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Cross sections of relativistic radiative electron capture by use of the strong-potential Born calculation

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The relativistically extended strong-potential Born (SPB) formalism is applied to the radiative electron capture process caused by the bombardment of a heavy and highly stripped charged particle with relativistically high velocity. The results are compared with those by use of nonrelativistic SPB calculations and with those by use of the relativistic Born calculation (Sauter's formula), which includes no distortion effects between a heavy projectile ion and an active electron. Even if the strong distortion effects are taken into consideration, the shapes of photon angular distributions in the laboratory frame still nearly depend on $\sin^2\theta_L$ (θ_L is the angle of the emitted photon) in the vicinity of the angle of 90°, which is the same as the results by use of Sauter's formula. The higher the charge of a projectile ion becomes, however, the greater the discrepancy between the angular shape of our results and that of Sauter's becomes at both smaller and larger angles than at 90°. As is expected, the magnitudes of the differential and the total cross sections are drastically influenced by the distortion effects ascribable to a large charge of a heavy projectile ion such as U^{92+}. Our results are in good agreement with recent experiments. In addition, the Coulomb off-shell factor introduced by the SPB theory is found playing important roles in the case of the relativistic radiative electron capture process because the results calculated by using the relativistic impulse approximation are too underestimated.

I. INTRODUCTION

The Coulomb parameter defined as $\nu = Z\alpha/v$ (in natural units, $\hbar = c = 1$, and α is the fine-structure constant) provides the criterion used to determine which approximation should be employed in the present problem of collisions between charged particles. Here, $Z\alpha$ represents the coupling constant between an electron and a particle with an electric charge of Ze, and v stands for velocity. As is well known, under the condition that $\nu \ll 1$, the (plane-wave) Born approximation is an excellent method to calculate some physical quantities. In the case that $\nu \lesssim 1$ or $\nu \gtrsim 1$, however, the Born approximation breaks down and then a much superior approximation must be introduced. The strong-potential Born (SPB) approximation[1] can be applied to a much wider range of the Coulomb parameter, namely, up to $\nu \lesssim 1$, than the Born approximation. Hence, the SPB theory is thought to be a great deal more effective for the problems of collisions caused by the bombardment of a projectile ion with a large electric charge as well as with high incident velocity.

The Coulomb parameters of fully stripped xenon and uranium ions amount to no less than 0.39 and 0.67, respectively, even when these ions travel at the speed of light. In this case, it is thought that the Born approximation no longer gives accurate information about collisions in such regions of the Coulomb parameter. However, apart from relativistic corrections, the SPB theory can adequately compensate for the faults ascribable to the Born theory. In this paper, we extend the SPB approximation relativistically in order to estimate the radiative electron capture (REC) processes brought about by heavy ions with high velocity. Hereafter, we use "REC process" to mean only "REC to the K shell of the projectile atomic states (K-REC)," unless otherwise stated.

By using the relativistic Born approximation proposed by Sauter,[2] it is indicated that the shape of the photon angular distribution for the relativistic REC process depends nearly on $\sin^2\theta_L$,[3] where θ_L is the angle of the emitted photon measured from the direction of the incident velocity $\mathbf{v}$ in the laboratory frame. The $\sin^2\theta_L$ dependence of the REC differential cross section is thought to be completely due to cancellation between the retardation effect of the emitted photon and the effect of the Lorentz transformation from the moving frame to the laboratory frame.[3,4,6] That is to say, the differential cross section of the REC process estimated in the moving frame

$$(d\sigma/d\Omega_M) \propto \sin^2\theta_M(1+\beta\cos\theta_M)^{-4} \tag{1.1}$$

is transformed into the laboratory frame by virtue of the Lorentz transformation

$$(d\sigma/d\Omega_L) = (1-\beta^2)(1-\beta\cos\theta_L)^{-2}(d\sigma/d\Omega_M) , \tag{1.2}$$

and then the following photon angular distribution can be obtained:

$$(d\sigma/d\Omega_L) \propto \sin^2\theta_L . \tag{1.3}$$

In Eq. (1.1), θ_M is the angle of the emitted photon in the moving frame and $\beta = |\mathbf{v}|/c$ (c is the velocity of light). Ω_M and Ω_L are solid angles in the moving and the laboratory frames, respectively. To get Eq. (1.3), the following relations have been made use of:

$$\sin\theta_M = (1-\beta^2)^{1/2}\sin\theta_L/(1-\beta\cos\theta_L)$$

and

$$\cos\theta_M = (\cos\theta_L - \beta)/(1-\beta\cos\theta_L) .$$

The angular dependence of the REC differential cross section calculated by the present SPB formulation is expected not to have so simple a form as Eq. (1.1) because the effects of both strong distortions and retardation have been included in a complicated manner. Hence, its angular shape in the laboratory frame will be modified to some extent from the $\sin^2\theta_L$ dependence of Eq. (1.3). We exemplify the deviation from the $\sin^2\theta_L$ dependence in the case of both $Xe^{54+}+Be$ at 197 MeV/amu and $U^{92+}+Be$ at 422 MeV/amu in Sec. III.

In Sec. II we provide the theory of calculating the REC process by virtue of the relativistic SPB approximation. The results and the discussion are given in Sec. III. Here, the theoretical results by the relativistic SPB calculation (RSPB) are compared with the recent experimental results.[4-6] Furthermore, by the comparison with the other theoretical calculations, such as the nonrelativistic SPB (NRSPB),[7] the relativistic Born (RB),[2] and the relativistic impulse approximation (RIA) calculations, it is estimated to what extent the relativistic effects, the distortion effects between the highly charged projectile ion and the active electron, and the effects of the off-shell factors[1] introduced by the SPB theory have an influence upon the relativistic REC processes, respectively. The summary is included in Sec. IV. We use the natural units throughout this paper, unless otherwise stated.

II. THEORY

A. SPB wave functions

It is, for the sake of simplicity, assumed that the collision system we are now taking notice of consists of three particles, namely, a projectile ion labeled A, a nucleus of a target atom B, and an electron initially bound on B as C. Furthermore, we define a collision channel as a label of a particle acting as a spectator. That is to say, the direct channel, $A+(B\text{-}C)$, is represented by the label of A and the rearrangement channel, $B+(A\text{-}C)$, by the label of B. Dirac's γ matrices of the particle N are defined as $\boldsymbol{\gamma}^{(N)} = -i\beta^{(N)}\boldsymbol{\alpha}^{(N)}$ and $\gamma_4^{(N)} = \beta^{(N)}$. $\boldsymbol{\alpha}^{(N)}$ and $\beta^{(N)}$ are the α and the β matrices of N.

The transition matrix element of the relativistic REC process is given as[8]

$$T^{\text{REC}} = e\langle a_\nu \overline{\Psi}_B | \gamma_\nu^{(C)} | \Psi_A\rangle , \qquad (2.1)$$

where $|\Psi_A\rangle$ and $\langle\overline{\Psi}_B|$ are the three-body wave functions of the channels A and B, respectively, and $\langle\overline{\Psi}_B|$ is defined as $\langle\overline{\Psi}_B| = (|\Psi_B\rangle)^\dagger \prod_N \gamma_4^{(N)}$. a_ν^* is the wave function of the emitted photon. Hereafter, it is understood that the summation and the direct product over the three particles A, B, and C, that is, $\sum_{N=A,B,C}$ and $\prod_{N=A,B,C}$, are simply denoted as $\sum_N$ and $\prod_N$, respectively, unless otherwise stated. According to the SPB formalism, $|\Psi_A\rangle$ and $\langle\overline{\Psi}_B|$ are approximated as[1]

$$|\Psi_A\rangle = (1+G_{AC}V_{AC})|\Phi_A\rangle \qquad (2.2)$$

and

$$\langle\overline{\Psi}_B| = \langle\overline{\Phi}_B|(1+V_{BC}G_{BC}) , \qquad (2.3)$$

respectively, where $|\Phi_A\rangle$ and $\langle\overline{\Phi}_B|$ are the (plane-wave) Born wave functions of the respecitive channels, and the relativistic Coulomb Green's functions G_{AC} and G_{BC} are given as

$$G_{AC} = \left[E - \sum_N H_N - V_{AC} + i\eta\right]^{-1} \qquad (2.4)$$

and

$$G_{BC} = \left[E - \sum_N H_N - V_{BC} + i\eta\right]^{-1} . \qquad (2.5)$$

Here, $H_N = -i\nabla_{\mathbf{x}_N}\cdot\boldsymbol{\alpha}^{(N)} + M_N\beta^{(N)}$ with M_N the mass of the particle N, E is the total energy of the collision system, V_{AC} and V_{BC} are the Coulomb potentials, and η is the infinitesimal positive number.

