"Coulomb logarithm" for inverse-bremsstrahlung laser absorption

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The "Coulomb logarithm" for inverse-bremsstrahlung laser absorption is examined for plasmas of different ionic charge, spanning the classical and quantum-mechanical limits. Previously, this term has not been calculated exactly for the conditions of interest in laser fusion experiments; it has only been estimated from physical considerations. For short-wavelength irradiation (e.g., 0.35 μm), uncertainties in the "logarithmic" factor can produce variations of 20–50% in the laser absorption coefficient. A more exact treatment of this term is presented here. For low-Z plasmas, a modified Born approximation is used that reproduces previous results for long-range interactions that cannot be described by a single electron-ion collision, and it simultaneously treats the short-range electron-ion encounters. For high-Z plasmas, the Coulomb logarithm is calculated in terms of the classical, nonlinear electron trajectory in a self-consistent electrostatic potential; strong ion-ion correlations are treated by the *nonlinear* Debye-Hückel model. There are no indeterminate quantities in the calculations.

I. INTRODUCTION

The classical coefficient for inverse-bremsstrahlung laser absorption is proportional to a logarithmic factor $\ln\Lambda_{ib}$ characteristic of Coulomb collisions. The argument Λ_{ib} generally is not calculated exactly but rather is estimated from physical considerations.[1,2] For low-density plasmas ($<10^{20}$ cm^{-3}) $\ln\Lambda_{ib}$ is sufficiently large (>10) that the error in estimating Λ_{ib} should produce less than a $\sim 10\%$ variation in the logarithm, which is acceptable for most calculations. However, at the high plasma densities characteristic of short-wavelength laser irradiation (e.g., $\sim 9\times 10^{21}$ cm^{-3} for 0.35-μm light), $\ln\Lambda_{ib}$ is <5, and uncertainties in $\ln\Lambda_{ib}$ can produce 20–50% modifications in the absorption coefficient. A more exact treatment of this term is presented here for both high- and low-Z plasmas. For low-Z plasmas, $\ln\Lambda$ is dominated by quantum-wave effects, and for high-Z plasmas, it is characterized by classical physics.

The first calculation of inverse bremsstrahlung in a plasma was performed by Dawson and Oberman.[3] Their results showed very different behavior for high and low frequencies ω of the electromagnetic wave relative to the plasma frequency ω_p. The low-frequency limit of ω includes the region around the critical density which typically dominates laser absorption, and this is the region of primary concern here. The other limit $\omega \gg \omega_p$ is already well understood in spite of the complication that the usual collision picture does not apply: The electron-ion (e-i) scattering time for distant collisions is long compared to the period of the electromagnetic wave. The electron-photon interaction distorts the e-i collision process, and this produces an ω dependence in the effective Coulomb logarithm. The ω dependence was obtained by Dawson and Oberman[3] and by Jones and Lee,[4] but with an additional indeterminant quantity $k_{\max}$ that resulted from use of linearized plasma equations, which could not adequately treat the nonlinear electron trajectory in close e-i collisions. However, this limit of ω corresponds to relatively low densities where plasma shielding and collective effects play an insignificant role in the interaction, and the problem could be well approximated by inverse bremsstrahlung for electrons scattering in the pure Coulomb field of an isolated ion. The pure Coulomb problem can be solved exactly, and an analytic expression has been obtained by Sommerfeld[5] for the case of bremsstrahlung emission (which is related to inverse bremsstrahlung by detailed balance). The Sommerfeld result gives guidance on the proper definition of $k_{\max}$ in the plasma calculations for $\omega \gg \omega_p$.

In contrast, for the low-frequency limit, near the critical density ($\omega \sim \omega_p$), plasma shielding is very important. As a result, the potential is no longer Coulombic, and the Sommerfeld solution does not apply. However, the collision approximation of factoring out the parameters characterizing e-i scattering is now valid, because the collision time is short compared to the period of the electromagnetic wave (and because the Dawson-Oberman calculation has shown that collective plasma processes do not significantly effect the absorption coefficient). The validity of the collision approximation is examined in Sec. IV for low-Z plasmas where an analytic solution is obtained over the entire range of ω. The collision approximation is then used for high-Z plasmas which require a numerical calculation.

To illustrate why bremsstrahlung from high- and low-Z plasmas should be treated differently (and to demonstrate the source of uncertainty in the determination of $\ln\Lambda$), we consider an example of the momentum-transfer cross section for electron-ion scattering, which is the process characterizing inverse bremsstrahlung near the plasma critical density. At the simplest level of approximation, we use only the result for small angle deflection of an electron incident on an isolated ion. Small momentum transfer $\Delta p \sim (1-\cos\theta)$ is inversely proportional to the square of the impact parameter[6] b. Integrating over all impact parameters, the total change in momentum is

$$\langle \Delta p \rangle \sim \int \Delta p \, b \, db \sim \int b^{-1} db = \ln(b_{\max}/b_{\min}) ,$$

where maximum and minimum values of b were introduced to prevent divergences of the integral. The divergence at large b is due to an incomplete statement of the problem. In a plasma, the ion will be shielded by surrounding electrons and ions, and the range of interaction will be limited to about the Debye length λ_D. (For inverse bremsstrahlung at low density, another mechanism can also affect $b_{\max}$; the collision time must be shorter than the period of the wave.[6]) At the other limit of impact parameter, the divergence at small b obviously results from breakdown in the approximation of a near-linear electron trajectory. An exact treatment of the electron hyperbolic trajectory in a pure Coulomb potential (Appendix A) shows that $b_{\min}$ can roughly be identified with the impact parameter for 90° scattering. A complete model for $\ln\Lambda$ must simultaneously treat both limits of the impact parameter. One such model, which is valid over a limited range of conditions, is to replace the Coulomb potential by a Debye-shielded potential. (The treatment in Secs. III and IV is more general.) Once the potential differs from the Coulomb form, then quantum-mechanical and classical treatments of the scattering can be different, and at this point high- and low-Z calculations deviate.

In a classical calculation for electrons scattering in a Debye-Hückel potential, Liboff[7] found the expected result: $b_{\max}=\lambda_D$ and $b_{\min}=b_{90^\circ}$ plus a small correction factor, for the limit $\lambda_D \gg b_{90^\circ}$. However, this approach is only valid for b_{90° greater than the electron de Broglie wavelength λ_q, otherwise the calculation must be performed quantum mechanically. Typically in low-Z, high-temperature plasmas, b_{90° is not large enough ($b_{90^\circ} \sim Z$) to justify a classical calculation of electron-ion scattering. A quantum-mechanical calculation can be performed relatively simply using the Born approximation for those cases when the electron kinetic energy is much larger than the potential energy.[8] This approximation is valid for forward scattering (large impact parameters), but for low-Z, high-temperature plasmas, the kinetic energy remains sufficiently large relative to potential energy, that the Born approximation is also valid down to the small impact parameters of interest for $\ln\Lambda$. Calculation of $\ln\Lambda$ with the Born approximation for a Debye-Hückel potential gives the expected result: $b_{\max}=\lambda_D$ and $b_{\min}=\lambda_q$ plus a small correction factor (Appendix B). In general, electron scattering in high-Z plasmas can be calculated from the classical electron trajectory, and low-Z, high-temperature plasmas require a quantum-mechanical treatment, with the Born approximation being valid for very low Z. A more detailed calculation of the Coulomb logarithm is given in the following sections, without assuming a Debye-Hückel form for the e-i potential.

For low-Z plasmas, a modified Born approximation is used to treat the quantum-mechanical electron-photon and electron-ion interactions. Previous Born approximation calculations[9] have not explicitly treated the effects of the surrounding plasma on the bremsstrahlung process. The new features here are that the time-dependent response of the plasma is modeled by including the plasma dielectric function as part of the Born-approximation treatment of the e-i interaction, and an explicit treatment of ion-ion correlations replaces the usual assumption that the electrons scatter in an average electrostatic potential determined by the average positions of neighboring ions (e.g., a Debye-Hückel potential). Since the ions are moving several orders of magnitude slower than the electrons, fluctuations around the average ion positions will not be smoothed during the collision, and use of an average potential may not be accurate. For this calculation, the inverse bremsstrahlung rate is calculated for an arbitrary ion configuration. The rate is then averaged over all ion configurations using the Debye-Hückel two-body correlation function (but this does not produce a static Debye-Hückel potential). This approach recovers the frequency dependence of $\ln\Lambda_{ib}$ obtained by Dawson and Oberman,[3] but it does not contain their indeterminate quantity $k_{\max}$, which results from close collisions. Close collisions, corresponding to $\sim 90^\circ$ scattering, are well-described by the Born approximation in terms of the electron de Broglie wavelength, for the low-Z, high-temperature plasmas.

