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Angular-distribution and linear-polarization correlation of photons induced by the relativistic radiative electron-capture process

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By using the Born approximation, calculations are made for photon angular distributions of the radiative electron capture to the *L* shell and for linear polarization of photons induced by the radiative electron capture to the *K* shell for relativistic incident velocities. The angular distributions have minimum value in the forward directions and maximum values in the backward directions relative to 90°. This behavior becomes significant with an increase of velocity. The polarization correlation of the emitted photon represents the crossover feature in the case of relativistically high energy impacts.

Spindler, Betz, and Bell[1] found for the first time that the radiation pattern of radiative electron capture (REC) to the *K* shell (*K* REC) exhibits the forward-backward symmetry independent of the projectile velocity by measuring the REC spectra caused by light-ion atom collisions. They attributed this effect to a cancellation between electron retardation effects and the Lorentz transformation from the projectile frame to the laboratory frame. Later, Anholt *et al.*[2] verified this effect in the case of heavy-ion atom collisions (197-MeV/u Xe^{54+} on Be). Recently, Pacher, González, and Miraglia[3] confirmed theoretically such behavior of the REC photon angular distribution by using the nonrelativistic impulse-approximation (IA) method with retardation corrections. Further, Hino and Watanabe[4] calculated the angular distributions of *K* REC photons for 197-MeV/u Xe^{54+} + Be and 422-MeV/u U^{92+} + Be collisions, respectively, by virtue of the relativistically extended strong-potential Born (SPB) approximation. The results for the former case were in good agreement with the experiments. As for the latter case, small discrepancies appeared mainly in the backward directions, which were thought to be due to the high-$Z\alpha$ corrections to the angular distributions (Z is the atomic number of the projectile ion and α is the fine-structure constant). As a whole, however, the forward-backward symmetry was valid. To our knowledge, there are no experimental data on this process.

Recently, it has become possible to measure the angular distribution for the *L* REC photons with the beam intensities of the high-energy heavy-ion accelerators increasing rapidly. As far as we know, however, there is not even a simple prediction for the angular distribution so far. In addition, it is expected that the measurements for the polarization effects of the REC photons will become available to understand more of the REC process from other aspects. In the present article, we discuss the angular distributions for *L* REC photons and, further, the linear polarization effects for *K* REC photons by utilizing the simple formulas for the photoelectric effect (PE) given by the Born approximation,[5] and by Lorentz transforming them to the laboratory frame. Hereafter, by the projectile frame it is meant that the origin of the coordinate system is centered on an incident projectile ion, and by the laboratory frame it is meant that the origin is chosen on the center of mass of an initial target atom. Thus, the coordinate system for the PE is always defined in the projectile frame. Herein, we use *L* REC to mean the radiative electron capture to the $2p$ state of a final projectile atom, unless otherwise stated. We have little interest in the radiation pattern for the REC to the $2s$ state because it is almost the same as that for *K* REC. Furthermore, it is assumed that the REC is nothing but the inverse process of PE, namely, the (two-body) radiative recombination process (RP).[5]

First we consider the shapes of the angular distributions for the *L* REC. The angular distribution for the PE from a $2p$ state (*L* PE) is given by using the Born approximation including the lowest-order retardation corrections as[6,7]

$$\frac{d\sigma_{\mathrm{PE}}^{L}}{d\Omega'}=\frac{\sigma_{\mathrm{PE}}^{L}}{4\pi}[1+2\beta\cos\theta'(1+\sin^2\theta')] , \tag{1}$$

where $\beta=v/c$ (v and c are velocities of an ejected electron and light, respectively), θ' and Ω' denote the angle between an incident photon and an ejected electron and the solid angle in the projectile frame, respectively, and σ_{PE}^{L} represents the total cross section for the *L* PE. If the retardation is negligible, i.e., if the second term in the square brackets of Eq. (1) can be dropped, the angular distributions for the *L* PE become isotropic as shown by Bethe and Salpeter.[5] In order to get the angular distribution for the *L* REC, we carry out the Lorentz transformation from the projectile frame to the laboratory frame after replacing β in Eq. (1) by $-\beta$. The transformation reads

$$\frac{d\sigma_{\mathrm{REC}}^{L}}{d\Omega}=\gamma^{-2}(1-\beta\cos)^{-2}\left[\frac{2\omega}{p}\frac{d\sigma_{\mathrm{PE}}^{L}}{d\Omega'}\right] , \tag{2}$$