Inserting the complete set $\{X_r\}$ into Eq. (2.2) leads to

$$|\Psi_A\rangle = \sum_r (1+G_{AC}V_{AC})|X_r\rangle\langle X_r|\Phi_A\rangle = \sum_r |\psi_{B,r}^{(+)}\rangle\langle X_r|\Phi_A\rangle , \qquad (2.6)$$

where by the subscript r we have suppressed all the states of three particles such as spins and momenta. In the second equality of Eq. (2.6), $(1+G_{AC}V_{AC})|X_r\rangle$ has been replaced by $|\psi_{B,r}^{(+)}\rangle$. The basis function $|X_r\rangle$ is given by the direct product of three plane-wave functions as follows:

$$|X_r\rangle = \prod_N (2\pi)^{-3/2} e^{i\mathbf{K}_N\cdot\mathbf{x}_N} u_N(\mathbf{K}_N) , \qquad (2.7)$$

where $\mathbf{K}_N$, $\mathbf{x}_N$, and $u_N(\mathbf{K}_N)$ are the momentum, the position, and the spinor of N. Moreover, the Born wave function $|\Phi_A\rangle$ is the solution of the differential equation

$$\left[E - \sum_N H_N - V_{BC}\right]|\Phi_A\rangle = 0 \qquad (2.8)$$

and is expressed as

$$|\Phi_A\rangle = (2\pi)^{-3/2} e^{i\mathbf{K}\cdot\mathbf{x}_A} u_A(\mathbf{K}) \times (2\pi)^{-3/2} e^{i\mathbf{P}\cdot\mathbf{X}_{BC}} \phi_{BC}(\mathbf{x}_{BC}) . \qquad (2.9)$$

Here, $\mathbf{K}$ and $\mathbf{P}$ represent the incident momentum of A and the barycentric momentum of the composite particle $(B\text{-}C)$, respectively. ϕ_{BC} stands for the wave function of $(B\text{-}C)$. $\mathbf{X}_{Bc}$ and $\mathbf{x}_{BC}$ are defined as follows: $\mathbf{X}_{BC} = \eta_B\mathbf{x}_B + \eta_C\mathbf{x}_C$ and $\mathbf{x}_{BC} = \mathbf{x}_C - \mathbf{x}_B$, with $\eta_B = M_B/(M_B+M_C)$ and $\eta_C = 1-\eta_B$. By using Eqs. (2.7) and (2.9), we get

$$\langle X_r|\Phi_A\rangle = \left[\prod_N u_N^\dagger(\mathbf{K}_N)\right] \times u_A(\mathbf{K}_A)\hat{\phi}_{BC}(\mathbf{k}_{BC})\delta^{(3)}(\mathbf{K}-\mathbf{K}_A) \times \delta^{(3)}(\mathbf{P}-\mathbf{K}_{BC}) , \qquad (2.10)$$

where $\mathbf{K}_{BC}=\mathbf{K}_B+\mathbf{K}_C$ and $\mathbf{k}_{BC}=\eta_B\mathbf{K}_C-\eta_C\mathbf{K}_B$. $\hat{\phi}_{BC}(\mathbf{k}_{BC})$ is the wave function of the momentum representation defined as

$$\hat{\phi}_{BC}(\mathbf{k}_{BC})=(2\pi)^{-3/2}\int d\mathbf{x}\, e^{-i\mathbf{k}_{BC}\cdot\mathbf{x}_{BC}}\phi_{BC}(\mathbf{x}_{BC}) .$$

Moreover, after simple calculations, $|\psi_{B,r}^{(+)}\rangle$ is reduced to

$$|\psi_{B,r}^{(+)}\rangle=(2\pi)^{-3/2}e^{i\mathbf{K}_B\cdot\mathbf{x}_B}u_B(\mathbf{K}_B)\times(2\pi)^{-3/2}e^{i\mathbf{K}_{AC}\cdot\mathbf{X}_{AC}}\phi_{AC}^{(+)}(\mathbf{x}_{AC};\varepsilon_{AC},\mathbf{k}_{AC}) . \tag{2.11}$$

Here, $\mathbf{X}_{AC}=\xi_A\mathbf{x}_A+\xi_C\mathbf{x}_C$, $\mathbf{x}_{AC}=\mathbf{x}_C-\mathbf{x}_A$, $\mathbf{K}_{AC}=\mathbf{K}_A+\mathbf{K}_C$, and $\mathbf{k}_{AC}=\xi_A\mathbf{K}_C-\xi_C K_A$, with $\xi_A=M_A/(M_A+M_C)$ and $\xi_C=1-\xi_A$. In Eq. (2.11), $\phi_{AC}^{(+)}(\mathbf{x}_{AC};\varepsilon_{AC},\mathbf{k}_{AC})$ is the off-shell Coulomb wave function defined as

$$\phi_{AC}^{(+)}(\mathbf{x}_{AC};\varepsilon_{AC},\mathbf{k}_{AC})=(1+g_{AC}V_{AC})(2\pi)^{-3/2}e^{i\mathbf{k}_{AC}\cdot\mathbf{x}_{AC}}u_A(\mathbf{K}_A)u_C(\mathbf{K}_C) , \tag{2.12}$$

where

$$g_{AC}=[\varepsilon_{AC}-(h_C+V_{AC})+i\eta]^{-1} . \tag{2.13}$$

The off-shell energy of ε_{AC} is given as $\varepsilon_{AC}=E-(\mathbf{K}_B^2+M_B^2)^{1/2}-(\xi_A\boldsymbol{\alpha}^{(A)}\cdot\mathbf{K}_{AC}+\xi_C\boldsymbol{\alpha}^{(C)}\cdot\mathbf{K}_{AC})-h_A$. h_A and h_C are expressed as $h_A=i\nabla_{\mathbf{x}_{AC}}\cdot\boldsymbol{\alpha}^{(A)}+M_A\beta^{(A)}$ and $h_C=-i\nabla_{\mathbf{x}_{AC}}\cdot\boldsymbol{\alpha}^{(C)}+M_C\beta^{(C)}$, respectively. The second term of the expression of ε_{AC} is the intermediate energy of B and the third the translational energy of the two-body system $A+C$. If h_A is allowed to be approximated as $h_A=-\boldsymbol{\alpha}^{(A)}\cdot\mathbf{k}_{AC}+M_A\beta^{(A)}$, ε_{AC} is reduced to $\varepsilon_{AC}=E-(\mathbf{K}_A^2+M_A^2)^{1/2}-(\mathbf{K}_B^2+M_B^2)^{1/2}$, where $\xi_C\boldsymbol{\alpha}^{(C)}\cdot\mathbf{K}_{AC}$ has been dropped because it is negligibly smaller than $\xi_A\boldsymbol{\alpha}^{(A)}\cdot\mathbf{K}_{AC}$. As a result, Eq. (2.12) becomes

$$\phi_{AC}^{(+)}(\mathbf{x}_{AC};\varepsilon_{AC},\mathbf{k}_{AC})=u_A(\mathbf{K}_A)\varphi_{AC}^{(+)}(\mathbf{x}_{AC};\varepsilon_{AC},\mathbf{k}_{AC}) , \tag{2.14}$$

where

$$\varphi_{AC}^{(+)}(\mathbf{x}_{AC};\varepsilon_{AC},\mathbf{k}_{AC})=(1+g_{AC}V_{AC})(2\pi)^{-3/2}e^{i\mathbf{k}_{AC}\cdot\mathbf{x}_{AC}}u_C(\mathbf{K}_C) . \tag{2.15}$$

