Off-resonant operation of a double-transition single-mode laser

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Scully and Lamb's quantum theory of the laser is extended to deal with nonresonant operation of a single-mode laser in a homogeneously broadened medium consisting of three-level atoms. In the case of weak-field intensity, the master equation and operation characters such as the threshold condition, the average photon number, and the half-width of the photon distribution are obtained and discussed.

In the past few years, some work[1-5] has been devoted to the double-transition two-mode laser within the quantum-mechanical framework.[6] In this paper, Scully and Lamb's quantum theory of the laser is generalized to study the double-transition single-mode laser operation with larger detunings or weaker fields. By using the density-operator method we derive the master equation with two different detunings Δ_1 and Δ_2, and then the threshold condition, the most probable photon number, and the half-width of the photon distribution are obtained and discussed. We also discuss the photon statistical properties in different cases. It is found that when the two detunings have opposite signs, the effects of the detunings on the operation properties will become much stronger due to the ac Stark shifts of the atomic levels.

As in Fig. 1, the active atoms have a common upper level $|a\rangle$ and two lower levels $|b\rangle$ and $|c\rangle$ (Refs. 1 and 2) and the decay rates for the three levels are the same γ.[2] The transitions $|a\rangle-|b\rangle$ and $|a\rangle-|c\rangle$ are coupled with the same cavity mode with frequency Ω. In the interaction picture, the interaction Hamiltonian of this system is

$$V^I=g_1e^{-i\Delta_1 t}aA_a^\dagger A_b+g_2e^{-i\Delta_2 t}aA_a^\dagger A_c+\text{H.c.}\ , \tag{1}$$

where $\Delta_1=\Omega-\omega_{ab}$ and $\Delta_2=\Omega-\omega_{ac}$ are the two detunings, $\omega_{ab}=\omega_a-\omega_b$ and $\omega_{ac}=\omega_a-\omega_c$ are the atomic

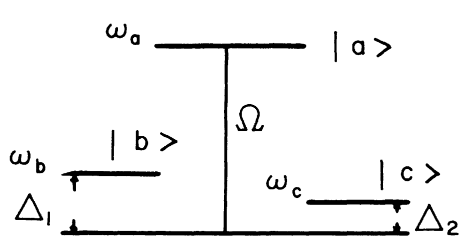

FIG. 1. Three-level atoms are coupled with the same cavity mode.

transition frequencies, $a^\dagger$ (a) are the creation (annihilation) operators of the laser mode, and the other symbols have been defined in Ref. 2. Following the same procedure as Refs. 5 and 6, we obtain the master equation[7] when $(B_i/A_i)\langle n\rangle \ll \delta_i$ (excluding the case when $B_1/A_1\delta_1=B_2/A_2\delta_2$),

$$\begin{aligned}\dot P(n)=&-[A_1(n+1)F(n)+A_2(n+1)G(n)]P(n)\\&+[A_1nF(n-1)+A_2nG(n-1)]P(n-1)\\&+C(n+1)P(n+1)-CnP(n)\ ,\end{aligned} \tag{2}$$

where the last two terms represent the cavity loss[6] and $P(n)=\rho_{nn}(t)$, $\langle n\rangle$ is the mean photon number, $\delta_i=\Delta_i/\gamma$,

$$F(n)=(1+\mu_{31}^2)^{-1},\quad G(n)=(1+\mu_{32}^2)^{-1}\ . \tag{3}$$

Let $\dot P(n)=0$. Through an iteration procedure, the photon-number probability is found to be[5,6]

$$P(n)=P(0)\prod_{j=1}^{n}\left\{\sum_{k=1}^{2}\frac{A_K}{C}\left[1+\left(\tfrac14 j\sum_{i=1}^{2}\frac{B_i}{A_i\delta_i}+\delta_K\right)^2\right]^{-1}\right\}. \tag{4}$$