The Born-approximation calculation shows that near the plasma critical density, inverse bremsstrahlung can in fact be calculated from the rate of electrons scattering in an average electrostatic potential, but with the addition of a small correction term resulting from ion correlations. The correction term is extrapolated here into the higher-Z region and used to calculate inverse bremsstrahlung from an average potential, where the Born approximation is no longer valid. For Z greater than ~ 10, the Born approximation is not valid over the entire range of scattering angles that determine $\ln\Lambda$; the quantum-mechanical treatment must include nonlinearities due to strong distortion of the electron wave function by the central ion, and in general a partial wave calculation is used.[10,11] However, at high Z, in the region of density and temperature of interest for laser absorption, the minimum impact parameter is no longer characterized by the de Broglie wavelength but by the classical impact parameter for 90° scattering, and a quantum-mechanical treatment is in fact not required. The Coulomb logarithm could be calculated using the classical nonlinear electron trajectory, and that is the approach taken here for moderate and high-Z plasmas.

For moderate-Z ions, the Debye-Hückel model adequately describes the scattering potential over the range of impact parameters contributing to $\ln\Lambda$. The Coulomb logarithm has been previously calculated for this potential over both the quantum-mechanical[12] and classical[7] regions with the approximation $\Lambda \gg 1$. We extend those calculations to the high-Z region by including the effects of strong ion-ion correlations, and numerically evaluating the electron trajectory in the resulting electrostatic potential. The ion correlations are described by a nonlinear Debye-Hückel model, which merges smoothly with the previous large-Λ results.

The outline of this paper is as follows: The physical parameters and background material related to $\ln\Lambda_{ib}$ are discussed in Sec. II. In Sec. III, the Born-approximation result, Eq. (29) is derived in terms of the plasma dielectric function. [The reader not wishing to follow the derivation of Eq. (29) can proceed to Sec. IV where the results are discussed.] In Sec. IV, the results are divided into "high-Z" and "low-Z" parts because of the difference in techniques used, and the transition from one to the other is discussed. Section IV A examines the new Born-approximation results, which apply to low-Z plasmas such as $(\mathrm{CH})_x$, and comparison is made with earlier work. Section IV B discusses the transition region between the classical and quantum-mechanical limits, appropriate for moderate-Z plasmas, and it also discusses the relation between $\ln\Lambda_{ib}$ and $\ln\Lambda_{ei}$. Section IV C presents the new results for high-Z plasmas that include strong ion-ion correlations described by the nonlinear Debye-Hückel model. A phenomenological fit to the results over the entire range of Z is presented in Eq. (46). Finally, the results are summarized in Sec. V.

II. PHYSICAL PARAMETERS AND DEFINITIONS

Following is a discussion of the physical parameters that should characterize $\ln\Lambda_{ib}$. A detailed calculation of the term is presented in Secs. III and IV. Since inverse bremsstrahlung is the process of light absorption induced by electron-ion collisions, the Coulomb logarithm generally is written in terms of the classical impact parameters characterizing e-i scattering,

$$\ln\Lambda=\ln(b_{\max}/b_{\min})+C , \tag{1}$$

where $b_{\max}$ is the maximum impact parameter, $b_{\min}$ is the impact parameter for 90° scattering, and C is a number containing the remainder of the term, which is generally on the order of 1. For laser absorption, a correct calculation of $\ln\Lambda$ should include (i) the response of plasma electrons to laser light in the presence of electron-ion scattering, (ii) plasma shielding of interacting charged particles, (iii) ion-ion correlations, and (iv) nonlinear orbit dynamics or quantum-mechanical wave effects for close collisions that result in $\sim 90°$ scattering. Various approximations have been used to determine the parameters in Eq. (1); no single approximation has determined all parameters self-consistently over the entire range of interest. (Of course $\ln\Lambda$ would be well-defined in a complete quantum-mechanical calculation.)

The classical plasma calculation[3] for laser absorption has determined $b_{\max}$ in terms of the plasma Debye length λ_D and the laser frequency ω. Physically, these parameters play the following role. In a plasma, each ion is shielded by neighboring electrons and ions; for a low to moderate-Z, high-temperature plasma the characteristic screening length is the Debye length

$$\lambda_D=[4\pi n_e e^2(1/T_e+Z/T_i)]^{-1/2} , \tag{2}$$

where n_e is the electron density, T_e the electron temperature, T_i the ion temperature, and Z the ionic charge. Typically, T_e can be two to three times larger than T_i, as the e-i equilibration time can be much longer than the electron-heating time by inverse bremsstrahlung. The results below use $T_e=T_i$ for simplification, but the modification of the shielding distance for unequal temperatures is straightforward. Often, only the electron contribution to shielding is used [i.e., $Z=0$ in Eq. (2)], which is based on the approximation of a uniform, ion background. But more realistic models, that include ion-ion correlations, show that the ion contribution to shielding can be dominant, as discussed below. For impact parameters much larger than the shielding distance, e-i scattering (and hence, inverse bremsstrahlung) is negligible. Besides shielding, an additional factor enters into the determination of $b_{\max}$; the electron collision time should not be much longer than the period of the electromagnetic wave, otherwise the interaction would be almost adiabatic and very little energy would be transferred to the electrons.[6] The interaction time for an electron with an impact parameter b is roughly b/v_t, where $v_t=(T_e/m)^{1/2}$. Combining these two factors, the maximum impact parameter is approximated by

$$b_{\max}=\min\{\lambda_D,\ v_t/\omega\} , \tag{3a}$$

which is characteristic of the detailed classical result.[3] Often only the high-frequency limit (low density) of the plasma calculation is quoted,[1] i.e., $b_{\max}=v_t/\omega$. This is not valid near the critical density where a majority of the laser light is absorbed. Near the critical density, λ_D more closely characterizes the maximum impact parameter; it is approximately a factor $(Z+1)^{1/2}$ smaller than v_t/ω.

The choice of λ_D as the shielding length is only valid when it is much larger than the average-ion radius R_0 defined as $(4\pi n_i/3)^{-1/3}$. For high-Z plasmas, λ_D can become smaller than R_0, and strong ion-ion correlations must be considered for evaluating the plasma shielding. In this case, often the larger of R_0 and λ_D is used.[2,13] This condition will be denoted here by an asterisk, i.e.,

$$b^*_{\max}=\min\{\max\{\lambda_D,R_0\},v_t/\omega\} . \tag{3b}$$

The minimum impact parameter $b_{\min}$ in Eq. (1) is left indeterminate in the classical plasma calculation.[3] It is often approximated by the impact parameter $b_{90°}$ for 90° scattering of an electron in a Coulomb potential (Appendix A),

$$b_{90°}=Ze^2/mv^2 , \tag{4}$$

where v is the electron velocity. If $b_{90°}$ is smaller than about the de Broglie wavelength, then quantum-mechanical effects must be considered. Typically, the quantum-mechanical "minimum-impact parameter" is defined as[6,14]

$$\lambda_q=\hbar/2mv . \tag{5}$$

The parameter $b_{\min}$ becomes

$$b_{\min}=\max\{b_{90°},\lambda_q\} \tag{6}$$

and is evaluated here at the effective velocity given by the energy relation

$$\tfrac{1}{2}mv^2=\tfrac{3}{2}T . \tag{7}$$

Other choices of the relation between v and T are possible, but they would simply result in a modification of the nonlogarithmic term C. The region where $b_{\min}=\lambda_q$ will be denoted here as quantum mechanical, and the remaining region will be called classical.

It is convenient to define a standard Coulomb logarithm $\ln\Lambda_s$ to compare with the new results discussed in Sec. IV. We use

$$\ln\Lambda_s = \ln(b_{\max}/b_{\min}) \tag{8}$$

with Eqs. (3) and (6) defining the impact parameters, and with C from Eq. (1) set equal to zero. We denote the classical and quantum-mechanical limits of Λ_s as Λ_c and Λ_q, respectively. The classical limit of Eq. (8) is

$$\ln\Lambda_c = \ln(12\pi n_e \lambda_D^3) \tag{9a}$$

evaluated near the critical density n_c with $\lambda_D < v_t/\omega$ and with the approximation $Z \approx Z+1$. For high density, Λ_c is modified by Eq. (3b) in which case the average-ion radius is used as the shielding distance,

$$\Lambda_c^* = \Lambda_c \max\{1, R_0/\lambda_D\} \ . \tag{9b}$$

In the quantum-mechanical limit, Eq. (8) becomes

$$\ln\Lambda_q = \ln(\sqrt{12mT}/\hbar k_D) \ , \tag{10}$$

where $k_D = 1/\lambda_D$.

Boundaries characterizing the different regions are sketched in Fig. 1, for Z versus T, at a plasma electron density of 9×10^{21} cm^{-3}. Temperatures around 1 keV are typical of laser irradiated plasmas. The boundary between the quantum-mechanical and classical regions is determined by the condition $b_{90^\circ}=\lambda_q$. Except for the lowest-Z materials, $\ln\Lambda$ is in the classical region and can be determined by classical orbit dynamics. Even for $(CH)_x$ ($Z\sim3$) at $T<1$ keV, $\ln\Lambda$ is nearly classical. For moderate and high-Z materials, Fig. 1 shows that approximations based on a linearized Debye-Hückel model may not be adequate, as $\lambda_D < R_0$. At high Z, approximations based on $\Lambda \gg 1$ are questionable.