Inserting Eqs. (2.10) and (2.11) into Eq. (2.6), we can get

$$|\Psi_A\rangle=(2\pi)^{-3}\sum_{\text{spins}}\int d\mathbf{K}_C u_B(\mathbf{K}_B)\phi_{AC}^{(+)}(\mathbf{x}_{AC};\varepsilon_{AC},\mathbf{k}_{AC})\left[\prod_N u_N^\dagger(\mathbf{K}_N)\right]u_A(\mathbf{K}_A)\hat{\phi}_{BC}(\mathbf{k}_{BC}) \times\exp\left[i\sum_N\mathbf{K}_N\cdot\mathbf{R}\right]\exp[i(-\xi_A M_B\mathbf{K}_{AC}+M_A\mathbf{K}_B)\cdot\mathbf{x}/M] \times\exp\{i[M_B\mathbf{K}_{AC}-(M_A+M_C)\mathbf{K}_B]\cdot\mathbf{r}/M\} , \tag{2.16}$$

where the summation of Eq. (2.16) with respect to r has been replaced by $\sum_{\text{spins}}\int\cdots\int\prod_N d\mathbf{K}_N$. $\sum_{\text{spins}}$ means the summation over the spin states of all the three particles A, B, and C. The total mass of the collision system $\sum_N M_N$ has been represented as M. In Eq. (2.16), we have obtained that $\mathbf{K}_A=\mathbf{K}$ and $\mathbf{K}_B=\mathbf{P}-\mathbf{K}_C$ by virtue of the momentum-conserving δ functions. Moreover, we have had the replacements $\mathbf{x}=\mathbf{x}_{AC}$, $\mathbf{r}=\mathbf{x}_{BC}$, and $\mathbf{R}=(\sum_N M_N\mathbf{x}_N)/M$. At this stage, we approximate the integrand of Eq. (2.16) except for $\hat{\phi}_{BC}(\mathbf{k}_{BC})$ by setting $\mathbf{k}_{BC}=0$, that is, $\mathbf{K}_B=\eta_B\mathbf{P}$ and $\mathbf{K}_C=\eta_C\mathbf{P}$. Thus, $|\Psi_A\rangle$ is reduced to

$$|\Psi_A\rangle=(2\pi)^{-3}\sum_{\text{spins}}\eta_B^{-1}u_B(\mathbf{K}_B)\phi_{AC}^{(+)}(\mathbf{x};\varepsilon_{AC},\mathbf{k}_{AC})\left[\prod_N u_N^\dagger(\mathbf{K}_N)\right]u_A(\mathbf{K}_A)\phi_{BC}(\mathbf{r}) \times\exp\left[i\sum_N\mathbf{K}_N\cdot R\right]\exp[i(M_B\mathbf{K}-\eta_B M_A\mathbf{P})\cdot(\mathbf{r}-\xi_A\mathbf{x})/M] . \tag{2.17}$$

By the way, it can be assumed that the relativistic off-shell Coulomb wave function $\varphi_{AC}^{(+)}$ of Eq. (2.15) is approximated as[1,9]

$$\varphi_{AC}^{(+)}(\mathbf{x};\varepsilon_{AC},\mathbf{k}_{AC})=M_{AC}(\nu_{AC},\varepsilon_{AC})\tilde{\varphi}_{AC}^{(+)}(\mathbf{x};\mathbf{k}_{AC}) \tag{2.18}$$

by following the SPB theory. Here, $M_{AC}(\nu_{AC},\varepsilon_{AC})$, the off-shell factor, has the form

$$M_{AC}(\nu_{AC},\varepsilon_{AC})=e^{\pi\nu_{AC}}\Gamma(1+i\nu_{AC})(|\mathbf{k}_{AC}|^2-2\varepsilon_{AC})^{-i\nu_{AC}}\times|4\mathbf{k}_{AC}|^{i\nu_{AC}}$$

with the Coulomb parameter $\nu_{AC}=Z_A\alpha(|\mathbf{k}_{AC}|^2+M_C^2)^{1/2}/|\mathbf{k}_{AC}|$. $\tilde{\varphi}_{AC}^{(+)}(\mathbf{x};\mathbf{k}_{AC})$ of Eq. (2.18) is the relativistic (on-shell) Coulomb wave function defined as

$$\tilde{\varphi}_{AC}^{(+)}(\mathbf{x};\mathbf{k}_{AC})=(1+\tilde{g}_{AC}V_{AC})(2\pi)^{-2/3}e^{i\mathbf{k}_{AC}\cdot\mathbf{x}}u_C(\mathbf{K}_C) . \tag{2.19}$$

Here,

$$\tilde{g}_{AC}=[(\mathbf{k}_{AC}^2+M_C^2)^{1/2}-(h_C+V_{AC})+i\eta]^{-1} . \tag{2.20}$$

$\tilde{\varphi}_{AC}^{(+)}$ of Eq. (2.19) can be further reduced to[10]

$$\tilde{\varphi}^{(+)}_{AC}(\mathbf{x};\mathbf{k}_{AC})=(2\pi)^{-3/2}N(\nu_{AC})e^{i\mathbf{k}_{AC}\cdot\mathbf{x}}\times[1+\gamma_4^{(C)}\boldsymbol{\gamma}^{(C)}\cdot(\mathbf{k}_{AC}^2+M_c^2)^{-1/2}(\nabla_{\mathbf{x}}/2)]\times{}_1F_1(i\nu_{AC},1;i(|\mathbf{k}_{AC}||\mathbf{x}|-\mathbf{k}_{AC}\cdot\mathbf{x}))\times u_C(\mathbf{K}_c)\,, \quad (2.21)$$

where $N(\nu_{AC})=e^{\pi\nu_{AC}}\Gamma(1-i\nu_{AC})$.

By employing Eqs. (2.8) and (2.9), we find that the wave function ϕ_{BC} must be satisfied with

$$\{E-(\mathbf{K}^2+M_A^2)^{1/2}-[(\eta_B\mathbf{P}+i\nabla_{\mathbf{r}})\cdot\boldsymbol{\alpha}^{(B)}+M_B\beta^{(B)}]-\eta_C\mathbf{P}\cdot\boldsymbol{\alpha}^{(C)}-h_C-V_{BC}\}\phi_{BC}=0\,. \quad (2.22)$$

Dropping $i\nabla_{\mathbf{r}}\cdot\boldsymbol{\alpha}^{(B)}$ and $\eta_C\mathbf{P}\cdot\boldsymbol{\alpha}^{(C)}$ because it is negligibly small and approximating $\eta_B\mathbf{P}\cdot\boldsymbol{\alpha}^{(B)}+M_B\beta(B)$ to be $[(\eta_B\mathbf{P})^2+M_B^2]^{1/2}$, the solution ϕ_{BC} of Eq. (2.22) is reduced to $u_B(\eta_B\mathbf{P})\varphi_{BC}(\mathbf{r})$, where $\varphi_{BC}(\mathbf{r})$ is the solution of the Dirac equation for the hydrogenlike atom (B-C) given as

$$(\varepsilon_i-h_C-V_{BC})\varphi_{BC}=0\,, \quad (2.23)$$

with $\varepsilon_i=M_C[1-(Z_B\alpha)^2]^{1/2}$. ε_i is nearly equal to $E-(\mathbf{K}^2+M_A^2)^{1/2}-[(\eta_B\mathbf{P})^2+M_B^2]^{1/2}$, because $E=(\mathbf{K}^2+M_A^2)^{1/2}+[\mathbf{P}^2+(M_B+M_C-\varepsilon_{\text{bind}})^2]^{1/2}$, where $\varepsilon_{\text{bind}}$ stands for the binding energy of the composite particle (B-C). By following the same procedure as that which led from Eq. (2.19) to (2.21), φ_{BC} can be approximated as[11]

$$\varphi_{BC}(\mathbf{r})=[1+\gamma_4^{(C)}\boldsymbol{\gamma}^{(C)}\cdot(2M_C)^{-1}\nabla_{\mathbf{r}}]\varphi_{BC}^{(0)}(\mathbf{r})u_C(\eta_C\mathbf{P})\,, \quad (2.24)$$

where $\varphi_{BC}^{(0)}$ is the solution of the Schrödinger equation for (B-C).