Therefore, the threshold condition is[6]

$$\sum_{k=1}^{2}\frac{A_K}{C}(1+\delta_k^2)^{-1}\geq 1\ . \tag{5}$$

When $\delta_1=\pm\delta_2=\delta$ and $A_1\approx A_2\approx A$, Eq. (5) becomes

$$2A/C(1+\delta^2)\geq 1\ . \tag{6}$$

From Eqs. (4) and (5) one can see that the operation properties of this laser are affected by both atomic transitions. Evidently, the parameters A_i and B_i as well as the detunings δ_i will play an important role.[8] All this is different from the case of single-transition laser.[6] From

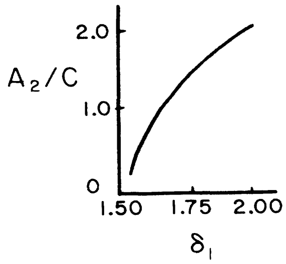

FIG. 2. A_2/C vs δ_1 at threshold of Eq. (5) when $A_1/C=2.6$ and $\delta_2=1.8$.

Eqs. (5) and (6) we can see that here the threshold condition is lower (50% lower when $\delta_1=\pm\delta_2$) than in the single-transition case,[6] i.e., it is easier for a laser with two transitions to start oscillating. This is because here both atomic transitions contribute to the same laser mode. It is not strange that the threshold condition is affected by more factors; e.g., when A_1/C and δ_2 are fixed but δ_1 changes, the laser can constantly oscillate just at the threshold by varying the value of A_2/C correspondingly. This example is shown in Fig. 2 exhibiting the "delasing" effect of the detuning δ_1. In Fig. 3 we plot the photon-number probability $P(n)$ as a function of n for some different values of the detuning δ_1. As δ_1 decreases from 1.8, the peak of the probability curve shifts to the right, i.e., the most probable photon number becomes larger or the intensity stronger. In Fig. 4 we plot the photon distribution for negative values of δ_1 when the value of δ_2 is the same as in Fig. 3, i.e., the two detunings have opposite signs. One can see that by increasing δ_1 from -1.825, the distribution curve shifts to the right more remarkably than in Fig. 3 and the slopes of these curves are flatter than those in Fig. 3 where δ_1 and δ_2 have the same sign; i.e., for the same magnitude but opposite signs of the detunings, the mean photon number and the half-width of distribution curves of $P(n)$ are larger. Therefore, the conclusion is that the effects of the detunings become much stronger when they have opposite signs.

Starting from Eq. (4) and noticing the weaker intensity assumption, we have the expression for $\langle n\rangle$,

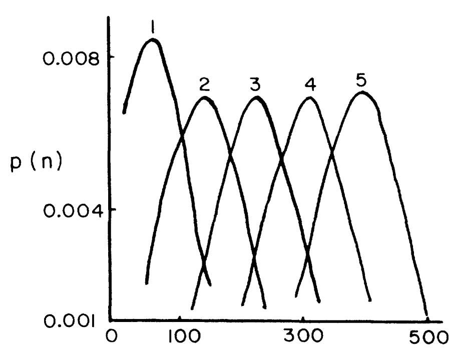

FIG. 3. Photon-number distribution of Eq. (4) for $A_1/C=2.6$, $A_2/C=1.7$, $\delta_2=1.8$, $B_1/A_1=0.0015$, $B_2/A_2=0.001$, and various δ_1's: curve 1, 1.8; curve 2, 1.75; curve 3, 1.7; curve 4, 1.65; curve 5, 1.6.

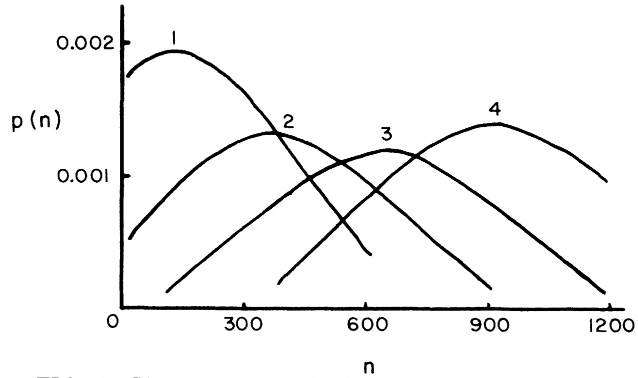

FIG. 4. Photon-number distribution of Eq. (4) for $\delta_2=1.8$, $A_1/C=2.6$, $A_2/C=1.7$, $B_1/A_1=0.0015$, $B_2/A_2=0.001$, and various δ_1's: curve 1, -1.825; curve 2, -1.82; curve 3, -1.815; curve 4, -1.81.