where $\gamma=(1-\beta^2)^{-1/2}$, and θ and Ω are the emission angle and the solid angle of a photon associated with the laboratory frame, respectively. Further, the quantity in the square brackets of Eq. (2) represent the differential cross section for the RP process into the *L* shell (the $2p$ states), which is the inverse process of the photoionization as stated above. The coefficient $(2\omega/p)$ is the transformation factor to obtain the RP cross section by employing the PE cross section, where ω and p stand for the photon energy and the momentum of an electron, respectively, and the factor 2 means the degree of freedom for the photon polarization. The relation between θ and θ' is given by

$$\sin\theta' = \gamma^{-1}\sin\theta(1-\beta\cos\theta)^{-1} , \quad \cos\theta' = (\cos\theta-\beta)(1-\beta\cos\theta)^{-1} . \tag{3}$$

The resultant formula for L REC becomes up to the lowest-order retardation corrections

$$\frac{d\sigma^L_{\rm REC}}{d\Omega} = \frac{\sigma^L_{\rm REC}}{4\pi}(1-2\beta\cos\theta\sin^2\theta) , \tag{4}$$

where $\sigma^L_{\rm REC}$ stands for the total cross section for the L REC. The nonrelativistic formula for $\sigma^L_{\rm REC}$ is obtained by utilizing the explicit expression for $\sigma^L_{\rm PE}$ given by Eq. (71.15) of Ref. 5.

It is easily found from Eq. (4) that the retardation effects of L REC are not completely cancelled by the Lorentz transformation, while those of K REC are cancelled.[1] Thus, the cancellation effects between the electron retardation and the Lorentz transformation do not apply to L REC case. The shapes of the angular distribution Eq. (4) are depicted for several velocities from $\beta=0.1$ to 0.4 in Fig. 1. Moreover, for comparison with them the shapes of Eq. (1) are illustrated in Fig. 2. We find that the angular distributions for the L REC have minimum values at 55° and maximum values at 135°. Furthermore, this asymmetric behavior becomes striking as the velocity increases. The observed photon spectra are given as the summation of both contributions of REC into $2s$ states and $2p$ states.[8] The radiation pattern of REC into $2s$ states is subjected to $\sin^2\theta$. Thus, it is expected that

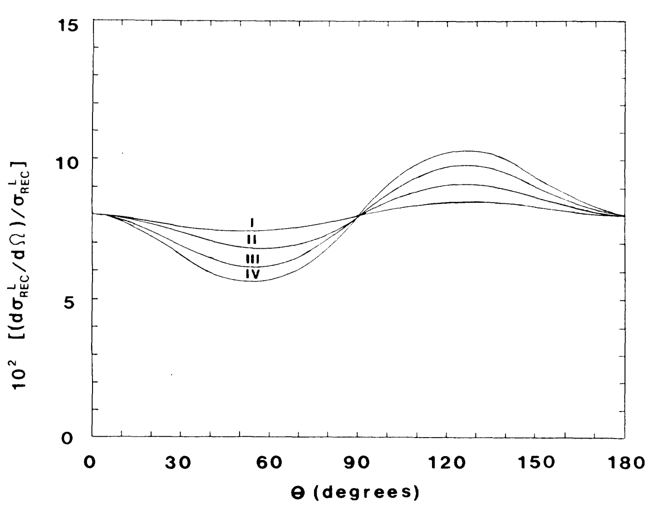

FIG. 1. Normalized photon angular distribution of radiative electron capture to the $2p$ state $(d\sigma^L_{\rm REC}/d\Omega)/\sigma^L_{\rm REC}$, predicted by the Born approximation for several velicities: I, $\beta=0.1$ (4.7 MeV/amu); II, 0.2 (19.4 MeV/amu); III, 0.3 (45.3 MeV/amu); and IV, 0.4 (85.5 MeV/amu).