According to the approximations made from Eqs. (2.14)-(2.24), we can reach the objective relativistic SPB wave function that

$$|\Psi_A\rangle=(2\pi)^{-3/2}\exp\left[i\sum_N\mathbf{K}_N\cdot\mathbf{R}\right](2\pi)^{-3/2}\exp\{i[(M_B+M_C)\mathbf{K}-M_A\mathbf{P}]\cdot(\eta_B\mathbf{r}-\mathbf{x})/M\}M_{AC}(\nu_{AC},\varepsilon_{AC})N(\nu_{AC})\times[1+\gamma_4^{(C)}\boldsymbol{\gamma}^{(C)}\cdot(\mathbf{k}_{AC}^2+M_C^2)^{-1/2}(\nabla_{\mathbf{x}}/2)]{}_1F_1(i\nu_{AC},1;i(|\mathbf{k}_{AC}||\mathbf{x}|-\mathbf{k}_{AC}\cdot\mathbf{x}))\times[1+\gamma_4^{(C)}\boldsymbol{\gamma}^{(C)}\cdot(2M_C)^{-1}\nabla_{\mathbf{r}}]\varphi_{BC}^{(0)}(\mathbf{r})\prod_N u_N(\mathbf{K}_N)\,, \quad (2.25)$$

where use has been made of $\prod_N\sum_{\text{spins}}u_N(\mathbf{K}_N)u_N^\dagger(\mathbf{K}_N)=1$, with $\mathbf{K}_A=\mathbf{K}$, $\mathbf{K}_B=\eta_B\mathbf{P}$, and $\mathbf{K}_C=\eta_C\mathbf{P}$.

Similarly, taking the same procedure by which the resultant formula of Eq. (2.25) has been obtained, we can get

$$\langle\bar{\Psi}_B|=(2\pi)^{-3/2}\exp\left[-i\sum_N\mathbf{K}'_N\cdot\mathbf{R}\right](2\pi)^{-3/2}\exp\{i[(M_A+M_C)\mathbf{K}'-M_B\mathbf{P}']\cdot(\xi_A\mathbf{x}-\mathbf{r})/M\}\times\prod_N\bar{u}_N(\mathbf{K}'_N)[1+\gamma_4^{(C)}\boldsymbol{\gamma}^{(C)}\cdot(2M_C)^{-1}\nabla_{\mathbf{x}}]\varphi_{AC}^{(0)*}(\mathbf{x})M_{BC}^*(\nu_{BC},\varepsilon_{BC})N^*(\nu_{BC})[1+\gamma_4^{(C)}\boldsymbol{\gamma}^{(C)}\cdot(\mathbf{k}_{BC}'^2+M_C^2)^{-1/2}(\nabla_{\mathbf{r}}/2)]\times{}_1F_1(i\nu_{BC},1;i(|\mathbf{k}'_{BC}||\mathbf{r}|+\mathbf{k}'_{BC}\cdot\mathbf{r}))\,. \quad (2.26)$$

The various notations employed in Eq. (2.26) are defined as follows. $\mathbf{K}'$ and $\mathbf{P}'$ represent the recoil momentum of B and the barycentric momentum of the composite particle (A-C), respectively. The intermediate momenta $\mathbf{K}'_A$, $\mathbf{K}'_B$, and $\mathbf{K}'_C$ are approximately given by $\mathbf{K}'_A=\xi_A\mathbf{P}'$, $K'_B=\mathbf{K}'$, and $K'_C=\xi_C\mathbf{P}'$, and $\mathbf{k}'_{BC}$ are defined as $\mathbf{k}'_{BC}=\eta_B\mathbf{K}'_C-\eta_C\mathbf{K}'_B$. The off-shell factor $M_{BC}(\nu_{BC},\nu_{BC})$ is provided as

$$e^{\pi\nu_{BC}}\Gamma(1+i\nu_{BC})(\mathbf{k}_{BC}'^2-2\varepsilon_{BC})^{-i\nu_{BC}}|4\mathbf{k}_{BC}|^{i\nu_{BC}}$$

with

$$\nu_{BC}=Z_B\alpha(\mathbf{k}_{BC}'^2+M_C^2)^{1/2}/|\mathbf{k}_{BC}|$$

and

$$\varepsilon_{BC}=E-(\mathbf{K}_A'^2+M_A^2)^{1/2}-(\mathbf{K}_B'^2+M_B^2)^{1/2}\,.$$

$\varphi_{AC}^{(0)}(\mathbf{x})$ is the solution of the Schrödinger equation for the hydrogenlike atom (A-C).

In addition, the wave function of the emitted photon a_ν^* is given as

$$a_\nu^*=(2\pi)^{-3/2}(2\omega)^{-1/2}e_\nu e^{-i\mathbf{k}\cdot(\mathbf{R}+(M_A/M)\mathbf{x}+(M_B/M)\mathbf{r})}\,, \quad (2.27)$$

where ω and $\mathbf{k}$ are the energy and the momentum of the photon.

B. Cross section

Inserting Eqs. (2.25), (2.26), and (2.27) into the transition matrix element T^{REC} of Eq. (2.1), we obtain

$$T^{\text{REC}}=t^{\text{REC}}\delta^{(3)}(\mathbf{K}+\mathbf{P}-\mathbf{K}'-\mathbf{P}'-\mathbf{k})\,, \quad (2.28)$$

where

$$t^{\rm REC}=(2\omega_M)^{-1/2}(2\pi)^{-9/2}eC_{AC}(\nu_{AC})C^*_{BC}(\nu_{BC})\times\bar{u}_A(0)u_A(\mathbf{K})\bar{u}_B(\mathbf{K}')u_B(\eta_B\mathbf{P})\times\bar{u}_C(0)I_C(\mathbf{q},\mathbf{p},\mathbf{k})u_C(\eta_C\mathbf{P})\ . \quad (2.29)$$

In Eq. (2.29), the moving frame $\mathbf{P}'=\mathbf{0}$ has been chosen as the frame of reference and we have defined that $C_{AC}(\nu_{AC})=M_{AC}(\nu_{AC},\varepsilon_{AC})N(\nu_{AC})$ and $C^*_{BC}(\nu_{BC})=M^*_{BC}(\nu_{BC},\varepsilon_{BC})N^*(\nu_{BC})$. ω_M represents the photon energy in the moving frame. Moreover, $I_C(\mathbf{q},\mathbf{p},\mathbf{k})$ has the form

$$I_C(\mathbf{q},\mathbf{p},\mathbf{k})=\int\int d\mathbf{x}\,d\mathbf{r}\,e^{i\mathbf{q}'\cdot\mathbf{x}}e^{i\mathbf{q}\cdot\mathbf{r}}\{1+\gamma_4^{(C)}\boldsymbol{\gamma}^{(C)}\cdot[\nabla_x/(2m)]\}\varphi^{(0)*}_{AC}(\mathbf{x})\times\{1+\gamma_4^{(C)}\boldsymbol{\gamma}^{(C)}\cdot[\nabla_r/(2p_0)]\}\,{}_1F_1(i\nu_{BC},1;i(|\mathbf{p}|\,|\mathbf{r}|+\mathbf{p}\cdot\mathbf{r}))(\mathbf{e}\cdot\boldsymbol{\gamma}^{(C)})\times\{1+\gamma_4^{(C)}\boldsymbol{\gamma}^{(C)}\cdot[\nabla_x/(2p_0)]\}\,{}_1F_1(i\nu_{AC},1;i(|\mathbf{p}|\,|\mathbf{x}|+\mathbf{p}\cdot\mathbf{x}))\{1+\gamma_4^{(C)}\boldsymbol{\gamma}^{(C)}\cdot[\nabla_r/(2m)]\}\varphi^{(0)}_{BC}(\mathbf{r})\ , \quad (2.30)$$

where $\mathbf{q}$, $\mathbf{q}'$, $\mathbf{p}$, and p_0 have been defined as $\mathbf{q}=\mathbf{K}'-\eta_B\mathbf{P}$, $\mathbf{q}'=-\mathbf{K}$, $\mathbf{p}=-\eta_B\eta_C\mathbf{P}-\eta_C\mathbf{q}\approx m\gamma\mathbf{v}$, and $p_0=(\mathbf{p}^2+m^2)^{1/2}\approx m\gamma$, with $m=M_C$ and $\gamma=(1-\mathbf{v}^2)^{-1/2}$, respectively. Here, $\mathbf{v}$ is the relative velocity of the present collision system defined as $\mathbf{v}=-\mathbf{P}/[\mathbf{P}^2+(M_B+m)^2]^{1/2}$.

