kick.

In the standard description of measurements of the first kind, the probability to obtain a result a from a measurement of the physical quantity represented by the operator $\hat{A}$ with a discrete spectrum is

$$P(a)=\mathrm{tr}[\hat{\rho}(t)\,|\,a\rangle\langle a\,|\,]\,, \tag{2.1}$$

where $|\,a\rangle$ is the corresponding eigenstate of $\hat{A}$ and $\hat{\rho}(t)$ represents the state of the system immediately prior to the measurement. The change in the state of the system is

$$\hat{\rho}(t)\rightarrow\hat{\tilde{\rho}}_a(t)=\frac{\mathrm{tr}[\,|\,a\rangle\langle a\,|\,\hat{\rho}(t)\,|\,a\rangle\langle a\,|\,]}{P(a)}$$

$$=|\,a\rangle\langle a\,|\,. \tag{2.2}$$

The interpretation of this transformation is as follows. One subjects each identically prepared element of an ensemble represented by $\hat{\rho}(t)$ to a measurement of $\hat{A}$ and "selects" all those elements which give result a to form a new ensemble represented by $\hat{\tilde{\rho}}_a$.

The above description of a perfectly accurate measurement is easily extended to treat inaccurate measurements.[6] The change in $\hat{\rho}(t)$ for a selective operation is then

$$\hat{\rho}(t)\rightarrow\hat{\tilde{\rho}}_a=\frac{\phi_a\hat{\rho}(t)}{P(a)}\,, \tag{2.3}$$

where ϕ_a is a positive operator on the space of density operators. Equation (2.3) generalizes Eq. (2.2). The

operators ϕ_a are known as superoperators, Liouville operators or operations. Clearly, one requires that the probability for a result a is

$$P(a)=\mathrm{tr}[\phi_a\hat{\rho}(t)] \ .$$

The precise form of ϕ_a depends on the measurement model. In Ref. 7 examples are constructed for a particular system-measuring apparatus interaction.

The above formalism may be used to describe kicked quantum systems (or classical systems). Let the probability of a kick in a sufficiently small time interval Δt be given by

$$P(t)\Delta t=\gamma\Delta t \tag{2.4}$$

with γ a positive constant. Further, assume the probability of more than one kick in time Δt is zero and that each kick is an independent event. That is, the probability for a kick to occur in time Δt is independent of kicks at all other times and of the intrinsic dynamics of the system. These assumptions define the kick arrivals to be Poisson distributed.[8] The kick arises from the coupling of the system to another system for a characteristic time τ, thus Δt though small on the time scale of system dynamics (γ^{-1}) is large compared to τ. In the model of this paper γ is an externally specified parameter independent of the state of the system.

Now consider the following selective operation. Let each element of the ensemble represented by $\hat{\rho}(t)$ interact with the kick apparatus. Select all those systems which actually are kicked in a time Δt to form a new ensemble described by $\hat{\rho}_1(t+\Delta t)$. This is determined by $\hat{\rho}(t)$ and a selective operator ϕ_τ as

$$\hat{\rho}_1(t+\Delta t)=\frac{\gamma\phi_\tau\hat{\rho}(t)\Delta t}{\mathrm{tr}[\gamma\phi_\tau\hat{\rho}(t)\Delta t]} \ . \tag{2.5}$$

The parameter τ characterizes an element of a class of operations, corresponding to different system-kick apparatus interaction times. The complementary selection procedure—that is, selection on the basis of no kick—described by an operation $\tilde{\phi}_\tau(t)$, is as follows:

$$\hat{\tilde{\rho}}_1(t+\Delta t)=\frac{\tilde{\phi}_\tau(\Delta t)\tilde{\rho}(t)}{\mathrm{tr}[\tilde{\phi}_\tau(\Delta t)\tilde{\rho}(t)]} \ . \tag{2.6}$$

Clearly, the probability of no kick in time Δt is

$$\mathrm{tr}[\tilde{\phi}_\tau(\Delta t)\hat{\rho}(t)]=1-\gamma\Delta t \ . \tag{2.7}$$