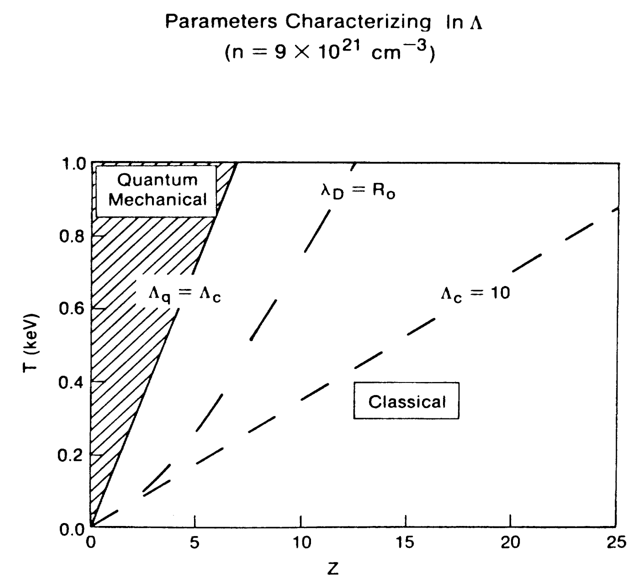

FIG. 1. Different regions and parameters characterizing $\ln\Lambda \approx \ln(b_{\max}/b_{\min})$. In the shaded region, the de Broglie wavelength, Eq. (5) determines $b_{\min}$; in the remainder b_{90°, Eq. (4), is the appropriate minimum impact parameter. For Z greater than ~25, the Debye length λ_D, Eq. (2) becomes comparable to the average-ion radius R_0 and strong ion correlations become important in determining $b_{\max}$; approximations based on $\Lambda \gg 1$ may become inaccurate.

Although the quantum-mechanical region (low Z) is relatively small, it is of considerable importance, because low-Z ablators are required for direct-drive laser fusion. In this region, an accurate expression for $\ln\Lambda$ can be obtained in a relatively simple fashion by using the Born approximation. This approximation is applicable[8] when the kinetic energy of the interacting electrons is much larger than the potential energy at approximately a de Broglie wavelength from the ion, i.e., $\frac{1}{2}mv^2 > Ze^2/(\hbar/mv)$, or using Eqs. (5) and (7),

$$T > Z^2 35 \text{ eV} \tag{11}$$

corresponding to a low-Z, high-temperature plasma. (This condition is equivalent to $b_{90^\circ} > \lambda_q$.) As discussed in Secs. III and IV A, the Born approximation determines all parameters in the interaction: the classical result for $b_{\max}$ is recovered in the Born approximation when the Coulomb potential is modified by the plasma dielectric function; $b_{\min}$ is obtained in terms of the de Broglie wavelength; and $C \sim -1$. Results similar to these were obtained by Cauble and Rozmus[15] who used a modified Coulomb potential that phenomenologically accounted for quantum wave effects in close electron-ion collisions.

The Born-approximation model is applicable to laser absorption for $(CH)_x$ ($Z\sim3$), but is invalid for SiO_2 ($Z=10$) and for higher-Z materials at the keV the temperatures characteristic of laser plasma interactions. We extend the calculation of $\ln\Lambda$ into the higher-Z region (Sec. IV B) by relating the inverse-bremsstrahlung Coulomb logarithm $\ln\Lambda_{ib}$ to the logarithm for electron-ion scattering $\ln\Lambda_{ei}$ in a shielded electrostatic potential. The Born approximation shows this relation to be

$$\ln\Lambda_{ib}(\text{Born}) = \ln\Lambda_{ei}(\text{Born}) + \tfrac{1}{2} + \frac{1}{Z}O(\tfrac{1}{2}) \tag{12}$$

near the critical density. The term $\frac{1}{2}$ is the result of averaging $\ln\Lambda_{ib}$ over all ion positions, compared to simply using an average electrostatic potential (Debye-Hückel) in the calculation of $\ln\Lambda_{ei}$, as discussed by Hubbard and Lampe.[16] Equation (12) is extrapolated into the high-Z region, beyond the validity of the Born approximation, according to

$$\ln\Lambda_{ib} = \ln\Lambda_{ei} + [\ln\Lambda_{ib}(\text{Born}) - \ln\Lambda_{ei}(\text{Born})] \ , \tag{13}$$

which is similar to Eq. (7) in Ref. 12. This extrapolation is probably the largest source of uncertainty for high Z. The term $\frac{1}{2}$ makes a 25% contribution to $\ln\Lambda_{ib}$ for $Z=50$. Using Eq. (13), $\ln\Lambda_{ib}$ can be determined by calculating e-i scattering in a spherically symmetric potential. An expression for $\ln\Lambda_{ei}$ that spans the quantum-mechanical and classical limits has been obtained by Williams and DeWitt,[12] for moderate Z. However, their results depend on the approximate solution by Liboff[7] for electrons scattering in a linearized Debye-shielded potential with $\Lambda \gg 1$, and is not valid at high Z.

For high-Z materials with Λ less than ~ 10, the potential around an ion can no longer be described by the linearized Debye-Hückel model, and stronger ion-ion correlations must be considered. Such correlations were examined by Cauble and Rozmus,[15] but with a model that produces only the quantum-mechanical minimum impact parameter. This is valid only at low Z where the strong ion correlations occur at very low temperatures, not characteristic of the laser absorption region. In this paper, strong ion-correlation effects are examined at the higher temperatures achieved in coronal plasmas. Strong ion correlations do not occur in the low-Z quantum-mechanical plasmas, but they do occur in the high-Z classical plasmas. A nonlinear Debye-Hückel (NLDH) model[17,18] is used to prevent the close approach of neighboring ions, which is the main effect of strong correlations. The Coulomb logarithm is evaluated by using the classical electron trajectory in the NLDH self-consistent electrostatic potential. This model is convenient for considering electrons and ions at different temperatures, and it merges smoothly with the low-Z (large Λ) results of Liboff for a linearized Debye-shielded potential.

The starting point for the calculation is Boltzmann's equation for the change in the electron distribution function f due to inverse bremsstrahlung,

$$\frac{\partial f}{\partial t}-\frac{e}{m}\mathbf{E}\cdot\nabla_{v}f=\int d^{3}\Delta v\,[W(\mathbf{v}-\Delta\mathbf{v}\Longrightarrow\mathbf{v})f(\mathbf{v}-\Delta\mathbf{v}) - W(\mathbf{v}\Longrightarrow\mathbf{v}+\Delta\mathbf{v})f(\mathbf{v})]\ . \tag{14}$$

The electric field $\mathbf{E}$ and the two-body interaction W can take on different meanings according to the particular model of the laser-plasma interaction. Three models are considered.

(i) The first model by Dawson and Oberman[3] treated all electron-ion scattering in terms of a self-consistent electrostatic potential, which was included in $\mathbf{E}$ together with the laser electric field. Close two-body interactions were considered negligible, and the term W was set equal to zero. This approach is able to calculate the collective plasma effects but not the close electron-ion encounters, which is reflected by an indeterminate quantity $k_{\max}$ in the effective Coulomb logarithm.

(ii) A second approach, based on the Born approximation, places both the laser electric field and e-i collisions into the term W, in terms of an inverse bremsstrahlung transition rate, and $\mathbf{E}$ is set equal to zero. Close e-i collisions are now treated accurately (within the range of validity of the Born approximation), and the collective plasma effects of (i) are recovered by using the plasma dielectric function to modify the vacuum Coulomb potential around an ion. There are no indeterminate parameters in this model,[19,20] but its validity is limited to very low ionic charge.

(iii) The third model assumes that electron oscillation in the laser electric field does not modify e-i scattering and can be separated from it: the laser electric field is included in $\mathbf{E}$, and e-i scattering (in an electrostatic potential) is in the term W. This model is used for high-Z plasmas.

The different models are discussed and compared below. In Sec. III, $\ln\Lambda_{ib}$ is derived in the Born approximation, and the results are discussed in Sec. IV A for low Z. For Z greater than ~ 10, quantum-mechanical effects on e-i scattering are negligible, and W can be evaluated using the classical trajectory of an electron in a self-consistent potential based on model (iii). Results for medium and high-Z materials, based on this model, are discussed in Secs. IV B and IV C.