We approximate $I_C(\mathbf{q},\mathbf{p},\mathbf{k})$ up to the first order with respect to $\nabla_x/(2p_0)$ and $\nabla_r/(2m)$. That is to say,

$$I_C(\mathbf{q},\mathbf{p},\mathbf{k})=\not{e}AA'+\gamma_4^{(C)}\not{V}\not{e}-\gamma_4^{(C)}\not{e}\not{V}\ , \quad (2.31)$$

where we have employed the abbreviated notations such as $\not{O}=\mathbf{O}\cdot\boldsymbol{\gamma}^{(C)}$ ($\mathbf{O}=\mathbf{e}$, $\mathbf{U}$, and $\mathbf{V}$). $\mathbf{U}$ and $\mathbf{V}$ are defined as $\mathbf{U}=\mathbf{B}'A+\mathbf{C}A'$ and $\mathbf{V}=\mathbf{B}A'+\mathbf{C}'A$, respectively, where

$$A=\int d\mathbf{r}\,e^{i\mathbf{q}\cdot\mathbf{r}}\varphi^{(0)}_{BC}(\mathbf{r})\,{}_1F_1(\mathbf{r})\ , \quad (2.32)$$

$$\mathbf{B}=(2m)^{-1}\int d\mathbf{r}\,e^{i\mathbf{q}\cdot\mathbf{r}}[\nabla_r\varphi^{(0)}_{BC}(\mathbf{r})]\,{}_1F_1(\mathbf{r})\ , \quad (2.33)$$

$$\mathbf{C}=(2p_0)^{-1}\int d\mathbf{r}\,e^{i\mathbf{q}\cdot\mathbf{r}}\varphi^{(0)}_{BC}(\mathbf{r})[\nabla_r\,{}_1F_1(\mathbf{r})]\ , \quad (2.34)$$

$$A'=\int d\mathbf{x}\,e^{i\mathbf{q}'\cdot\mathbf{x}}\varphi^{(0)*}_{AC}(\mathbf{x})\,{}_1F_1(\mathbf{x})\ , \quad (2.35)$$

$$\mathbf{B}'=(2m)^{-1}\int d\mathbf{x}\,e^{i\mathbf{q}'\cdot\mathbf{x}}[\nabla_x\varphi^{(0)*}_{AC}(\mathbf{x})]\,{}_1F_1(\mathbf{x})\ , \quad (2.36)$$

and

$$\mathbf{C}'=(2p_0)^{-1}\int d\mathbf{x}\,e^{i\mathbf{q}'\cdot\mathbf{x}}\varphi^{(0)*}_{AC}(\mathbf{x})[\nabla_x\,{}_1F_1(\mathbf{x})]\ . \quad (2.37)$$

Here, we have used the abbreviations for the confluent hypergeometric functions ${}_1F_1(\mathbf{x})={}_1F_1(i\nu_{AC},1;i(|\mathbf{p}|\,|\mathbf{x}|+\mathbf{p}\cdot\mathbf{x}))$ and ${}_1F_1(\mathbf{r})={}_1F_1(i\nu_{BC},1;i(|\mathbf{p}|\,|\mathbf{r}|+\mathbf{p}\cdot\mathbf{r}))$. A, $\mathbf{B}$, and $\mathbf{C}$ are quantities dependent upon one another and connected by the identity

$$i\mathbf{q}A+2m\mathbf{B}+2p_0\mathbf{C}=\mathbf{0}\ . \quad (2.38)$$

Likewise, A', $\mathbf{B}'$, and $\mathbf{C}'$ are connected with one another by

$$i\mathbf{q}'A'+2m\mathbf{B}'+2p_0\mathbf{C}'=\mathbf{0}\ . \quad (2.39)$$

Equations (2.32)–(2.37) can be estimated by virtue of the Nordsieck integrals[12]

$$I_r=\int d\mathbf{r}\,e^{-\lambda|\mathbf{r}|+i\mathbf{q}\cdot\mathbf{r}}|\mathbf{r}|^{-1}\times{}_1F_1(i\nu_{BC},1,i(|\mathbf{p}|\,|\mathbf{r}|+\mathbf{p}\cdot\mathbf{r}))=2\pi a^{-1+i\nu_{BC}}(a+b)^{-i\nu_{BC}} \quad (2.40)$$

and

$$I_x=\int d\mathbf{x}\,e^{-\lambda'|\mathbf{x}|+i\mathbf{q}'\cdot\mathbf{x}}|\mathbf{x}|^{-1}\times{}_1F_1(i\nu_{AC},1;i(|\mathbf{p}|\,|\mathbf{x}|+\mathbf{p}\cdot\mathbf{x}))=2\pi a'^{-1+i\nu_{AC}}(a'+b')^{-i\nu_{AC}}\ , \quad (2.41)$$

where $a=(\mathbf{q}^2+\lambda^2)/2$, $b=\mathbf{p}\cdot\mathbf{q}-i\lambda|\mathbf{p}|$, $a'=(\mathbf{q}'^2+\lambda'^2)/2$, and $b'=\mathbf{p}\cdot\mathbf{q}'-i\lambda'|\mathbf{p}|$, with $\lambda=mZ_B\alpha$ and $\lambda'=mZ_A\alpha$. By the use of Eq. (2.40), A and $\mathbf{C}$ are given as

$$A=-N(\partial I_r/\partial\lambda) \quad (2.42)$$

and

$$\mathbf{C}=N(2p_0)^{-1}|\mathbf{p}|(\partial I_r/\partial\mathbf{p})\ , \quad (2.43)$$

respectively. Likewise, we get

$$A'=-N'(\partial I_x/\partial\lambda') \quad (2.44)$$

and

$$\mathbf{C}'=N'(2p_0)^{-1}|\mathbf{p}|(\partial I_x/\partial\mathbf{p})\ . \quad (2.45)$$

Here, N and N' are the normalization constants of $\varphi^{(0)}_{BC}$ and $\varphi^{(0)}_{AC}$ given by $N=(\lambda^3/\pi)^{1/2}$ and $N'=(\lambda'^3/\pi)^{1/2}$, respectively. $\mathbf{B}$ and $\mathbf{B}'$ are obtained by using Eqs. (2.38) and (2.39).

The photon angular distribution in the moving frame is provided as follows:

$$(d\sigma/d\Omega_M)=2(2\pi)^4|\mathbf{v}|^{-1}\int d\omega_M\omega_M^2\int d\mathbf{q}\langle|t^{\rm REC}|^2\rangle\delta(\omega_M-\omega_M^{(0)}-\mathbf{v}\cdot\mathbf{q})\ . \quad (2.46)$$

The factor 2 in front of the right-hand side of Eq. (2.46) is attributed to the summation with respect to the directions of polarization of the emitted photon. Inside the energy-conserving δ function, $\omega_M^{(0)}$ stands for the peak energy of the photon given as $\omega_M^{(0)}=m(\gamma\xi-\xi')$, with $\xi=[1-(Z_B\alpha)^2]^{1/2}$ and $\xi'=[1-(Z_A\alpha)^2]^{1/2}$, and $\gamma\mathbf{v}\cdot\mathbf{q}$ represents the Doppler-broadening effect of the photon spectrum. Moreover, $\langle |t^{\mathrm{REC}}|^2\rangle$ is defined as

$$\langle |t^{\mathrm{REC}}|^2\rangle=8^{-1}\sum_{\mathrm{spins}}|t^{\mathrm{RED}}|^2\ , \tag{2.47}$$

where we have taken the average with respect to the initial spin states and the summation with respect to the final spin states of the three particles. Equation (2.47) is expressed by employing Eq. (2.29) as

$$\langle |t^{\mathrm{REC}}|^2\rangle=(2\pi)^{-9}(e^2/2\omega_M)|C(\nu_{AC},\nu_{BC})|^2\prod_N J_N\ , \tag{2.48}$$

with $C(\nu_{AC},\nu_{BC})=C_{AC}(\nu_{AC})C_{BC}^*(\nu_{BC})$. Here,

$$J_A=2^{-1}\mathrm{Tr}\left[\left[\sum_{\text{spins of }A}u_A(0)\bar{u}_A(0)\right]\times\left[\sum_{\text{spins of }A}u_A(\mathbf{K})\bar{u}_A(\mathbf{K})\right]\right]\ , \tag{2.49}$$

$$J_B=2^{-1}\mathrm{Tr}\left[\left[\sum_{\text{spins of }B}u_B(\mathbf{K}')\bar{u}_B(\mathbf{K}')\right]\times\left[\sum_{\text{spins of }B}u_B(\eta_B\mathbf{P})\bar{u}_B(\eta_B\mathbf{P})\right]\right]\ , \tag{2.50}$$

and

$$J_C=2^{-1}\mathrm{Tr}\left[\left[\sum_{\text{spins of }C}u_C(0)\bar{u}_C(0)\right]I_C(\mathbf{q},\mathbf{p},\mathbf{k})\times\left[\sum_{\text{spins of }C}u_C(\eta_C\mathbf{P})\bar{u}_C(\eta_C\mathbf{P})\right]^{\gamma_4^{(C)}}\times I_C^\dagger(\mathbf{q},\mathbf{p},\mathbf{k})^{\gamma_4^{(C)}}\right]\ . \tag{2.51}$$