Furthermore, in the absence of kicks the state of the system must change according to ordinary Schrödinger evolution. Thus

$$\tilde{\phi}_\tau(\Delta t)\hat{\rho}=(1-\gamma\Delta t)e^{-(i/\hbar)\hat{H}_0\Delta t}\hat{\rho}e^{(i/\hbar)\hat{H}_0\Delta t} \ . \tag{2.8}$$

If no account is taken of whether the system is kicked or not—that is, both kicked and nonkicked systems are mixed to form the postinteraction ensemble—this ensemble is described by the density operator

$$\hat{\rho}(t+\Delta t)=\gamma\phi_\tau\hat{\rho}(t)\Delta t+\tilde{\phi}_\tau(\Delta t)\hat{\rho}(t) \ . \tag{2.9}$$

This is an example of a "nonselective operation." An evolution equation for the state of the system may then be defined by

$$\frac{d\hat{\rho}(t)}{dt}=\lim_{\Delta t\to 0}\left[\frac{\hat{\rho}(t+\Delta t)-\hat{\rho}(t)}{\Delta t}\right] \ . \tag{2.10}$$

Substituting Eq. (2.8) and (2.9) into (2.10), the evolution equation becomes

$$\frac{d\hat{\rho}(t)}{dt}=\frac{-i}{\hbar}[\hat{H}_0,\hat{\rho}(t)]+\gamma[\phi_\tau\hat{\rho}(t)-\hat{\rho}(t)] \ . \tag{2.11}$$

This equation is expected to describe a wide variety of kicked dynamical problems with different choices for ϕ_τ. It is similar to an equation recently obtained by Ghirardi, Rimini, and Weber[9] to describe the emergence of a classical phase-space description from underlying quantum processes. A similar equation also results from various models of time continuous measurements.[3,4] It should be noted that Eq. (2.11) describes nonunitary evolution. This is to be expected as a kicked quantum system is an open system. The problem now is to calculate ϕ_τ for a specific model. In Sec. III this is carried out for the kicked quantized cavity mode model of Filipowicz *et al.*

The formalism presented above enables one to calculate a number of fundamental probabilities.[10] If no kick occurs in the interval $[0,t]$, the change in the state of the system is given by $\tilde{\phi}(t)\hat{\rho}(0)$ where

$$\tilde{\phi}(t)\equiv\lim_{\substack{n\to\infty\\ \Delta t\to 0}}[\tilde{\phi}_\tau(\Delta t)]^n$$

and $n\Delta t=t$. Using Eq. (2.8) this becomes

$$\tilde{\phi}(t)\hat{\rho}=e^{-(i/\hbar)\hat{H}_0t-(\gamma/2)t}\hat{\rho}e^{(i/\hbar)\hat{H}_0t-(\gamma/2)t} \ . \tag{2.12}$$

One may now calculate, for example, the probability for one kick to occur at the end of the time interval $[0,t]$. It is given by

$$\mathrm{tr}[\gamma\phi_\tau\tilde{\phi}(t)\hat{\rho}(0)]=\gamma e^{-\gamma t} \ .$$

Thus the mean arrival time for the first kick is found to be γ^{-1}. This is a typical result for a Poisson process.

III. THE KICKED CAVITY MODEL

In this model the system of interest is a single quantized cavity mode. The kicking apparatus is a stream of two-level atoms. Each atom interacts with the intracavity field for a time τ and enters the cavity at Poisson distributed times. Initially all interaction times will be assumed to be equal. In Sec. V this assumption will be relaxed; the interaction times each atom experiences will be taken to be an independent statistical event described by a Gaussian distribution. Thus the formalism of Sec. II may be applied. On resonance the interaction picture Hamiltonian describing the interaction is

$$H_I=\hbar g(c_2^\dagger c_1 a+c_1^\dagger c_2 a^\dagger) \ , \tag{3.1}$$

where $a^\dagger,a$ are Bose creation and destruction operators for the intracavity field, $c_i^\dagger,c_i$ are Fermi creation and destruction operators for the two levels, and g is a coupling constant. Level $|1\rangle$ is the ground state while $|2\rangle$ is the