III. BORN APPROXIMATION

For low-Z elements where the Born approximation is valid, the transition rate W in Eq. (14) is calculated in terms of a transition amplitude $|a|$ according to[9,19]

$$W(\mathbf{v}\Longrightarrow\mathbf{v}+\Delta\mathbf{v})d^{3}\Delta\mathbf{v} = V(2\pi/\hbar)\,|a|^{2}\delta(\pm\hbar\omega+E_{p}-E_{p+k})d^{3}k/(2\pi)^{3}\ , \tag{15}$$

where p and k are wave numbers related to the electron momentum ($\hbar\mathbf{p}=m\mathbf{v}$ and $\hbar\mathbf{k}=m\Delta\mathbf{v}$), E_p is the electron energy $\hbar^2p^2/(2m)$, V is the plasma volume, and the $\pm$ sign corresponds to absorption and emission. The amplitude $|a|$ is calculated from second-order perturbation theory in terms of both the operator for photon absorption (and emission) H_1, and the operator for electron-ion scattering H_2,

$$a=V\int[\langle\mathbf{p}+\mathbf{k}\,|\,H_1\,|\,\mathbf{p}'+\mathbf{k},\hbar\omega\rangle\langle\mathbf{p}'+\mathbf{k},\hbar\omega\,|\,H_2\,|\,\mathbf{p},\hbar\omega\rangle/(E_p-E_{p+k}) + \langle\mathbf{p}+\mathbf{k}\,|\,H_2\,|\,\mathbf{p}'\rangle\langle\mathbf{p}'\,|\,H_1\,|\,\mathbf{p},\hbar\omega\rangle/\hbar\omega]d^{3}p'/(2\pi)^{3}\ . \tag{16}$$

The photon momentum is neglected here compared to the electron momentum. Induced emission is obtained by replacing ω with $-\omega$. The processes being considered are shown schematically in Fig. 2. The electrons are described by plane waves

$$|\,\mathbf{p}\rangle=V^{-1/2}\exp(i\mathbf{p}\cdot\mathbf{r}-iE_{p}t/\hbar)\ . \tag{17}$$

The photon interaction term is

$$\langle\mathbf{p}'\,|\,H_1\,|\,\mathbf{p},\hbar\omega\rangle=-V^{-1}(\hbar e/mc)\,\mathbf{A}\cdot\mathbf{p}\delta(\mathbf{p}-\mathbf{p}')\ , \tag{18}$$

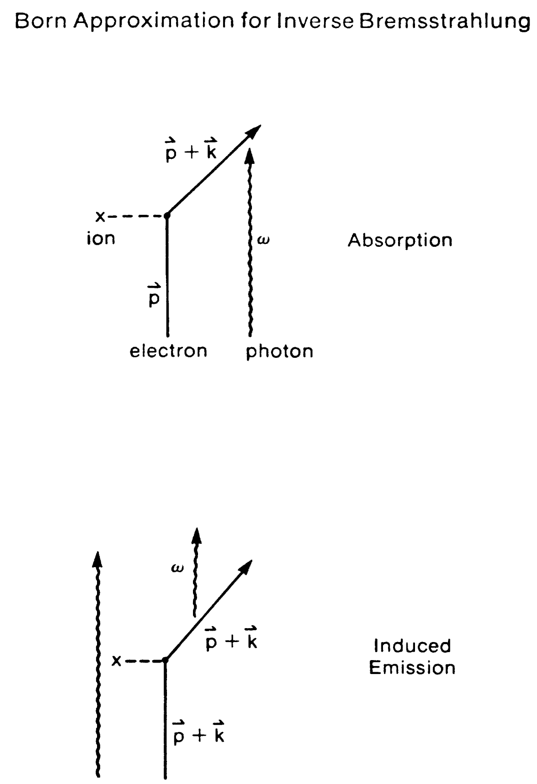

FIG. 2. Schematic of the processes contributing to inverse bremsstrahlung in the Born approximation. Momentum transfer by the photon is neglected.

where $\mathbf{A}$ is related to the laser electric field by $\mathbf{E}=-c^{-1}\partial A/\partial t$, and $\mathbf{E}=\mathbf{E}_0 e^{i\omega t}$. The electron-ion scattering term H_2 is related to the vacuum electrostatic potential $V(\mathbf{r})$, modified by the plasma dielectric function[19] ϵ. The vacuum potential is

$$V(\mathbf{r})=-Ze^2\sum_j \frac{1}{|\mathbf{r}-\mathbf{r}_j|} , \tag{19}$$

where the summation is over all ion positions. The resulting e-i matrix element is

$$\langle \mathbf{p}+\mathbf{k} \,|\, H_2 \,|\, \mathbf{p} \rangle = V^{-1} V(k,\Delta\omega)/\epsilon(k,\Delta\omega) , \tag{20}$$

where $V(k,\Delta\omega)=\langle \mathbf{p}+\mathbf{k} \,|\, V(\mathbf{r}) \,|\, \mathbf{p}\rangle$ and $\Delta\omega=(E_{p+q}-E_p)/\hbar$. Using

$$V(k,\Delta\omega)=4\pi Ze^2\frac{1}{k^2}e^{i\Delta\omega t}\sum_j e^{-i\mathbf{k}\cdot\mathbf{r}_j} , \tag{21}$$

together with Eqs. (16), (18), and (20), the transition rate in Eq. (15) is obtained,

$$\begin{aligned} W(\mathbf{v}\Longrightarrow \mathbf{v}+\hbar\mathbf{k}/m)d^3\hbar\mathbf{k}/m &= \delta(\pm\omega-\mathbf{v}\cdot\mathbf{k}-\hbar k^2/2m) \\ &\times \frac{1}{V}\frac{2\pi}{\hbar^2}\left|\frac{4\pi Ze^2}{k^2\epsilon(k,\omega)}\right|^2\left[\frac{e}{m\omega^2}\mathbf{k}\cdot\mathbf{E}\right]^2 \\ &\times \sum_{ij}\exp[i\mathbf{k}\cdot(\mathbf{r}_j-\mathbf{r}_i)]d^3k/(2\pi)^3 . \end{aligned} \tag{22}$$

Substituting the different rate processes into the rhs (right-hand side) of Eq. (14), we obtain the change in the distribution function from inverse bremsstrahlung,

$$\begin{aligned} \left[\frac{\partial f}{\partial t}\right]_{ib} = \frac{1}{\hbar^2}\left[\frac{2Ze^3}{m\omega^2}\right]^2 \int \left|\frac{\mathbf{k}\cdot\mathbf{E}}{k^2}\right|^2 \{ & [f(\mathbf{v}-\hbar\mathbf{k}/m)\delta(\omega-\mathbf{v}\cdot\mathbf{k}+\hbar k^2/2m)-f(v)\delta(\omega-\mathbf{v}\cdot\mathbf{k}-\hbar k^2/2m)]/|\epsilon(k,\omega)|^2 \\ & -[f(\mathbf{v}-\hbar\mathbf{k}/m)\delta(-\omega-\mathbf{v}\cdot\mathbf{k}+\hbar k^2/2m) \\ & +f(v)\delta(-\omega-\mathbf{v}\cdot\mathbf{k}-\hbar k^2/2m)]/|\epsilon(k,-\omega)|^2\} V^{-1}\sum_{ij}\exp[-i\mathbf{k}\cdot(\mathbf{r}_j-\mathbf{r}_i)]d^3k , \end{aligned} \tag{23}$$

where the first two terms in { } account for photoabsorption and the last two account for induced emission.

To proceed, we assume that the electron distribution function f is Maxwellian, which is a good approximation for a low-Z plasma with short wavelength illumination. We separate $\mathbf{v}$ into a component parallel to $\mathbf{k}$, $v_\parallel$, and a component perpendicular $\mathbf{v}_\perp$. Using the relations for $\mathbf{v}_\parallel$ from the δ functions, Eq. (23) reduces to

$$\begin{aligned} \left[\frac{\partial f}{\partial t}\right]_{ib} = \frac{1}{\hbar^2}\left[\frac{2Ze^3}{m\omega^2}\right]\frac{\hbar\omega}{T}f(v_\perp)\int\left|\frac{\mathbf{k}\cdot\mathbf{E}}{k^2\epsilon(k,\omega)}\right|^2 \exp\left[-\frac{m}{2T}\left[\frac{\omega^2}{k^2}+\frac{\hbar k^2}{4m^2}\right]\right] \\ \times\frac{1}{V}\sum_{ij}\exp[-i\mathbf{k}\cdot(\mathbf{r}_j-\mathbf{r}_i)][\delta(\omega-\mathbf{v}\cdot\mathbf{k}+\hbar k^2/2m-\delta(\omega-\mathbf{v}\cdot\mathbf{k}-\hbar k^2/2m)]d^3k , \end{aligned} \tag{24}$$

where we have used $\hbar\omega \ll T$; replaced $\mathbf{k}$ by $-\mathbf{k}$ in the last two terms in { } in Eq. (23); used the relation[14] $\epsilon(-k,-\omega)=\epsilon^*(k,\omega)$; and assumed $f(v)=f(v_\perp)\exp(-mv_\parallel^2/2T)$.