Equations (2.49)–(2.51) can be carried out by using the Casimir operator

$$\sum_{\text{spins of }N}u_N(\mathbf{K}_N)\bar{u}_N(\mathbf{K}_N)=(-i\mathbf{K}_N\cdot\boldsymbol{\gamma}^{(N)}+K_{N0}\gamma_4^{(N)}+M_N)/(2K_{N0})\ ,$$

with $K_{N0}=(\mathbf{K}_N^2+M_N^2)^{1/2}$. Therefore, J_A and J_B are reduced to

$$J_A=(K_0+M_A)/(2K_0) \tag{2.52}$$

and

$$J_B=[(M_B+m)/P_0]^2\ , \tag{2.53}$$

respectively, where $K_0=(\mathbf{K}^2+M_A^2)^{1/2}$ and $P_0=[\mathbf{P}^2+(M_B+m)^2]^{1/2}$. In Eq. (2.53), we have approximated $\mathbf{K}'$, which is equal to $\mathbf{q}+\eta_B\mathbf{P}$, to $\eta_B\mathbf{P}$ by dropping $\mathbf{q}$. Since $|\mathbf{K}|<<M_A$ and $\mathbf{P}=-(M_B+m)\gamma\mathbf{v}$ in the moving frame, J_A and J_B are further reduced to

$$J_A=1 \tag{2.54}$$

and

$$J_B=\gamma^{-2}\ , \tag{2.55}$$

respectively. After performing simple trace calculations of J_C, we get

$$J_C=\sum_{i=1,2}S_i(\mathbf{q},\mathbf{p},\mathbf{k})\ , \tag{2.56}$$

where

$$S_1(\mathbf{q},\mathbf{p},\mathbf{k})=-2(1+\gamma^{-1})\mathrm{Re}[(\mathbf{e}\cdot\mathbf{U})(\mathbf{e}\cdot\mathbf{V})^*]-[2/(m\gamma)]\mathrm{Im}[(\mathbf{e}\cdot\mathbf{p})(\mathbf{e}\cdot\mathbf{U})^*AA'] \tag{2.57}$$

and

$$S_2(\mathbf{q},\mathbf{p},\mathbf{k})=[(1-\gamma^{-1})/2]|A|^2|A'|^2+[(1+\gamma^{-1})/2]|\mathbf{U}+\mathbf{V}|^2+(m\gamma)^{-1}\mathrm{Im}\{[\mathbf{p}\cdot(\mathbf{U}+\mathbf{V})^*]AA'\}\ . \tag{2.58}$$

Here, $\eta_C\mathbf{P}$ has been replaced approximately by $-\mathbf{p}$. Re($\cdots$) and Im($\cdots$) represent the real and the imaginary parts of ($\cdots$), respectively.

Inserting Eqs. (2.54)–(2.56) into Eq. (2.48) yields the resultant formula

$$\langle |t^{\mathrm{REC}}|^2\rangle=(2\pi)^{-8}\alpha(\gamma^2\omega_M)^{-1}|C(\nu_{AC},\nu_{BC})|^2\times[S_1(\mathbf{q},\mathbf{p},\mathbf{k})+S_2(\mathbf{q},\mathbf{p},\mathbf{k})]\ , \tag{2.59}$$

where the fine-structure constant α is defined in the Heaviside unit as $\alpha=e^2/4\pi$. At the sight of Eqs. (2.57) and (2.58), it is found that $S_1(\mathbf{q},\mathbf{p},\mathbf{k})$ is dominant to $S_2(\mathbf{q},\mathbf{p},\mathbf{k})$ because the former includes the factor ascribable to the dipole radiation, namely, the factor of $\sin^2\theta_M$ with $\theta_M=\cos^{-1}[(\mathbf{k}\cdot\mathbf{v})/(|\mathbf{k}|\,|\mathbf{v}|)]$. In other words, taking the nonrelativistic approximation that $|\mathbf{v}|$ is allowed to approach zero leads $S_2(\mathbf{q},\mathbf{p},\mathbf{k})$ to zero. Thus, apart from the ultrarelativistic energy range, in general, $S_1(\mathbf{q},\mathbf{p},\mathbf{k})$ has greater contributions to $\langle |t^{\mathrm{REC}}|^2\rangle$ than $S_2(\mathbf{q},\mathbf{p},\mathbf{k})$. Substituting Eq. (2.59) into Eq. (2.46), we obtain the angular distribution for the REC process in the moving frame. Furthermore, we can reach the objective differential cross section in the laboratory frame by employing the Lorentz transformation of Eq. (1.2) and the energy of the emitted photon in the laboratory frame ω_L given by $\omega_L=\gamma^{-1}(1-|\mathbf{v}|\cos\theta_L)^{-1}\omega_M$.

III. RESULTS AND DISCUSSION

Firstly, we investigate the angular shapes of the emitted photon for the relativistic REC processes. The normalized differential cross sections and the deviation from the differential cross sections at 90° multiplied by $\sin^2\theta_L$ are indicated for the collision of 197 MeV/amu Xe^{54+} on Be in Fig. 1 and for that of 422 MeV/amu U^{92+} on Be in Fig. 2, respectively. In the practical calculations, the effective nuclear electric charge of Be is set to be 1.95 by using the Slater rules. Thus, the magnitude of the optimized orbital exponent of a Be valence electron is 0.98. We compare the results of the present relativistic SPB calculation with that of the relativistic Born calculation by Sauter. In the case of the Xe^{54+}-Be system, the angular shapes of both calculations are nearly subjected to $\sin^2\theta_L$

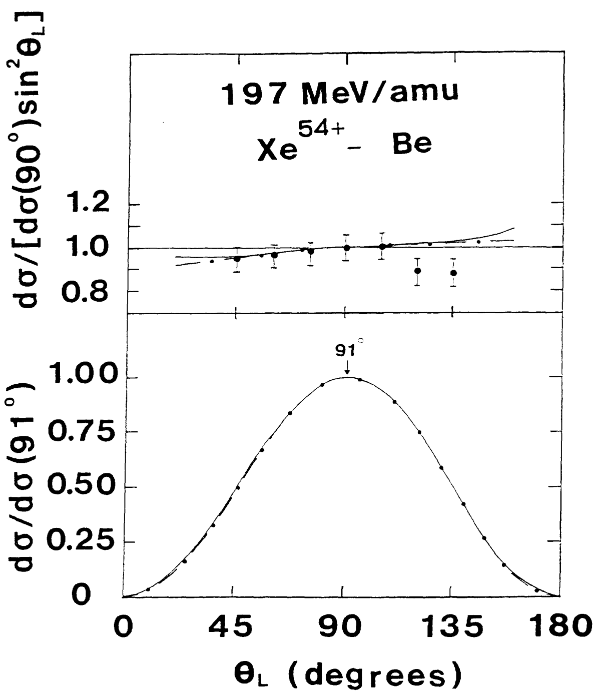

FIG. 1. Angular shape of REC photons and the ratio to $\sin^2\theta_L$ normalized at $\theta_L=90°$ in the 197 MeV/amu Xe^{54+}-Be collisions. Solid lines, the present calculations. Dashed-dotted curve, the relativistic Born calculations of Sauter (Ref. 2). Experimental results are quoted from Ref. 4.

over all the ranges of the emission angles, in spite of the fact that the maximum values of the differential cross sections are both at 91°. Our theoretical results are in good agreement with the recent experiments by Anholt *et al.*,[4] except for the two measured points at both 120° and 135°. Thus, the effects of the Coulomb distortion between an Xe^{54+} ion and an active electron at the incident energy 197 MeV/amu seem not to play quite an important role to determine the shape of the angular distribution of the emitted photon. The Coulomb parameter for this case is 0.72.