excited state. Let $\hat{\rho}_F(t)\otimes\hat{\rho}_A(t)$ represent the state of the total field-atom system immediately prior to an interaction period. The state of the field at time $t+\Delta t$ assuming one atom has interacted with the field for a time τ is

$$\hat{\rho}_F(t+\Delta t)=\mathrm{tr}_A[U(\tau)\hat{\rho}_A(t)\otimes\hat{\rho}_F(t)U^\dagger(\tau)] , \tag{3.2}$$

where

$$U(\tau)=\exp[-ig\tau(c_2^\dagger c_1 a+c_1^\dagger c_2 a^\dagger)] .$$

The selective operation ϕ_τ is then defined by

$$\phi_\tau\hat{\rho}_F(t)\equiv\mathrm{tr}_A[U(\tau)\hat{\rho}_A(t)\otimes\hat{\rho}_F(t)U^\dagger(\tau)] . \tag{3.3}$$

Let the state of the atoms injected into the cavity be the mixed state

$$\hat{\rho}_A(t)=\lambda_1|1\rangle\langle 1|+\lambda_2|2\rangle\langle 2| , \tag{3.4}$$

where $\lambda_i\geq 0$ with $\lambda_1+\lambda_2=1$. In Ref. 1, λ_1 and λ_2 were determined by a thermal distribution,

$$\lambda_1=(1-\lambda_2)=(1+e^{-\beta})^{-1} , \tag{3.5}$$

where $\beta=\hbar\omega_A/kT$, with ω_A the atomic transition frequency. Expanding $\hat{\rho}_F(t)$ in a number-state basis Eq. (3.3) together with Eq. (3.4) may be used to show

$$\begin{aligned}\phi_\tau\hat{\rho}_F(t)=\sum_{n,m=0}^{\infty} P_{nm}(t)[&\alpha(n,m)|n\rangle\langle m| \\ &+\beta_1(n,m)a|n\rangle\langle m|a^\dagger \\ &+\beta_2(n+1,m+1)a^\dagger|n\rangle\langle m|a] ,\end{aligned} \tag{3.6}$$

where

$$\alpha(n,m)=\lambda_1\cos(\Omega_n\tau)\cos(\Omega_m\tau)+\lambda_2\cos(\Omega_{n+1}\tau)\cos(\Omega_{m+1}\tau) , \tag{3.7}$$

$$\beta_i(n,m)=\lambda_i(nm)^{-1/2}\sin(\Omega_n\tau)\sin(\Omega_m\tau) , \tag{3.8}$$

and

$$\Omega_n=g\sqrt{n} . \tag{3.9}$$

One then easily verifies that $1=\mathrm{tr}[\phi_\tau\hat{\rho}_F(t)]$ as required. Equation (3.6) when substituted into Eq. (2.11) fully determines the field dynamics.

The evolution equation for the intracavity field $P(n)=\langle n|\hat{\rho}_F(t)|n\rangle$ is easily found to be the master equation

$$\frac{d}{dt}P(n,t)=\omega^+(n-1)P(n-1,t)+\omega^-(n+1)P(n+1,t)-[\omega^+(n)+\omega^-(n)]P(n,t) , \tag{3.10}$$

where

$$\omega^+(n)=\gamma\lambda_2\sin^2(\Omega_{n+1}\tau) , \tag{3.11}$$

$$\omega^-(n)=\gamma\lambda_1\sin^2(\Omega_n\tau) . \tag{3.12}$$

This equation has also been obtained in Ref. 11. As discussed in Ref. 1 a steady state exists if $\lambda_2<\lambda_1$ and is given by

$$P_s(n)=\left[1-\frac{\lambda_2}{\lambda_1}\right]\left[\frac{\lambda_2}{\lambda_1}\right]^n . \tag{3.13}$$