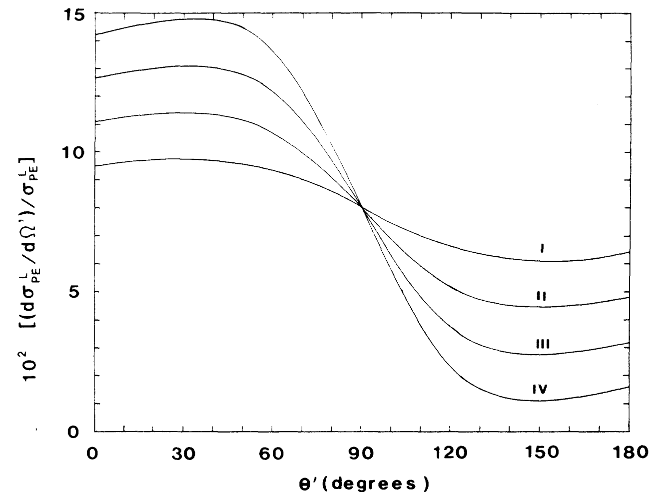

FIG. 2. Normalized angular distribution of photoelectron from the $2p$ state $(d\sigma^L_{\rm PE}/d\Omega')/\sigma^L_{\rm PE}$, predicted by the Born approximation for several velocities: I, $\beta=0.1$ (4.7 MeV/amu); II, 0.2 (19.4 MeV/amu); III, 0.3 (45.3 MeV/ amu); and IV, 0.4 (85.5 MeV/amu).

the asymmetric behavior represented by Eq. (4) is verified by measuring the deviations of REC photon angular distribution from the $\sin^2\theta$ dependence.

The formula of Eq. (4) cannot be applied at least for the velocity β above 0.4 because the higher-order retardation corrections become more important. In fact, Eq. (1) has meaningless negative values in such a high velocity region. The present predictions for the shapes of the L REC photon angular distributions are based on the Born approximation including the lowest-order retardation corrections as mentioned above. Thus, the electron-spin correlations and the projectile charge dependence (the Coulombic distortion effects between a projectile and an electron and the relativistic $Z\alpha$ corrections to the nonrelativistic wave functions), as well as the higher-order retardation corrections, are absent in our simple theory. In the L PE, the electron-spin corrections become important in the small forward directions,[7,9] where the difference of the shapes between the $L_{\rm II}$ PE (where the subscript II represents the angular momentum $j=\frac{1}{2}$) and the $L_{\rm III}$ PE (where the subscript III stands for $j=\frac{3}{2}$) angular distributions is clarified. Therefore, the spin corrections to the L REC are thought to appear at backward angles relative to 90° because the relative velocity v of the PE should be replaced by $-v$ to obtain the photon angular distributions of the REC by using those of the PE. Similarly, inferring from the present and other calculations[7,9] for the L PE, the minimum and the maximum values of the L REC angular distributions would be shifted slightly to the forward or backward directions and the intensities would be changed from our predictions to some extent. Equation (4) does not enable us to obtain detailed information on the L REC in the case that the corrections mentioned above have significant influences. However, our predictions are expected to give satisfactory results in general as long as $\beta \lesssim 0.4$ and $Z\alpha/\beta \ll 1$.

Next, the linear-polarization correlations for the K REC photon are taken into consideration. The differential cross section for the PE from the K shell (K PE) is of the form[10,11]

$$\frac{d\sigma_{\rm PE}^{K}}{d\Omega'}=2^{-1}\left(\frac{d\sigma_{\rm PE}^{K}}{d\Omega'}\right)_{\rm unpol}\sum_{i,j=0}^{3}\xi_i\zeta_j C'_{ij} , \qquad (5)$$