The expression (24) clearly conserves electrons, i.e., $\int(\partial f/\partial t)_{ib}d^3v=0$, because after integration over $v_\parallel$, the δ functions cancel. To find the rate of energy increase from photon absorption, we evaluate

$$\frac{\partial E}{\partial t}=\int\left[\frac{\partial f}{\partial t}\right]_{ib}\left[\frac{1}{2}mv^2\right]d^3v \ . \tag{25}$$

The summation over ions in Eq. (24) is evaluated using the thermal equilibrium ion-ion correlation factor,[21]

$$\frac{1}{V}\sum_{ij}\exp[-ik\cdot(\mathbf{r}_j-\mathbf{r}_i)]=n_i\frac{k_e^2+k^2}{(1+Z)k_e^2+k^2} \ , \tag{26}$$

where k_e is the reciprocal of the electron Debye length ($k_e^2=4\pi n_e e^2/T$). Substituting into Eq. (25), we obtain

$$\frac{\partial E}{\partial t}=\frac{16\pi^2}{3}\left[\frac{Ze^3}{m\omega}\right]^2\frac{1}{m}f(v=0)n_iE_0^2 \times\int\frac{1}{|\epsilon(k,\omega)|^2}\frac{k_e^2+k^2}{(1+Z)k_e^2+k^2} \times\exp\left[-\frac{m}{2T}\left[\frac{\omega^2}{k^2}+\frac{\hbar^2k^2}{4m^2}\right]\right]\frac{dk}{k} \ . \tag{27}$$

Factoring out the laser intensity I ($=cE_0^2/8\pi$) from the rhs of Eq. (27), we are left with the absorption coefficient.

The integral in Eq. (27) is identified as $\ln\Lambda_{ib}$; it is the same as the classical expression[3] except for the exponential factor $\exp[-(\hbar k)^2/8mT]$. This additional factor is characteristic of quantum-mechanical plasma calculations[20] and has the effect of preventing divergence of the integral for large k (corresponding to small impact parameter). The usual divergence at small k, i.e., large impact parameter, is eliminated by both the Debye shielding in $\epsilon(k,\omega)$ and the frequency dependent exponential. The integral can be evaluated analytically if $\epsilon(k,\omega)$ is replaced by its static result,[14]

$$\epsilon(k,\omega)\approx 1+k_e^2/k^2 \ . \tag{28}$$

This approximation introduces only a few percent error compared to the exact result which was evaluated numerically.

Using the static approximation for $\epsilon(k,\omega)$ and the condition $\hbar\omega \ll T$ (characteristic of the laser irradiated plasma), the integral in Eq. (27) becomes

$$\ln\Lambda_{ib}(\text{Born})=\ln\Lambda_q+\ln[(Z+1)^{1/2}/\tilde{\omega}]-\gamma+\tfrac{1}{2}\ln(\tfrac{4}{3}) +\frac{1}{2Z}e^{\tilde{\omega}^2/2}\mathrm{E}_1(\tilde{\omega}^2/2) -\frac{Z+1}{2Z}e^{\tilde{\omega}^2/2(Z+1)}\mathrm{E}_1(\tilde{\omega}^2/2(Z+1)) \ , \tag{29}$$

where $\ln\Lambda_q$ is the standard quantum-mechanical Coulomb logarithm defined in Eq. (9), γ is Euler's constant ($\gamma=0.577$), E_1 is the exponential integral, and $\tilde{\omega}=\omega/\omega_p$ (where ω_p is the local plasma frequency: or in terms of the critical density, $\tilde{\omega}^2=n_c/n$).

IV. RESULTS

In this section, we discuss the following results.

(1) The new Born-approximation result (with plasma response), Eq. (29), is evaluated for $Z\sim 3$ at conditions relevant to 0.35 μm irradiation (Sec. IV A).

(2) In Sec. IV B, the relation [Eq. (13)] between $\ln\Lambda_{ib}$ and $\ln\Lambda_{ei}$ is discussed in terms of an extrapolation from the Born approximation result. Based on this relation, $\ln\Lambda_{ei}$ can be calculated in the classical region ($Z>10$) by evaluating the trajectory of an electron in a shielded electrostatic potential. This has been done previously, using the approximation $\Lambda \gg 1$ (which is appropriate for moderate-Z plasmas), and no improvements are presented where this approximation is valid.

(3) The approximation $\Lambda \gg 1$ is removed in Sec. IV C, by numerically calculating the electron trajectory in a self-consistent electrostatic potential determined by the nonlinear Debye-Hückel equations. This approach qualitatively models the onset of strong ion-ion correlations as high Z is approached, and it merges smoothly with previous results at moderate Z.

A phenomenological fit to the results over the entire range of Z is presented in Eq. (46).

A. Low Z

Two features characterize the Coulomb logarithm for laser absorption in low-Z materials: (1) the plasma shielding distance λ_D is sufficiently large, that the ω/v_t contribution to $b_{\max}$ can introduce an ω dependence to Λ [Eq. (3a)]; and (2) the impact parameter for 90° electron scattering is sufficiently small that quantum wave effects can contribute to $b_{\min}$ [Eq. (6)]. The first effect has been calculated by Dawson and Oberman.[3] Both effects are simultaneously addressed by the modified Born approximation result derived in Sec. III. To compare the Born-approximation result with Dawson and Oberman, we examine two limits: (1) absorption near the critical density [$\tilde{\omega}\sim 1$ in Eq. (29)], and (2) absorption at very low density ($\tilde{\omega}\gg 1$). It is the latter limit which is most often quoted,[1] but it is the former which is most relevant to laser fusion experiments.

In the high frequency limit ($\tilde{\omega}\gg 1$) appropriate for low density absorption, both exponential integrals in Eq. (29) approach zero, leaving

$$\ln\Lambda_{ib}\approx\ln(4T/\hbar\omega)-\gamma \ . \tag{30}$$

(The same result has been obtained for bremsstrahlung emission[22] in the Born approximation with $\hbar\omega \ll T$, for a pure Coulomb potential.) To compare with the Dawson-Oberman result, $\ln\Lambda_{\mathrm{DO}}$, the indeterminate quantity $k_{\max}$ ($=1/b_{\min}$) in Ref. 3 is replaced by the suggested quantum-mechanical expression, $k_{\max}=(mT)^{1/2}/\hbar$. The result is listed in Table I A. The difference from Eq. (30) is $\ln\Lambda_{ib}-\ln\Lambda_{\mathrm{DO}}\approx 0.75$, representing a 15% correction for conditions attained in short-wavelength laser irradiation of $(\mathrm{CH})_x$, characterized by $\ln\Lambda\approx 5$.

For the region around the critical density ($\tilde{\omega}\sim 1$), Eq. (29) reduces to

$$\ln\Lambda_{ib}\approx\ln\Lambda_q+\tfrac{1}{2}\ln(\tfrac{2}{3})-\tfrac{1}{2}\gamma-\frac{1}{2Z}\ln(Z+1) \ , \tag{31}$$

where we have also assumed $Z^{1/2}/\tilde{\omega}\gg 1$. This should

TABLE I. Quantum-mechanical limit of $\ln\Lambda_{ib}$ ($T > Z^2 35$ eV). The effective Coulomb logarithm is written in terms of Λ_q [Eq. (10)] and a remainder C. The expressions for C are compared.

	C	Numerical value
	A. $\omega \gg \omega_p$: $\ln\Lambda = \ln(\Lambda_q \omega_p/\omega) + \frac{1}{2}\ln(1+Z) + C$	
Dawson and Oberman	$-\frac{1}{2}\ln(6) - \gamma/2$	-1.18
Cauble and Rozmus	$\frac{1}{2}\ln(\pi/3) - \gamma$	-0.55
Bremsstrahlung ($\hbar\omega \gg T$)	$\frac{1}{2}\ln(\frac{4}{3}) - \gamma$	-0.43
$\ln\Lambda_{ib}$ (Born), Eq. (30)	$\frac{1}{2}\ln(\frac{4}{3}) - \gamma$	-0.43
	B. $\omega \approx \omega_p$: $\ln\Lambda = \ln\Lambda_q - \frac{1}{2Z}\ln(1+Z) + \frac{1}{Z}O(1/2) + C$	
Dawson	$-\frac{1}{2}\ln(12)$	-1.24
Cauble and Rozmus	$\frac{1}{2}\ln(6/\pi)$	-0.32
$\ln\Lambda_{ei}$(Born), Eq. (36)	$\frac{1}{2}\ln(\frac{2}{3}) - \gamma/2 - \frac{1}{2}$	-0.49
$\ln\Lambda_{ib}$(Born), Eq. (31)	$\frac{1}{2}\ln(\frac{2}{3}) - \gamma/2$	-0.49

be compared with the $\omega \ll \omega_p$ case of Dawson in Ref. 21, $\ln\Lambda_D$, listed here in Table I B, and evaluated at $\omega=\omega_p$. (When the effects of ion shielding are included, the results for large and small ω are no longer equal at $\omega=\omega_p$ as they were in Ref. 3, which used only electron shielding. The $\omega \ll \omega_p$ result is the one that best approximates the correct solution for $\omega=\omega_p$.) Again, the suggested replacement $k_{\max}=(mT)^{1/2}/\hbar$ was used, with the same result for the difference in solutions: $\ln\Lambda_{ib} - \ln\Lambda_D \approx 0.75$. In this region that dominates laser absorption, the ω dependence of $\ln\Lambda_{ib}$ is found to be negligible.