When $\lambda_1>\lambda_2$ a steady state may not occur; however, as noted by Filipowicz *et al.*, an interesting situation arises when $\lambda_2=1$ ($\lambda_1=0$). In this case $\omega^-(n)=0$ for all n and only birth transitions are possible. It may then arise that for suitable interaction times $\omega^+(n)=0$ for some n, say, n_0, as $\sin(\Omega_{n_0+1}\tau)=0$. The level $|n_0\rangle$ becomes a "trap" in the number-state energy ladder in which population gradually accumulates due to transitions from occupied lower levels. In fact, there will be more than one level trap due to the multiple zeros of the sine function. Consideration of $P(n,t)$ in this case permits the conclusion that the field evolves to either a linear superposition of number states or a classical mixture of number states. In order to decide the issue, one must examine the off-diagonal elements of $\hat{\rho}(t)$. Assume that the initial state is a linear superposition of two adjacent trapping levels, i.e.,

$$\hat{\rho}(0)=\tfrac{1}{2}(|n_1\rangle+|n_2\rangle)(\langle n_1|+\langle n_2|) , \tag{3.14}$$

where $\omega^+(n_1)=\omega^+(n_2)=0$. A formal solution of Eq. (2.11) may be written as

$$\hat{\rho}(t)=\sum_{m=0}^{\infty}\mathcal{N}_t(m)\hat{\rho}(0) , \tag{3.15}$$

where

$$\mathcal{N}_t(m)=\gamma^m\int_0^t dt_m\int_0^{t_m}dt_{m-1}\cdots\int_0^{t_2}dt_1\tilde{\phi}_\tau(t-t_m)\phi_\tau\tilde{\phi}(t_m-t_{m-1})\cdots\phi_\tau\tilde{\phi}(t_1)\hat{\rho}(0) . \tag{3.16}$$

Using Eqs. (2.12) and (3.6), one may show the initial superposition state in Eq. (3.14) does not change, i.e.,

$$\hat{\rho}(t)=\hat{\rho}(0)=\tfrac{1}{2}(|n_1\rangle+|n_2\rangle)(\langle n_1|+\langle n_2|) .$$

Thus coherences between trapping levels do not decay. Consider now the coherence between a trapping level n_0 and the next highest level n_0+1. Using Eq. (3.15) one may show

$$\langle n_0|\hat{\rho}(t)|n_0+1\rangle=\exp\{-\gamma t[1-\cos(\Omega_{n_0+2}\tau)]\}\times\langle n_0|\hat{\rho}(0)|n_0+1\rangle ,$$

which decays to zero in general.

It would appear that the system evolves towards a coherent superposition of number states at the trapping level numbers. This is a uniquely quantum feature due to the discrete nature of the intracavity energy-level spectrum.

The decay of off-diagonal coherence in a particular basis is characteristic of the evolution of systems subject to measurement. Zurek[5] has shown that the existence of such a basis, called the "pointer basis," is essential in establishing unambiguously what physical quantity is measured and further prevents macroscopic pointer superposition states from occurring. More generally, the rapid decay of off-diagonal coherence provides a mechanism for the appearance of classical behavior in open quantum systems, as the initial states of the system approach a suitable macroscopic limit.[9,12] The situation of a periodically kicked quantum system exhibits this effect. However, there is an important additional feature. As shown above, certain coherences in the number-state basis are "immune" to the effect of the kicks. This is due to the deterministic nature of the kicks. Similar features arise in other models of kicked quantum systems and examples will be given in a future paper. The presence of these special coherences is likely to lead to observable quantum effects as the macroscopic level is approached despite the decay of off-diagonal coherence typical of open quantum systems.

Before considering the description of postinteraction atomic measurements, it will be useful to derive some results for the mean photon number of the field. It follows directly from the identification of Eq. (3.10) as a master equation that[8]

$$\frac{d\langle n\rangle}{dt}=\langle\omega^{+}(n)\rangle-\langle\omega^{-}(n)\rangle\ , \tag{3.17}$$

where $\langle n\rangle$ is the mean photon number for the field. Using Eqs. (3.11) and (3.12), it is clear that

$$-\gamma\lambda_1\le\frac{d\langle n\rangle}{dt}\le\gamma\lambda_2\ . \tag{3.18}$$