where $\xi_0=\zeta_0=1$, $C'_{00}=1$, and $(d\sigma_{\rm PE}^{K}/d\Omega)_{\rm unpol}$ is the differential cross section from unpolarized photons, summed over final photoelectron spins. The photon polarization is described with three parameters (the Stokes parameters) ξ_i ($i=1, 2, 3$).[12] Defining e_1 as the component of the photon polarization vector $\mathbf{e}$, which lies in the plane of the incoming photon $\mathbf{k}$ and the outgoing photoelectron $\mathbf{p}$, and e_2 as the component perpendicular to the plane (along $\mathbf{k}\times\mathbf{p}$), $\xi_1=e_1^*e_1-e_2^*e_2$, $\xi_2=e_1e_2^*+e_2e_1^*$, and $\xi_3=i(e_1e_2^*-e_2e_1^*)$, where $(e_1,e_2,\mathbf{k})$ form a right-hand set. Photons linearly polarized parallel to or perpendicular to the production plane are characterized by a nonvanishing ξ_1, and circularly polarized photons by a nonvanishing ξ_3. Similarly, the polarization state of the photoelectron in the rest frame is described in terms of ζ_j ($j=1,2,3$). ζ_1 lies in the scattering plane constructed by $\mathbf{k}$ and $\mathbf{p}$, ζ_2 perpendicular to it, and ζ_3 is taken along the direction $\mathbf{p}$. The quantities C'_{ij}, satisfying $|C'_{ij}|\leq 1$, are the polarization correlations for the PE. But the symmetrical invariance considerations[10,11] permit only seven nonvanishing correlations besides C'_{00}, namely, C'_{02}, C'_{10}, C'_{12}, C'_{21}, C'_{23}, C'_{31}, and C'_{33}. In this paper we are concerned only with the unpolarized electron-spin states (i.e., only ζ_0 is nonzero for electron polarization parameters).[13] Thus, the allowed polarization correlation is only C'_{10}, representing the effect of the linear polarized photon. After replacing C'_{10} by $P'(\theta')$, the corresponding differential cross section is of the form

$$\frac{d\sigma_{\rm PE}^{K}}{d\Omega'}=2^{-1}\left(\frac{d\sigma_{\rm PE}^{K}}{d\Omega'}\right)_{\rm unpol}[1+P'(\theta')\cos 2\phi] , \qquad (6)$$

where ϕ is the angle between the scattering plane and the polarization plane constructed by $\mathbf{k}$ and $\mathbf{e}$. On the other hand, following the Sauter formula[5,12,14] based on the relativistic Born approximation for the K PE, the differential cross section of Eq. (6) reads

$$(d\sigma_{\rm PE}^{K}/d\Omega')=C(\theta')[A(\theta')+B(\theta')\cos^2\phi] , \qquad (7)$$

where

$$A(\theta')=\gamma(\gamma-1)^2(1-\beta\cos\theta')/2 ,$$

$$B(\theta')=2-\gamma(\gamma-1)(1-\beta\cos\theta') ,$$

and

$$C(\theta')=Z^5\alpha^4r_e^2\gamma^{-4}(\gamma+1)^{\frac{3}{2}}\times(\gamma-1)^{-\frac{7}{2}}\sin^2\theta'(1-\beta\cos\theta')^{-4} ,$$

with r_e the classical electron radius. Comparing Eqs. (6) and (7), we get

$$P'(\theta')=B(\theta')/[2A(\theta')+B(\theta')]=[2-\gamma(\gamma-1)(1-\beta\cos\theta')]/[2+\gamma(\gamma-1)(\gamma-2)(1-\beta\cos\theta')] . \qquad (8)$$

We can obtain the linear-polarization correlation for the K REC, denoted by $P(\theta)$, replacing β of Eq. (8) by $-\beta$ and successively employing the Lorentz transformation of Eq. (3) on $P'(\theta')$. The resultant formula is of the form

$$P(\theta)=[-(\gamma-1)+2\gamma(1-\beta\cos\theta)]/[(\gamma-1)(\gamma-2)+2\gamma(1-\beta\cos\theta)] . \qquad (9)$$

$P(\theta)$ represents the polarization correlation for the linear-polarized K REC photon, averaged over initial target electron-spin states and summed over final projectile electron-spin states. $P(\theta)$ and $P'(\theta')$ are depicted in Figs. 3 and 4, respectively. One of the features for $P'(\theta')$ is the presence of the "crossover" angle θ'_c. This is defined as the angle at which the sign of $P'(\theta')$ is changed. θ'_c is given by

$$\cos^{-1}\{\beta^{-1}[1-2\gamma^{-1}/(\gamma-1)]\} .$$

On the basis of the simple Sauter's prediction, the crossover feature first appears at $\beta=0.8$ with the velocity increasing.

Similarly, $P(\theta)$, the polarization correlation for the K REC photon, represents the crossover feature at the angle

$$\theta_c=\cos^{-1}[(2\beta)^{-1}(1+\gamma^{-1})] . \qquad (10)$$

As shown in Fig. 3, $P(\theta)$ always gives the crossover feature at the velocity above 0.8. Following Eq. (10), the crossover angle θ_c varies from 0° to 60° as the incident velocity β changes its values from 0.8 to 1. The polarization direction for the K REC photon lies almost in the scattering plane at the velocity below 0.7, but tends to change to the direction perpendicular to the scattering plane along $\mathbf{k}\times\mathbf{p}$ at the forward angles with an increase in velocity.