Equation (31) is similar to the result obtained by Cauble and Rozmus[15] who did not use the Born approximation but rather a modified Coulomb potential which approximates quantum effects at small distances. The main difference numerically is that they obtain the factor $k_q^2/(k_q^2+k^2)$, where k_q is the thermal de Broglie wave number $(2\pi mT)^{1/2}/\hbar$, instead of the factor $\exp(-\hbar^2 k^2/8mT)$ in Eq. (27). Their resulting Coulomb logarithm differs from the one here by only ~ 0.1 for $Z=3$. Cauble and Rozmus note that there is a substantial difference between their results (with linear, Debye-Hückel ion correlations) and the Dawson-Oberman results from Ref. 3 which indeed did not include the ion contribution to shielding. However, if comparison had been made with Ref. 21 instead, where Dawson has removed the assumption of a random ion distribution and imposed Debye-Hückel correlations, then very little difference would have been found (using the $\omega \ll \omega_p$ result for the region around the critical density). The remaining difference could be removed by modifying the choice of $k_{\max}$, which does not depend on ion correlations.

The relationship between $\ln\Lambda_{ib}$ and the classical high and low-frequency limits is shown in Fig. 3 over the density range from $0.1n_c$ to n_c, for a 1-keV plasma with $n_c = 9\times10^{21}$ cm^{-3}. Although $\ln\Lambda_{DO}(\omega \ll \omega_p)$ was derived for $n > n_c$, it is evaluated here at the subcritical density indicated. Over this density range where laser absorption predominantly occurs, $\ln\Lambda_{ib}$ is well approximated by the $\omega \ll \omega_p$ result. Only at densities well below $0.1n_c$ does $\ln\Lambda_{ib}$ approach the high-frequency limit. Use of the high-frequency limit around n_c results in a $\sim 20\%$ error for low Z.

B. Moderate Z

To extend the calculation of the inverse-bremsstrahlung Coulomb logarithm to moderate and

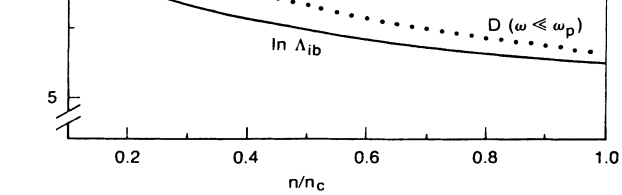

FIG. 3. The density dependence of $\ln\Lambda_{ib}$(Born) for $Z=3$, $T=1$ keV in terms of the critical density n_c (9×10^{21} cm^{-3}). Compared are Eq. (8) for $\ln\Lambda_s$, the Dawson and Oberman (DO) result (Ref. 3) for $\omega \gg \omega_p$, and Dawson's (D) result (Ref. 21) derived for $\omega \ll \omega_p$ but evaluated at the $\omega > \omega_p$ density indicated. Over the region shown, $\ln\Lambda_{ib}$ is best approximated by the $\omega \ll \omega_p$ result. Only at densities below $\sim 0.1n_c$ does $\ln\Lambda_{ib}$ reach the $\omega \gg \omega_p$ result.

high Z, we first examine the relationship between $\ln\Lambda_{ib}$ and the Coulomb logarithm for electron scattering in a shielded, electrostatic potential around each ion, $\ln\Lambda_{ei}$. The change in the distribution function due to e-i scattering is found from Eq. (14) with the transition rate W written in terms of the e-i cross section σ,

$$W(\mathbf{v}\Longrightarrow\mathbf{v}')d^3v = vn_i\sigma(\theta,\mathbf{v})d\Omega \ . \tag{32}$$

Since only elastic scattering is considered, we have $|\mathbf{v}'|=|\mathbf{v}|$ and $\mathbf{v}\cdot\mathbf{v}'=v^2\cos\theta$. There are a number of approximations implicit in using Eq. (32) with Eq. (14) to calculate inverse bremsstrahlung. (1) Modification of the scattering by the laser interaction, which produces the ω dependence in $b_{\max}$, has been neglected. (2) The time-dependent response of the plasma is neglected. (3) The static response of the plasma is approximated with a self-consistent electrostatic potential, i.e., the potential rather than the transition rate is averaged over ion positions. The contributions of the first two processes were found to be small or negligible for $(CH)_x$ ($Z\sim3$) near n_c and to further decrease as $1/Z$ (Secs. III and IV A). The third approximation is discussed in more detail below.

Equations (14) and (32) are now used to extract an expression for $\ln\Lambda_{ib}$ in terms of σ. We treat the laser electric field in Eq. (14) as a perturbation of the form $\mathbf{E}=\mathbf{E}_0 \cos(\omega t)$ and expand the electron distribution function as

$$f(\mathbf{v})=f_0(v)+\hat{\mathbf{E}}_0\cdot\hat{\mathbf{v}}f'(v) \ , \tag{33}$$

where the zeroth-order distribution function is a Maxwellian normalized to the electron density. (The zeroth-order distribution function can deviate from a Maxwellian[23] at high laser intensities for high-Z materials, but this effect will not be considered here.) The laser absorption rate is evaluated in Appendix A, yielding the following quantity as the effective e-i Coulomb logarithm appropriate for inverse bremsstrahlung:

$$\ln\Lambda_{ei}=\tfrac{1}{8}\int dv\ vf_0(v) \times \int_{-1}^{1}\bar{\sigma}(\theta,v)(1-\cos\theta)\,d(\cos\theta)\Big/\int f_0 v\,dv \ . \tag{34}$$

The dimensionless quantity $\bar{\sigma}$ is defined as

$$\bar{\sigma}=\sigma(\theta,v)/\sigma_c(180^\circ,v) \tag{35}$$

in terms of σ_c, the cross section for 180° scattering in a pure Coulomb potential: $\sigma_c=(Ze^2/2mv^2)^2$. The first velocity moment of the distribution function is required for inverse bremsstrahlung. (Other e-i processes such as electron diffusion would require higher-order moments.)

Equation (34) is a general expression spanning the Born and classical limits. Its accuracy can be tested in the quantum-mechanical limit by using the Born approximation to evaluate $\sigma(\theta,v)$ in a Debye-shielded potential and then comparing the result with $\ln\Lambda_{ib}$ (Born) of Eq. (31) which did not make any assumptions about the potential. The calculation of $\ln\Lambda_{ei}$ (Born) is performed in Appendix B, with the result

$$\ln\Lambda_{ei}(\text{Born})=\ln\Lambda_q+\tfrac{1}{2}\ln(\tfrac{2}{3})-\tfrac{1}{2}\gamma-\tfrac{1}{2} \ . \tag{36}$$

Comparing Eqs. (31) and (36), one difference is that $\ln\Lambda_{ib}$ has an ω dependence; however this dependence was found to be small near the critical density for $Z=3$ and to further decrease for higher Z (Sec. III A). The only other difference is that $\ln\Lambda_{ib}$ has the term $C_{ib}=\ln(1+Z)/2Z$ compared to $C_{ei}=\frac{1}{2}$ in $\ln\Lambda_{ei}$. This difference arises because $\ln\Lambda_{ib}$ was averaged over ion positions, whereas for $\ln\Lambda_{ei}$ the electrostatic potential was averaged. These two terms, C_{ib} and C_{ei}, have the same limit for $Z\Longrightarrow0$, but they rapidly diverge for nonzero Z. For $Z=3$, C_{ib} is already reduced to 0.2. This term represents perhaps the greatest source of uncertainty in relating $\ln\Lambda_{ib}$ to $\ln\Lambda_{ei}$, and further investigation is required to determine the magnitude of the error. For the remainder of this paper, Eq. (13) will be used to extrapolate into the high-Z region, beyond the limit of validity of the Born approximation.

Using Eq. (13), $\ln\Lambda_{ib}$ is now evaluated for moderate Z plasmas ($\Lambda\gg1$), using a previously calculated expression for $\ln\Lambda_{ei}$. Liboff[7] obtained a result for $\ln\Lambda_{ei}$ by calculating the classical trajectory of an electron in a Debye shielded potential. The result corresponding to Eq. (34) is

$$\ln\Lambda_{ei}(\text{classical})=\ln\Lambda_c+\ln\tfrac{4}{3}-2\gamma-\tfrac{1}{2} \ , \tag{37}$$

which is valid for $\Lambda\gg1$. Inserting into Eq. (13), we obtain for $\Lambda\gg1$ and $\omega\approx\omega_p$

$$\begin{aligned}\ln\Lambda_{ib}(\text{classical})&=\ln\Lambda_{ib}(\text{Born})+\ln(\Lambda_c/\Lambda_q)-\tfrac{3}{2}\gamma+\tfrac{1}{2}\ln(\tfrac{8}{3})\\&=\ln\Lambda_c-2\gamma+\ln(\tfrac{4}{3})-\frac{1}{2Z}\ln(1+Z)\end{aligned} \tag{38}$$

in the classical region.