If all the atoms are injected in the ground state, the mean photon number decays in such a way that

$$\langle n(t)\rangle\ge\langle n(0)\rangle-\gamma t\ . \tag{3.19}$$

If all the atoms are injected in the excited state, the mean photon number grows in such a way that

$$\langle n(t)\rangle\le\langle n(0)\rangle+\gamma t\ . \tag{3.20}$$

IV. ATOMIC MEASUREMENTS

As pointed out in Ref. 1, the model discussed here may be experimentally realized by injecting a low density beam of Rydberg atoms into a high-Q microwave cavity. In such experiments measurements are usually made directly on the atomic systems after interaction rather than on the intracavity field. The formalism of operations is ideally suited to describing such measurements.

In Sec. III the operation ϕ_τ was defined. This operation determines the change in the state of the field when no attempt is made to monitor the postinteraction atomic states. To take account of atomic measurements, one must decompose ϕ_τ into selective operations with selection based on whether the atom is found in the excited or ground state upon leaving the cavity.

Assume that the atomic measurements are ideal, that is, the atomic measurements project the atom into either the excited state or the ground state. The selective operations describing selection according to the results of such measurements are

$$\phi_\tau^{(i)}\hat{\rho}_F\equiv\mathrm{tr}_A[\,|\,i\rangle\langle i\,|\,\hat{U}(\tau)\hat{\rho}_A\otimes\hat{\rho}_F\hat{U}^{\dagger}(\tau)]\ , \tag{4.1}$$

where $i=1$ describes selection if the atom is found in the ground state and $i=2$ describes selection if the atom is found in the excited state. Using Eqs. (3.4) and (3.6), one finds

$$\phi_\tau^{(1)}\hat{\rho}(t)=\sum_{n,m=0}^{\infty}P_{nm}(t)[\lambda_1\cos(\Omega_n\tau)\cos(\Omega_m\tau)\,|\,n\rangle\langle m\,|+\beta_2(n+1,m+1)a^{\dagger}\,|\,n\rangle\langle m\,|\,a]\ , \tag{4.2}$$

$$\phi_\tau^{(2)}\hat{\rho}(t)=\sum_{n,m=0}^{\infty}P_{nm}(t)[\lambda_2\cos(\Omega_{n+1}\tau)\cos(\Omega_{m+1}\tau)\,|\,n\rangle\langle m\,|+\beta_1(n,m)a\,|\,n\rangle\langle m\,|\,a^{\dagger}\,|\,]\ . \tag{4.3}$$

Not surprisingly,

$$\phi_\tau=\phi_\tau^{(1)}+\phi_\tau^{(2)}\ , \tag{4.4}$$

which says that the operation ϕ_τ is the sum of the mutually exclusive possibilities for selection according to atomic measurements.

The probability for detecting an atom in state (i) per unit time (i.e., atomic detection rate) is

$$P_A(i,t)=\gamma\,\mathrm{tr}[\phi_\tau^{(i)}\hat{\rho}(t)]\ . \tag{4.5}$$

Using Eqs. (4.2) and (4.3),

$$P_A(1,t)=\gamma\lambda_1\langle\cos^2(\Omega_n\tau)\rangle+\gamma\lambda_2\langle\sin^2(\Omega_{n+1}\tau)\rangle\ , \tag{4.6}$$

$$P_A(2,t)=\gamma\lambda_1\langle\sin^2(\Omega_n\tau)\rangle+\gamma\lambda_2\langle\cos^2(\Omega_{n+1}\tau)\rangle\ . \tag{4.7}$$

These equations may be put in a more useful form using Eq. (3.17) for the mean photon number:

$$P_A(1,t)=\gamma\lambda_1+\frac{d\langle n\rangle}{dt}\ , \tag{4.8}$$

$$P_A(2,t)=\gamma\lambda_2-\frac{d\langle n\rangle}{dt}\ . \tag{4.9}$$

Thus $P_A(1,t)+P_A(2,t)=\gamma=P_A(t)$, the atomic detection rate regardless of atomic state.

There are three cases of interest.