At the angles of 0° and 180°, the photon polarization correlation $P(\theta)$ gives nonzero values by employing the Born expression, Eq. (9). As a matter of fact, such behavior indicates that the Born prediction breaks down in both the forward and backward directions. From the physical point of view, it is required that $P(0°)=P(180°)=0°$, and this zero behavior can be interpreted as follows: The scattering plane is formed by the two vectors $\mathbf{p}$ (the momentum of an active electron) and $\mathbf{k}$ (that of an emitted photon), and the emission angle of REC photon is defined as the angle between these vectors. Thus, the scattering plane cannot be uniquely defined in the case that $\mathbf{p}$ and $\mathbf{k}$ are parallel ($\theta=0°$) and antiparallel ($\theta=180°$), respectively. At these angles, it is possible that the photon polarization vector turns to all directions and, hence, the photon polarization correlation must become averaged to zero. This is because $P(0°)$ and $P(180°)$ must be zero. However, this zero behavior can be recovered by utilizing the approximation methods su-

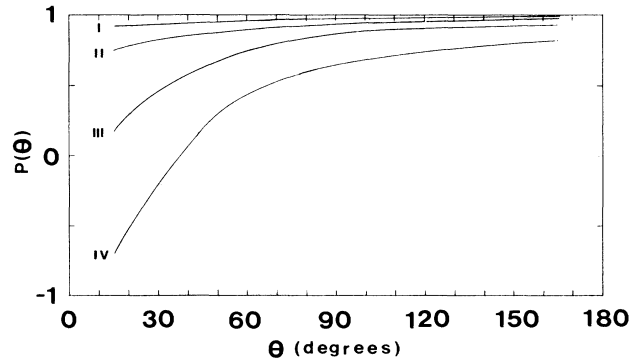

FIG. 3. Linear-polarization correlation $P(\theta)$ for radiative electron capture to the K shell for several velocities: I, $\beta=0.6$ (235 MeV/amu); II, 0.7 (376 MeV/amu); III, 0.8 (626 MeV/amu); and IV, 0.9 (1.22 GeV/amu).

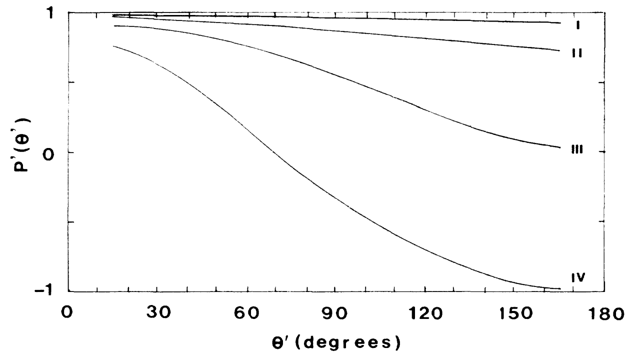

FIG. 4. Linear-polarization correlation $P'(\theta')$ for photoelectric effect from the K shell for several velocities: I, $\beta=0.6$ (235 MeV/amu), II, 0.7 (376 MeV/amu); III, 0.8 (626 MeV/amu); and IV, 0.9 (1.22 GeV/amu).

perior to the Born approximation, for example, the relativistic IA or SPB approximations.[15] It is thought, therefore, that $Z\alpha$ corrections to the photon polarization correlation $P(\theta)$ become important at both the forward and the backward directions. In reality, the crossover angle θ_c must also depend on the projectile charge Z in a certain manner. Comparing the results obtained by the relativistic IA and SPB approximations,[15] however, the Z dependence of θ_c is quite mild and then the Born prediction of Eq. (10) holds nearly true as a whole.

In this article, we have made the estimation for the differential cross section of the L REC photon and for the linear-polarization correlation of the K REC photon on the basis of the Born approximations. As mentioned above, some significant correlations are absent in the present prediction. We shall achieve the more detailed calculations, including them by virtue of the relativistically extended IA and SPB approximation[4,15] in the near future.

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