It now remains to treat the transition region between the quantum-mechanical and classical limits, for moderate-Z plasmas. The transition region, as a function of Z, is sufficiently small (Fig. 1) and the logarithms are sufficiently slowly varying, that generally only a few percent error is made by simply using

$$\ln\Lambda=\min\{\ln\Lambda(\text{classical}),\ln\Lambda(\text{Born})\} \ . \tag{39}$$

A more exact treatment is obtained using the transition formula derived by Williams and DeWitt,[12]

$$\ln\Lambda_{ei}=\ln\Lambda_{ei}(\text{classical})-\tfrac{1}{2}e^zE_1(z) \ , \tag{40}$$

where E_1 is the exponential integral, $\gamma=0.577$, and z is related to the ratio Λ_q and Λ_c [defined by Eqs. (9) and (10)] according to

$$z=\tfrac{3}{8}e^{2\gamma}(\Lambda_q/\Lambda_c)^2 \ . \tag{41}$$

In the limits of large and small z, the classical and quantum-mechanical limits are obtained, respectively. For the example of $Z=3$, $n_e=9\times10^{21}$ and $T=1$ keV, the interpolation parameter z is ~0.4, which is only marginally in the quantum-mechanical limit. The result is $\ln\Lambda_{ei}=\ln\Lambda_{ei}(\text{Born})-0.35$, representing about a 7% correction to the Born approximation, at these conditions.

C. High Z

For high Z, where Λ is less than ~ 10, we continue to use Eq. (13) to calculate $\ln\Lambda_{ib}$ from e-i scattering in an average, self-consistent, electrostatic potential, but we do not assume that the potential is given by the linear Debye-Hückel model. In this region, corresponding roughly to $Z > 25$, Fig. 1 shows that it would be questionable to use λ_D as the shielding distance, or to use approximations dependent on $\Lambda \gg 1$. Here we evaluate $\ln\Lambda_{ei}$ by numerically calculating the electron trajectory in the nonlinear Debye-Hückel potential to determine the relationship between the impact parameter b and the scattering angle θ. Using $\sigma\, d\cos\theta = b\, db$, the double integral in Eq. (34) is calculated numerically, and finally, $\ln\Lambda_{ib}$ is evaluated from $\ln\Lambda_{ei}$ using Eq. (13).

The electrostatic potential $V(r)$ was calculated from Poisson's equation, without linearization,

$$\nabla^2 V = -4\pi e(Zn_i - n_e)\ , \tag{42}$$

$$n_e = Z\langle n_i\rangle \exp(eV/T_e)\ , \tag{43}$$

$$n_i = \langle n_i\rangle \exp(-eZV/T_i)\ , \tag{44}$$

with the boundary conditions $V(\infty)=0$, $V(r \Longrightarrow 0) = Ze/r$. The Fermi-Dirac form of the electron distribution function was also used, because n_e can become large (and degenerate) near the nucleus; however the degeneracy effect on $\ln\Lambda$ was found to be insignificant for the conditions considered here. If Eqs. (42)–(44) are linearized in terms of V, the usual Debye-Hückel shielding length is obtained. However, linearization is not valid within the average ion-sphere radius R_0 where V becomes large. Near the central ion, Eq. (44) forces the ion density of neighboring ions to rapidly approach zero, and only electrons remain for shielding.

The classical trajectory for an electron scattering in an arbitrary potential $V(r)$ is given by[24]

$$\theta = \pi + 2\int_0^{u_0} [1 - V(1/u)/E - (bu)^2]^{-1/2} b\, du\ , \tag{45}$$

where θ is the scattering angle, b is the impact parameter, and u is the inverse radius between the electron and the ion. The upper limit to the integral is given by the zero of the square-root factor and corresponds to the distance of closest approach.

This nonlinear Debye-Hückel model for the ions is valid for values of the ion-ion coupling parameter Γ ($= Z^2e^2/R_0T$) less than ~ 1. For the example of $Z=50$, $T=0.5$ keV, and $n_e = 9\times10^{21}$, we have $\Gamma = 6$, which suggests that NLDH may be only marginally applicable. To test the sensitivity of $\ln\Lambda_{ib}$ to the model, an alternate potential was tried: the potential was determined assuming a uniform electron density, which does not permit neighboring ions inside R_0. (The NLDH model does permit a small amount of neighboring ions to penetrate R_0.) For $Z \sim 50$, there was less than $\sim 2\%$ difference between the models for the calculation of $\ln\Lambda_{ei}$; and both gave values about a factor of 2 higher than the linearized Debye-Hückel model result. (For low Z, the NLDH model reproduces the linearized results of Liboff to within a few percent, while the uniform electron model is $\sim 50\%$ low and would not be applicable in this region.) This suggests the applicability of using the NLDH model to calculate $\ln\Lambda_{ei}$ over the entire classical region, at the conditions considered here.

To test the numerical procedure, comparison was made with the free-free Gaunt factor calculated by Lamoureux *et al.*[11] They performed a quantum-mechanical partial wave calculation of bremsstrahlung emission, produced by 1 keV electrons in a Ce ($Z=55$) plasma at an ion density of 8.6×10^{21} cm^{-3}. The Gaunt factor G is related to the Coulomb logarithm by $G = (\pi^{1/2}/3)\ln\Lambda$. Lamoureux *et al.* observe that G is relatively insensitive to the shapes of potentials with roughly the same range, as above. Their effective $\ln\Lambda$ in the soft photon limit is 1.2. The classical model used here is in close agreement predicting 1.3 for the NLDH potential.

The NLDH results for $\ln\Lambda_{ib}$, as a function of Z, are presented in Fig. 4 for $T=1$ keV and in Fig. 5 for $T=0.5$ keV (both are at the critical density 9×10^{21} cm^{-3} for 0.35 μm light). Also shown in the figures are (1) $\ln\Lambda_{ei}$ from Liboff's calculations corresponding to the moments in Eq. (34), (2) $\ln\Lambda_s$ defined in Eq. (9a) which uses the Debye length λ_D as the shielding distance, and (3) $\ln\Lambda_s^*$ which uses the average-ion radius as the shielding distance whenever it is larger than λ_D [Eq. (9b)]. We make the following observations. The deviation between $\ln\Lambda_s$ and $\ln\Lambda_s^*$ becomes apparent for Z greater than ~ 10, corresponding to the region of $R_0 > \lambda_D$ in Fig. 1. Both $\ln\Lambda$ (Liboff) and $\ln\Lambda_s$ have the wrong functional form in this region, and they would become negative at higher Z or lower T. [The two are related by Eq. (37).] Nevertheless, over the region shown, $\ln\Lambda_s$ is able to ap-

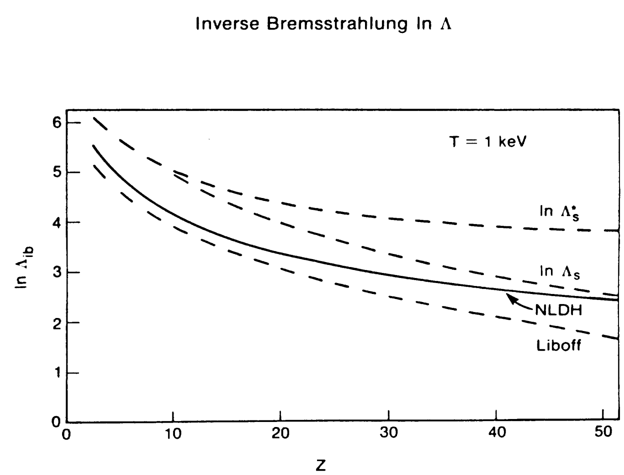

FIG. 4. $\ln\Lambda_{ib}$, using nonlinear Debye-Hückel ion correlations (NLDH), compared with the approximate $\ln\Lambda_{ei}$ of Liboff (Ref. 7), $\ln\Lambda_s$ of Eq. (9a) which uses λ_D as the shielding length, and $\ln\Lambda_s^*$ which uses the maximum of the ion-sphere radius R_0 and λ_D as the shielding length. At higher Z (or lower temperature) both $\ln\Lambda_s$ and Liboff's result would become negative. The results are for $n_e = 9\times10^{21}$ cm^{-3} and $T_e = 1$ keV. Equation (46) is a good approximation to the NLDH results.

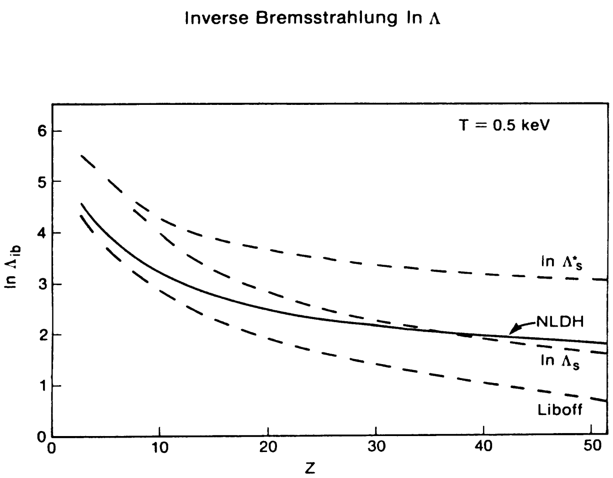

FIG. 5. Same as Fig. 4 but with $T_e = 0.5$ keV.

proximate the NLDH results to within $\sim 10\%$.