(i) $\lambda_1=0$ (all atoms excited):

$$P_A(1,t)=\frac{d\langle n\rangle}{dt}\ , \tag{4.10}$$

$$P_A(2,t)=\gamma-\frac{d\langle n\rangle}{dt}\ . \tag{4.11}$$

Equation (3.18) indicates that $P_A(2,t)$ will be always positive as required. If a level trap occurs at some photon number $\langle \dot{n} \rangle \to 0$ and $P_A(1,t) \to 0$ with $P_A(2,t) \to \gamma$. This indicates that in such a situation all the atoms will be found in the excited state upon leaving the cavity. This is due to the fact that for the initial state $|n_0\rangle \otimes |2\rangle$ where $|n_0\rangle$ is the photon trapping level, the Rabi frequency is of just the right size to return the atom to the state $|2\rangle$ in an interaction time τ.

(ii) $\lambda_2 = 0$ (all atoms in the ground state):

$$P_A(1,t) = \gamma + \frac{d\langle n \rangle}{dt} , \tag{4.12}$$

$$P_A(2,t) = -\frac{d\langle n \rangle}{dt} . \tag{4.13}$$

Once again Eq. (3.18) indicates that $P_A(2,t)$ and $P_A(1,t)$ will be always positive. As the system evolves towards the steady state, the rate of detecting atoms in the ground state approaches γ, indicating that the atomic systems feel no effect from the cavity.

(iii) $\lambda_2 \langle \lambda_1; \lambda_1, \lambda_2 \neq 0$ (a steady state occurs):

$$P_A(1,t) = \gamma \lambda_1 , \tag{4.14}$$

$$P_A(2,t) = \gamma \lambda_2 . \tag{4.15}$$

The detected atoms in this case duplicate the statistics of the injected atoms.

Other interesting questions may easily be answered by the formalism. For example, what is the probability that one atom is detected in the interval $(0,t)$ and that it is in the excited state? The result is

$$P_A[2;(0,t)] = -\gamma \int_0^t \mathrm{tr}[\tilde{\phi}_\tau(t-t_1)\phi_\tau^{(2)}\tilde{\phi}_\tau(t_1)\hat{\rho}(0)]dt_1$$

$$= \gamma \lambda_2 t e^{-\gamma t} - [\langle n(t) \rangle - \langle n(0) \rangle] e^{-\gamma t} . \tag{4.16}$$

Equations (3.19) and (3.20) indicate that this remains positive for all time.

V. INCLUSION OF ATOMIC VELOCITY PROFILE

In Sec. II the operation ϕ_τ was introduced. There it was assumed that the interaction times experienced by each atom were equal. In an experiment this is an unreasonable assumption. The interaction times for each atom are determined by its velocity while passing through the cavity and these velocities may vary from one atom to the next in a completely random fashion. Ultimately, the distribution of atomic velocities is determined by the Maxwell-Boltzmann distribution of atomic velocities in the atomic source, and thus is Gaussian. If we assume that the distance traveled by each atom in the cavity is the same, this would imply a Gaussian distribution for the interaction times. Of course, the actual velocity distribution of the atoms entering the cavity is open to some experimental control. However, for the purposes of this discussion, I will assume it to be Gaussian and that the interaction times for each atom are an independent statistical event.

To treat this situation in the context of the formalism presented here, one may regard ϕ_τ as a selective operation for a selection procedure based on the interaction times τ. For the reasons discussed above, however, the interaction times τ for each atom are not usually known to the experimenter and are thus not available for use as a selection procedure. (If they were known, questions such as "What is the probability for a postinteraction atom to be found in the excited state and with a particular velocity v?" may reasonably be asked.) A typical experiment may then be described by a nonselective operation ϕ formed by summing the selective operations ϕ_τ with appropriate weights determined by the distribution of interaction times. The decomposition of ϕ into selective operations ϕ_τ is analogous to the decomposition of ϕ_τ into the selective operations $\phi_\tau^{(i)}$ as discussed in Sec. IV. Thus

$$\phi = \int_{-\infty}^{\infty} I(\tau)\phi_\tau d\tau , \tag{5.1}$$

where $I(\tau)$ is the probability distribution for interaction times. The evolution equation for the state of the field is then given by

$$\frac{d\hat{\rho}(t)}{dt} = -\frac{i}{\hbar}[\hat{H}_0, \hat{\rho}] + \gamma(\phi\hat{\rho} - \hat{\rho}) . \tag{5.2}$$