In the case of the U^{92+}-Be system of Fig. 2, the normalized differential cross section by the relativistic SPB calculation is somewhat greater than that by the relativistic Born calculation at angles both larger and smaller, above all at larger, than 90°. At the sight of the upper graph of Fig. 2, the deviation from the $\sin^2\theta_L$ dependence becomes drastic as the emission angle goes from 90° to 180°, which is thought to be mainly due to the large distortion effects between an U^{92+} ion and an active electron. In this case, the Coulomb parameter between the two particles amounts to no less than 0.94. Nevertheless, the angular shape is still nearly dependent on $\sin^2\theta_L$ in the vicinity of 90°, the contributions from which, in fact, determine the main part of the REC total cross section.

Figures 3 and 4 indicate the absolute values of the differential cross sections for the Xe^{54+}-Be and the U^{92+}-Be systems, respectively. The differences between the RSPB and the RB calculations are mainly attributed to the Coulomb distortion effects of the active electron

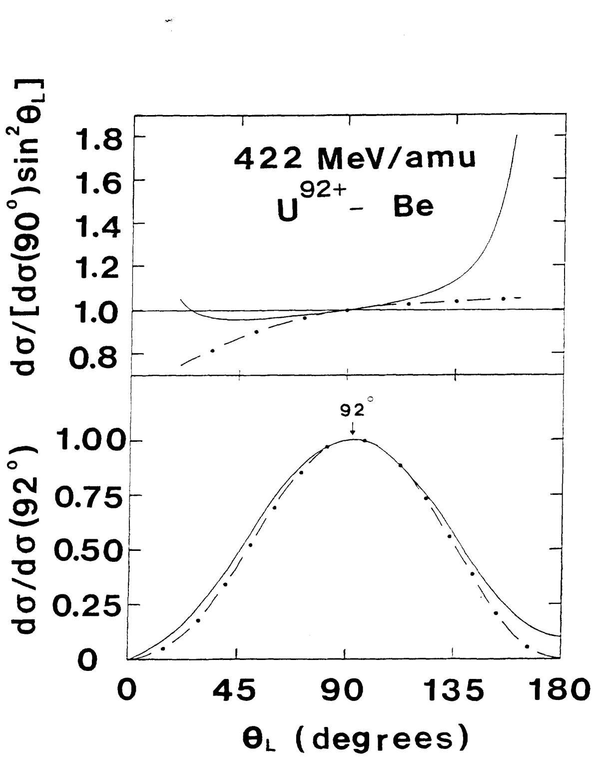

FIG. 2. Same as Fig. 1 for the 422 MeV/amu U^{92+}-Be collisions.

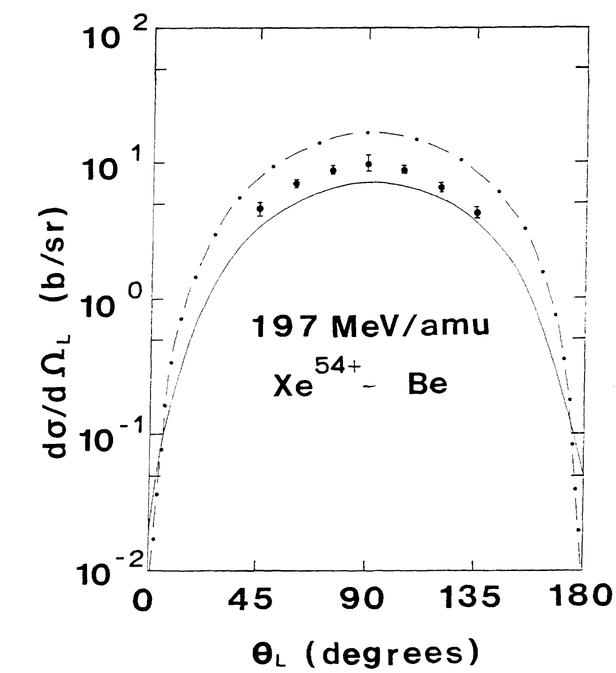

FIG. 3. Angular distributions of REC photons for the 197 MeV/amu Xe^{54+}-Be collisions. Solid lines, the present calculations. Dashed-dotted curve, the relativistic Born calculations of Sauter (Ref. 2). Experimental results are quoted from Ref. 4.

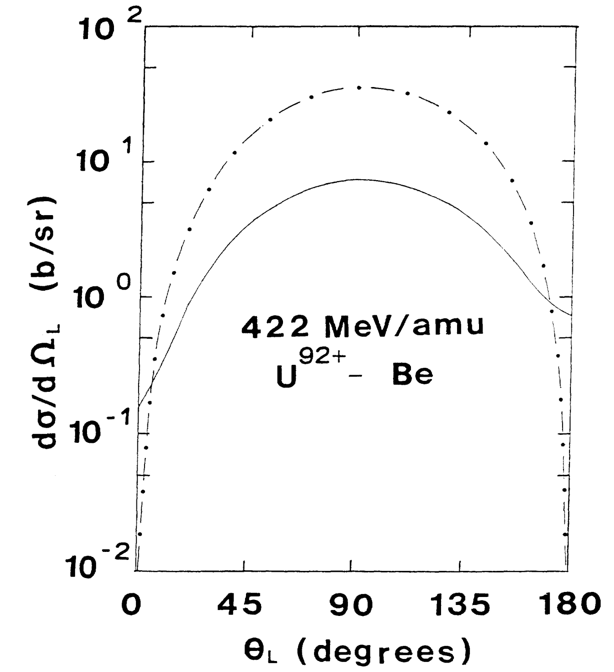

FIG. 4. Same as Fig. 3 for the 422 MeV/amu U^{92+}-Be collisions.

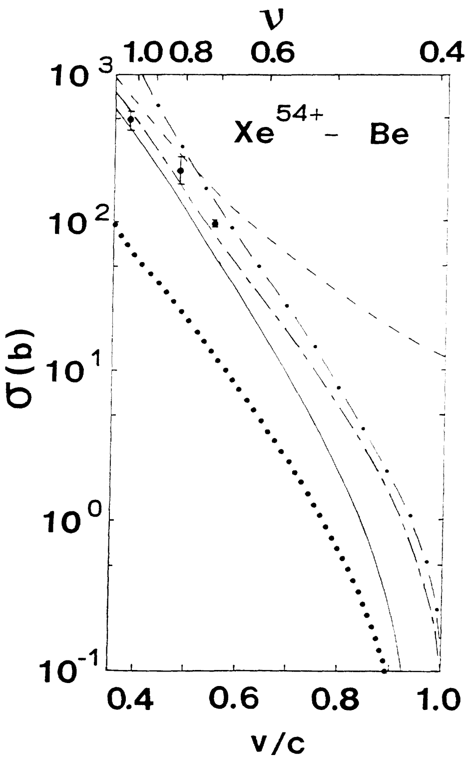

FIG. 5. Total cross sections for the Xe^{54+}-Be collisions vs the incident velocity v/c and the Coulomb parameter between an active electron and the projectile ion ν. Solid lines (——), the present calculations (RSPB-I). Dashed chain lines (— — — — —), the relativistic SPB calculations without the Lorentz contraction factor based on the motion of a target nucleus in the moving frame (RSPB-II). Dashed-dotted lines (—·—·—·), the relativistic Born calculations of Sauter (RB). Dashed lines (— — —), the nonrelativistic SPB calculation of Ref. 7 (NRSPB). Dotted lines (· · · ·), the relativistic impulse approximation calculations (RIA). Experimental results are quoted from Ref. 5.

against the highly charged projectile ion. These effects reduce the results by the plane-wave Born calculations at 90° by 2.3 times for Xe^{54+}-Be and by 4.7 times for U^{92+}-Be, respectively. The experimental results[4] are also plotted in the former case.

We provide the total cross section for the Xe^{54+}-Be and the U^{92+}-Be systems versus the incident velocities and the Coulomb parameters between the projectile ions and active electrons in Figs. 5 and 6, respectively. Several theoretical curves are obtained by the NRSPB, the RB, and the RIA calculations as well as by the present relativistic SPB calculations. The cross section by the RIA can be obtained by replacing $C(\nu_{AC},\nu_{BC})$ of Eq. (2.59) with $N(\nu_{AC})N(\nu_{BC})$, in other words, by removing the effects of the Coulomb off-shell factors from the RSPB cross section. Figures 5 and 6 include two types of the relativistic SPB calculations, one of which comes from Eq. (2.59). We refer to it as the relativistic SPB calculation I (RSPB-I) for convenience. The other, which is referred to as the relativistic SPB calculation II (RSPB-II), is defined as what is obtained by multiplying the cross section derived by using Eq. (2.59) by γ^2. The factor γ^2 is ascribable to J_B of Eq. (2.55). In a word, the effect of the Lorentz contraction based on the target nucleus B traveling in the moving frame is neglected in the RSPB-II.