The often quoted high-frequency limit of Dawson and Oberman is not shown in the figures. Effectively, it uses only electron shielding for $b_{\max}$ and is related to $\ln\Lambda_s$ by $\ln\Lambda(\omega \gg \omega_p) \approx \ln\Lambda_s + \frac{1}{2}\ln(1+Z) - 1$ For $Z = 50$, it would be in error by about 50% compared to $\ln\Lambda$(NLDH).

The NLDH solution decreases very slowly with Z, and does not fall much below 2. (The results quoted by Lee and More[10] also have a minimum value of 2. However, the agreement is probably only coincidental as Lee and More refer to the Coulomb logarithm for thermal conductivity which uses a much higher velocity moment than $\ln\Lambda_{ib}$.) One result of the NLDH calculation is to support the use of $\max(R_0, \lambda_D)$ as the effective shielding distance. The $\ln\Lambda_s^*$ curve, which has this constraint, very closely follows the functional form of $\ln\Lambda$(NLDH) into the high-Z, low-temperature region. An approximation to $\ln\Lambda$(NLDH), to within a few percent, is

$$\ln\Lambda_{ib}(\text{NLDH}) = \ln\Lambda_s^* - 1.25 \tag{46}$$

for the conditions of temperature and density considered here, including the quantum-mechanical region. The nonlogarithmic term 1.25 contains all the details of the calculation. It corresponds to a $\sim 50\%$ effect at high Z.

For high Z, $\ln\Lambda_{ib}$ is obtained by adding ~ 0.5 to $\ln\Lambda_{ei}$ [Eqs. (12) and (13)]. This attempt to reduce the effect of different ion configurations to an average electrostatic potential, represents a 25% variation for $Z \sim 50$. It is probably the greatest source of uncertainty in the calculation and needs further investigation.

V. SUMMARY

The "Coulomb logarithm" for laser absorption has been calculated for conditions achieved in short-wavelength laser irradiation: $n_e \sim 10^{22}$ cm^{-3} and $T \sim 1$ keV. At these conditions $\ln\Lambda_{ib}$ is <5, and uncertainties in previously used models can produce variations in this term of 20–50 %.

For low-Z materials, $\ln\Lambda_{ib}$ was calculated quantum mechanically using a modified Born approximation. Collective plasma effects were included by multiplying the e-i interaction term by the plasma dielectric function. Unlike the classical calculation,[3] the "minimum impact parameter" was well determined and, of course, related to the de Broglie wavelength. The effective "maximum impact parameter" was the same as the classical result. The ω dependence of $\ln\Lambda_{ib}$ was found to be negligible near the critical density n_c (Fig. 3) where absorption predominantly occurs. Use of the often-quoted high-frequency limit of $\ln\Lambda_{ib}$ in this region can lead to a $\sim 20\%$ error. Near n_c, $\ln\Lambda_{ib}$ is found to be closely related to $\ln\Lambda_{ei}$, the "Coulomb logarithm" for electrons scattering in a shielded electrostatic potential around an ion; Eq. (13) was used to extrapolate that relationship beyond the range of validity of the Born approximation into the high-Z region.

For Z greater than ~ 10, the minimum impact parameter is no longer quantum mechanical (Fig. 1) and is determined by the distance of closest approach for the classical electron trajectory around an ion. From the trajectory, an effective e-i Coulomb logarithm, $\ln\Lambda_{ei}$, was calculated using Eq. (34), and Eq. (13) was then used to determine $\ln\Lambda_{ib}$. To bridge the classical and quantum-mechanical regions, the results of Williams and DeWitt[12] were used. For moderate Z plasmas, where the approximation $\Lambda_{ei} \gg 1$ is applicable, a previously calculated expression[7] was used for $\ln\Lambda_{ei}$.

However, at high Z, the calculation of $\ln\Lambda_{ei}$ does not permit approximations based on $\Lambda \gg 1$ or the use of λ_D as the shielding length (Fig. 1). We have extended the calculation into the high-Z region by using the nonlinear Debye-Hückel model. The dominant high-Z effect is that neighboring ions are strongly repelled at distances smaller than the average-ion radius R_0. The NLDH model was found to reproduce results for $\ln\Lambda_{ei}$ at high Z (as calculated from the uniform electron model), and to also merge smoothly to moderate-Z results (calculated by Liboff[7]). Use of Eq. (13) to relate the average of $\ln\Lambda_{ib}$ over all ion configurations to the result obtained from an average, spherical, electrostatic potential is probably the greatest source of uncertainty in the calculation, and the resulting error requires further investigation.

Results for $\ln\Lambda_{ib}$ are shown in Fig. 4 at $n_e = 9 \times 10^{21}$ cm^{-3} for $T = 1$ keV, and in Fig. 5 for $T = 0.5$ keV. A good fit to the numerical results for all Z is given by Eq. (46). The calculation supports the use of replacing λ_D by R_0 as the effective shielding distance, Eq. (3b), whenever $\lambda_D < R_0$. The nonlogarithmic term 1.25 in Eq. (46) represents a 50% correction at high Z.

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APPENDIX A: EFFECTIVE *e-i* COULOMB LOGARITHM

When Eqs. (32) and (33) are substituted into Eq. (14), the following first-order equation is obtained:

$$\frac{\partial f'}{\partial t}+\frac{eE_0 v}{T}\cos(\omega t)f_0 =2\pi n_i f' v\int_{-1}^{1}\sigma(1-\cos\theta)\,d(\cos\theta)\ . \qquad (A1)$$

The solution of Eq. (A1) is used to find the average rate of laser absorption which is given by

$$\langle \mathbf{E}\cdot\mathbf{j}\rangle=\int \mathbf{E}\cdot\mathbf{v}\, ef'\,d^3v =\frac{2\pi}{3}\left[\frac{eE_0}{\omega}\right]^2\frac{1}{T}\int\frac{1}{\tau}\frac{1}{1+(\omega\tau)^{-2}}f_0v^4\,dv\ , \qquad (A2)$$

where τ is defined in terms of the right-hand side of Eq. (A1),

$$\frac{1}{\tau}=2\pi n_i\left[\frac{Ze^2}{2mv^2}\right]^2 v\int_{-1}^{1}\bar{\sigma}(1-\cos\theta)\,d(\cos\theta) \qquad (A3)$$

and $\bar{\sigma}$ is defined in Eq. (35). Typically in laser plasmas, the results are only slightly modified by the approximation $\omega\tau \gg 1$. Comparing Eqs. (A2) and (A3) with the usual expression for laser absorption,[23] the expression in Eq. (34) is identified as the effective $\ln\Lambda$ factor, neglecting the $\omega\tau$ factor.

As an example, we evaluate the θ integral in Eq. (34) for the case of scattering in a Coulomb potential: $\bar{\sigma}=\sin^{-4}(\theta/2)$. Without plasma shielding, it is necessary to impose a lower bound $\theta_{\min}$ on the θ integration to prevent divergence of the integral. The result is $\ln\Lambda=\ln(2/\theta_{\min})$ with the approximation $\sin\theta\approx\theta$ and neglect of any velocity dependence in $\theta_{\min}$. Using the classical relation between θ and impact parameter b for a Coulomb potential $[\sin^2(\theta/2)=1+(b/b_{90°})^2$ obtained from solution of Eq. (45)], we relate $\theta_{\min}$ to $b_{\max}$ and obtain

$$\ln\Lambda(\text{Coulomb})\approx\tfrac{1}{2}\ln(1+b_{\max}^2/b_{90°}^2) \qquad (A4)$$

with $b_{90°}$ defined in Eq. (4).

APPENDIX B: $\ln\Lambda_{ei}$ (BORN)

For electrons scattering in a Debye shielded potential, the scattering cross section in the Born approximation is[25]

$$\bar{\sigma}_B(\theta,v)=[\sin^2(\theta/2)+\lambda_q^2/\lambda_D^2]^{-2}\ , \qquad (B1)$$

where $\lambda q=\hbar/2mv$. Substituting into Eq. (34), we obtain after the θ integration

$$\ln\Lambda_{ei}(\text{Born}) =\int f_0\tfrac{1}{2}\left[\ln(1+\Lambda^2)-\frac{\Lambda^2}{1+\Lambda^2}\right]v\,dv\Big/\int f_0 v\,dv\ , \qquad (B2)$$

with $\Lambda=\lambda_D/\lambda_q$. The velocity integration can be written in terms of the exponential integral E_1 for f_0 equal to a Maxwellian. In the limit of $\Lambda\gg 1$, Eq. (36) is obtained.

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