$$\langle n\rangle=\frac{2\left[\sum_{i=1}^{2}\frac{A_i/C}{1+\delta_i^2}-1\right]}{\sum_{i,j=1}^{2}\frac{B_i}{A_i\delta_i}\frac{\delta_j A_j/C}{(1+\delta_j^2)^2}}\ . \tag{7}$$

When $\delta_1=\pm\delta_2=\delta$, (7) is reduced to

$$\langle n\rangle=\frac{(1+\delta^2)[2(A_1+A_2)-C(1+\delta^2)]}{(B_1/A_1\pm B_2/A_2)(A_1\pm A_2)}\ . \tag{8}$$

Considering that the laser is not very far above the threshold, we can see from Eq. (8) that $\langle n\rangle$ decreases as δ increases. This is what we expected. Meanwhile, $\langle n\rangle$ becomes larger when the two detunings have opposite signs, which is in agreement with the above discussion related to Figs. 3 and 4.

An approximate expression for the width of the photon distribution probability can be obtained in the same way as used by Riska and Stenholm.[9] So from (4) we have

$$k^2=\bar{n}\Big/\left[\sum_{i=1}^{2}\frac{A_i/C}{1+\delta_i^2}-1\right]\ . \tag{9}$$

In Fig. 5 we plot the linewidth of the photon distribution given by Eq. (9) as a function of δ_1 for $\delta_2=1.8$,

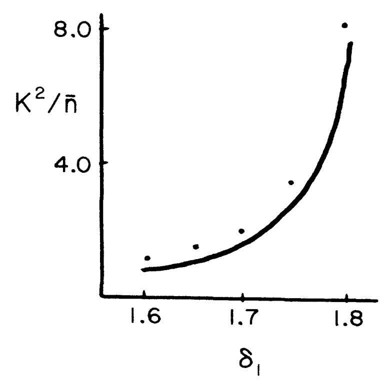

FIG. 5. Curve, approximate values of the width $k^2/\bar{n}$ from Eq. (9); points, respective values computed from the photon distribution curves.

$A_1/C = 2.6$, $A_2/C = 1.7$, and compare it with the values obtained from the photon distribution curves in Fig. 3. We see that Eq. (9) shows the correct trend of the linewidth, even if the actual values are slightly lower. It is clear from Eqs. (7) and (9) that the half-width as well as the mean photon number are larger when the two detunings have opposite signs, i.e., opposite signs of the detunings will lead to higher intensity but larger deviation from the coherent state. This novel phenomenon has also been shown in the above discussion related to Figs. 3 and 4 and is related to the following fact: in the presence of the ac Stark effect,[5,10] the transition frequencies for the two atomic transitions should be modified. It is easy to show that the modified detunings (i.e., the differences between the field frequency and the modified atomic-transition frequencies) will become smaller when Δ_1 and Δ_2 (detunings between the field and the free atom) have opposite signs, and vice versa. Therefore, the saturation effects are weakened.

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[7]Similar to Ref. 5, here we define $A_i = 2R_a(g_i/\gamma)^2$, $B_i = 4(g_i/\gamma)^2 A_i$, $\mu_i = \delta_i$, $\mu_3 = -\frac{1}{4}(n+1)\sum_{j=1}^{2} B_j/(A_j\delta_j)$, $\mu_{3i} = \mu_3 - \mu_i$ $(i = 1,2)$.
[8]The case when $B_1/(A_1\delta_1) \approx -B_2/(A_2\delta_2)$ will lead to a very large mean photon number but will also be in contradiction with our weak-field assumption. Therefore, it can be naturally excluded from our discussion.
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