Comparing the results by the RSPB-I with the NRSPB[7] shows the magnitude of the relativistic effects. Likewise, the difference between the results by the RSPB-I and by the RB indicates the magnitude of the distortion effects. The total cross sections by the RIA are found too underestimated for both cases of Figs. 5 and 6. The discrepancies between the RSPB-I and the RIA are mainly due to the Coulomb off-shell factors representing radiative transitions from the Rydberg states of the electron-projectile binding system into the final state of the K shell. The SPB wave function includes the contributions from intermediate bound states as well as from the continuum states. The former contributions are absent in the impulse approximation (IA) wave function. As compared with the experimental results, it is found that the Coulomb off-shell factors play significant roles to enhance the intensity of the REC cross section. The RSPB-I cross sections of the 197 MeV/amu Xe^{54+}-Be and 422 MeV/amu U^{92+}-Be systems are 4.5 and 6.0 times greater than the RIA ones, respectively.

The results by the RSPB-I are, in general, rather small, by γ^{-2} times, in comparison with those by the RSPB-II. The reductions of the RSPB-I results arise from considering the Lorentz contraction factor of the target nucleus traveling at the velocity of $|\mathbf{v}|$ in the moving frame. From the theoretical point of view, this factor can by no means be neglected. As the velocity increases, the RSPB-II cross sections get away from the RSPB-I ones and approach the RB ones gradually. It is usually as-

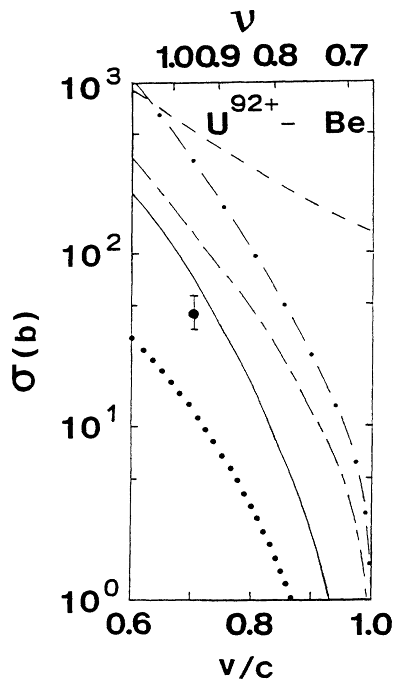

FIG. 6. Same as Fig. 5 for the U^{92+}-Be collisions. The experimental result is quoted from Ref. 4.

sumed that the REC is the inverse process of the photoelectric effects of atoms[3,4,6] besides the Compton profile reflecting the initial momentum distribution of a target electron. The present RB cross sections are obtained on the basis of such a model. According to this conventional model, a target nucleus is regarded as playing no role other than loosely binding an active electron. Such an assumption is though to be satisfactory, at least in the case of the nonrelativistic energy range.[7] The effects of the Lorentz contraction coming from the target nucleus reduce the total cross sections of the RSPB-II by 1.4 and 2.0 times in the case of Xe^{54+}-Be at 197 MeV/amu ($v=0.55$) and U^{92+}-Be at 422 MeV/amu ($v=0.71$), respectively.

Finally, we mention the target nuclear charge dependences of the REC total cross sections. It is well known that the REC differential and total cross sections are independent of the effective charge of a target atom,[13] which is important only to estimate the full width at half maximum (FWHM) of the photon spectrum based on the Doppler-broadening. In the general case of the REC process, the target nuclear charge is smaller than the projectile one. Thus, the Coulomb parameter ν_{BC} is usually smaller than ν_{AC}. In the case of the collisions of 197 MeV/amu Xe^{54+}-Be and 422 MeV/amu U^{92+} on Be, especially, $\nu_{AC} \gg \nu_{BC} \gtrsim 0$. Hence, it is quite possible to neglect the effects of the distortion and the Coulomb off-shell factor caused by the target effective charge in the range of the relativistic incident energy. However, these effects are thought to be somewhat important in the range of the nonrelativistic incident energy. We exemplify the cases of collisions of a F^{9+} ion at the velocity of 10 a.u. on target atoms with the effective nuclear charges $Z_B=27/16$ (He), 1.95 (Be), and 0 (a free electron), respectively. The ratios of the total cross sections of the F^{9+}-He and the F^{9+}-Be systems to the total cross section of F^{9+} impinging on a free electron amount to 1.6 and 1.3, respectively. As is expected, such effects of the target nucleus are small, but not negligible in the nonrelativistic energy region.

IV. SUMMARY

We summarize the present article as follows:

(i) The SPB wave function has been extended in the case of the relativistic Coulomb three-body problem. Moreover, on the basis of this wave function the cross section of the relativistic REC process has been calculated.

(ii) The shapes of the angular distributions are nearly subjected to the $\sin^2\theta_L$ dependence in spite of strong distortion effects caused by the projectile ions in both the case of 197 MeV/amu Xe^{54+} and that of 422 MeV/amu U^{92+} colliding on Be. However, the absolute values of the photon angular distributions are drastically influenced by the distortion effects.

(iii) The total cross sections for the Xe^{54+}-Be and the U^{92+}-Be systems have been calculated by use of several methods such as the nonrelativistic SPB, the relativistic Born, and the relativistic impulse approximations, as well as the present relativistic SPB approximation. Comparing the results of the relativistic SPB calculations with those by others leads to estimating the degrees of the magnitudes of various effects such as the relativistic corrections, the distortions, and the Coulomb off-shell factors included within the relativistic REC processes. Especially, the effects of the Coulomb off-shell factors play outstanding roles. Without them the cross sections are too underestimated in the comparison with the experimental results.

(iv) We have introduced the Lorentz contraction factor based on the motion of a target nucleus in the moving frame. This effect modifies to some extent the magnitude of the REC cross section derived on the basis of the model that the REC is assumed to be only the inverse process of the photoelectric effect of an atom.

(v) The effects of the Coulomb distortion and the Coulomb off-shell factor between an active electron and a recoiling target nucleus have some influences on the REC process in the nonrelativistic energy region.

(vi) As a whole, our theoretical results by the relativistic SPB calculations are in good agreement with the experimental ones.

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[9]As a matter of fact, the appearance of the Coulomb off-shell factor $M_{AC}(\nu_{AC},\varepsilon_{AC})$ of Eq. (2.18) must be proved mathematically by solving the differential equation for the relativistic off-shell Coulomb wave function. But we avoid such mathematical procedures and assume that the relativistic Coulomb off-shell wave function is simply constructed by mulitplying the relativistic Coulomb on-shell wave function by the off-shell factor derived from the nonrelativistic SPB theory. Thus, all of the relativistic corrections are included only in the relativistic on-shell wave function.

[10]In the relativistic Coulomb problem, the separation of the wave equation in the parabolic coordinates is impossible and then the relativistic pure Coulomb wave function cannot be obtained in a closed analytic form. The approximate Coulomb wave function of Eq. (2.21) relies on the data of V. B. Berestetskii, E. M. Lifshitz, and L. P. Pitaevskii, Vol. 4 of *Quantum Electrodynamics, Course of Theoretical Physics,* 2nd ed. (Pergamon, New York, 1982), p. 144.

[11]The exact discrete solution of Eq. (2.23) has been, of course, known already. Nevertheless, we make use of its approximate solution for the reason of maintaining the same precision of approximations as that of the continuum solution given by Eq. (2.21). From the viewpoint of practical calculations, in addition, the form of Eq. (2.24) is very convenient.

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[13]As is stated in Ref. 4, in fact, the REC cross sections are nearly proportional to the number of available target electrons per atom. Thus, as the atomic number of a target atom increases, the cross section increases as well. As far as our three-body model is concerned, however, the cross section is free from such an effect. Small discrepancies between our results and the experimental ones might arise from the fact that there are some available electrons in a target atom.