One can now proceed to derive an equation of motion for, say, the photon number distribution. As in Sec. III, a master equation results but the transition probabilities are modified to

$$\bar{\omega}^+(n) = \gamma \lambda_2 \int_{-\infty}^{\infty} I(\tau) \sin^2(\Omega_{n+1}\tau) d\tau , \tag{5.3}$$

$$\bar{\omega}^-(n) = \gamma \lambda_1 \int_{-\infty}^{\infty} I(\tau) \sin^2(\Omega_n \tau) d\tau . \tag{5.4}$$

The inclusion of the distribution of atomic velocities through the distribution of interaction times τ prevents the possibility of "level trapping" in the photon number dynamics as $\bar{\omega}^+(n)$ cannot be zero unless the support of $I(\tau)$ is the set $\{k\pi/\Omega_{n+1}\}$, $k = 0, \pm 1, \ldots$. However, for sufficiently narrow velocity distribution, a "bottleneck" could occur at a trapping level which may be observed as a decrease in the detection rate for ground-state atoms [see Eq. (4.10)].

VI. CONCLUSION

In this paper techniques from the quantum theory of open systems have been applied to the model of a kicked quantized cavity mode. This example demonstrates that such an approach may be useful in elucidating the dynamics of periodically kicked quantum systems. The main advantage of this approach is being able to work with a continuous evolution equation while retaining the essential features of the discrete map dynamics.

In the case of this model, the formalism provides a direct method to calculate experimentally accessible quantities such as the rate of detecting postinteraction atoms in the excited state. Of course, there may be other ways to obtain these quantities but the method used here is of quite general utility. Similar techniques have been used to discuss photon counting experiments, for example.[10]

The kicks have two effects on the dynamics of the kicked system. Firstly, they cause a general decay of off-diagonal coherence in the photon number-state basis.

This is typical behavior for open quantum systems.[5] Secondly, due to the deterministic nature of the kicks (and despite their Poisson distributed times of application), certain coherences in the number-state basis do not decay. This second feature is expected to be characteristic of kicked quantum systems. The destruction of off-diagonal coherence in open systems ensures that they behave like classical systems at the macroscopic level. The presence of coherences which do not decay in kicked systems may lead to observable departures of the dynamics from that expected classically, even at the macroscopic level. Further investigations of these features will be presented in a forthcoming paper.

---

[1]P. Filipowicz, J. Javanainen, and P. Meystre, J. Opt. Soc. Am. B **3**, 906 (1986).

[2]E. B. Davies, *Quantum Theory of Open Systems* (Academic, New York, 1976).

[3]A. Barchielli, L. Lanz, and G. M. Prosperi, in *Proceedings of the International Symposium on Foundations of Quantum Mechanics*, edited by S. Kamefuchi *et al.* (Physical Society of Japan, Tokyo, 1984), p. 165.

[4]C. M. Caves and G. J. Milburn (unpublished).

[5]W. H. Zurek, Phys. Rev. D **26**, 1862 (1982).

[6]K. Kraus, *States, Effects and Operations* (Springer, Berlin, 1983).

[7]C. M. Caves, Phys. Rev. D **33**, 1643 (1986).

[8]C. W. Gardiner, *Handbook of Stochastic Methods for Physics, Chemistry and the Natural Sciences* (Springer, Berlin, 1985).

[9]G. C. Ghirardi, A. Rimini, and T. Weber, Phys. Rev. D **34**, 470 (1986).

[10]M. D. Srinivas and E. B. Davies, Opt. Acta **28**, 981 (1981).

[11]J. Kraus, M. O. Scully, and H. Walther, Phys. Rev. A **34**, 2032 (1986).

[12]G. J. Milburn and C. A. Holmes, Phys. Rev. Lett. **56**, 2237 (1986).