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Generation of squeezed radiation from a free-electron laser

I. Gjaja and A. Bhattacharjee
Department of Applied Physics, Columbia University, New York, New York 10027
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We demonstrate that squeezed states of the radiation field in a free-electron laser are possible only in the small gain and inverse regimes. Moreover, in these regimes, the feasibility of generating squeezed radiation is severely constrained by the quantum fluctuations in the initial states of the electron beam.

In squeezed states of the radiation field, the quantum fluctuations in one Hermitian component are reduced below its vacuum fluctuation level at the expense of the enhanced fluctuations in the other conjugate component. Squeezed states have been realized in the laboratory by using optical materials which exhibit parametric gain.[1-3] Since the optical nonlinearities suitable for the generation of squeezed states are generally weak,[4] it is worth examining whether high-intensity squeezed radiation can be produced using alternate methods. One such approach has been proposed using free-electron lasers (FEL's).[5,6] The existing studies, however, have investigated the squeezing of combined radiation field-electron operators, so that it is not clear what their implications are for the squeezing of radiation-field operators alone. In this paper, we give a novel treatment of radiation squeezing in FEL's, and demonstrate that squeezing can occur only in the small gain and inverse FEL regimes. Furthermore, we show that in these regimes, squeezing is limited by the ability to prepare the initial electron beam in a required state.

We define squeezing as follows:[7,8] let a and $a^\dagger$ be the creation and annihilation operators for a mode of the radiation field, $[a,a^\dagger]=1$. We represent the operator a in terms of Hermitian operators A_1 and A_2 as $ae^{i\theta}=A_1+iA_2$; θ is the phase of the radiation field with respect to the measuring apparatus, as, for example, with respect to the local oscillator in a homodyne detector. Then $[A_1,A_2]=\frac{1}{2}i$; the associated uncertainty relation is $(\Delta A_1)^2(\Delta A_2)^2\geq\frac{1}{16}$, where $(\Delta A)^2\equiv\langle A^2\rangle-\langle A\rangle^2$ for any Hermitian operator A and $\langle\,\rangle$ represents the expectation value. For a coherent state of the field (including vacuum) $(\Delta A_1)^2=(\Delta A_2)^2=\frac{1}{4}$, independent of θ, whereas, for a squeezed state there exists a phase θ for which ($j=1$ or 2)

$$(\Delta A_j)^2<\tfrac{1}{4}\ . \tag{1}$$

In order to investigate squeezing in FEL's, we use a Hamiltonian in the laboratory frame obtained from the Dirac Hamiltonian.[9] In the conventional Bambini-Renieri analysis,[10] it is assumed that the electron dynamics is nonrelativistic in the beam frame. We have shown[9] that such an assumption leads to the omission of certain terms in the FEL equations. In the present paper, we therefore use the correct Hamiltonian in the laboratory frame. We emphasize, however, that the qualitative conclusions of this paper would remain unaltered even if we used the Bambini-Renieri Hamiltonian.

We work in the Heisenberg picture and examine if, starting from a radiation field which does not obey inequality (1), an FEL can generate radiation in a squeezed state. The Hamiltonian for the linearized one-dimensional, one-mode FEL problem is given by[9]

$$H=\alpha y^\dagger y-Da'^\dagger a'+\beta(ia'y^\dagger-ia'^\dagger y+a'x^\dagger+a'^\dagger x)\ . \tag{2}$$

x and y are non-Hermitian operators for the electron collective variables,

$$x=\left[\frac{p_0^2+M^2}{Np_0(k+k_w)}\right]^{1/2}\sum_j\delta\phi_j e^{-i\phi_{0j}}\ , \tag{3a}$$

$$y=\left[\frac{p_0}{N(p_0^2+M^2)(k+k_w)}\right]^{1/2}\sum_j\delta p_j e^{-i\phi_{0j}}\ , \tag{3b}$$

where $\delta\phi_j=\phi_j-\phi_{0j}$, $\delta p_j=p_j-p_0$, and ϕ_{0j} and p_0 are the equilibrium expectation values of the electron momentum and phase relative to the radiation field, $\phi_j=(k+k_w)z_j-\omega t-Dt$. p_0 is shown to be the expectation value of the average initial-electron momentum, whereas ϕ_{0j} satisfies $\sum_j e^{in\phi_{0j}}=0$, $n=\pm1,\pm2,\ldots$ (Refs. 11 and 12). Other quantities are defined as follows: k, k_w, $\omega=k$, $\omega_w=k_w$ are the wave numbers and frequencies of the radiation and wiggler field, respectively, N is the total number of electrons, D is the detuning parameter

$$D=(k+k_w)\{\langle p(0)\rangle/[\langle p(0)\rangle^2+M^2]^{1/2}\}-\omega\ ,$$

$$a'=ae^{iDt}\ ,$$

α and β are constants defined as

$$\alpha=M^2(\omega+D)/p_0^2\ ,$$

$$\beta=e^2A_w(p_0^2+M^2)^{-1}[(k+k_w)p_0N\pi/(V\omega)]^{1/2}\ ,$$

V is the quantization volume, M^2 is the effective mass $M^2=m^2+e^2A_w^2$, and A_w is the strength of the vector potential of the wiggler field. For compactness in notation, we have set $\hbar=c=1$. The equations of motion for x, y, and a' are obtained using the commutation relations $[x,y^\dagger]=[x^\dagger,y]=i$, $[a',a'^\dagger]=1$, and $[x,y]=[x,a']=[y,a']=0$.

In order to study squeezing of radiation, it is convenient to introduce new operators $b_1=2^{-1/2}(x+iy)$ and $b_2=2^{-1/2}(x^\dagger+iy^\dagger)$.[13] Then $[b_i,b_j^\dagger]=\delta_{ij}$, $[b_i,b_j]=0$, and $[b_i,a'^\dagger]=[b_i,a']=0$ for $i,j=1,2$. The Hamiltonian for the linearized problem now becomes

$$H=\frac{\alpha}{2}(b_1^\dagger b_1+b_2^\dagger b_2-b_1^\dagger b_2^\dagger-b_1 b_2)$$
$$-Da'^\dagger a'+\sqrt{2}\beta(a'b_2+a'^\dagger b_2^\dagger)\ , \qquad (4)$$

where the equations of motion

$$\dot{b}_1=-\frac{i\alpha}{2}(b_1-b_2^\dagger)\ , \qquad (5a)$$

$$\dot{b}_2^\dagger=\frac{i\alpha}{2}(b_2^\dagger-b_1)+i\sqrt{2}\beta a'\ , \qquad (5b)$$

$$\dot{a}'=iDa'-i\sqrt{2}\beta b_2^\dagger\ . \qquad (5c)$$

Here the dot indicates derivatives with respect to time. It is easy to verify that these equations are equivalent to the ones for $\dot{x}$, $\dot{y}$, and $\dot{a}'$ that result from the Hamiltonian given by Eq. (2). From Eqs. (5), we obtain the solution for $a'(t)$, from which it follows that

$$a(t)=G_1'(t)b_1(0)+G_2'(t)b_2^\dagger(0)+F_3'(t)a(0)\ .$$

G_1', G_2', and F_3' are functions of time determined from the linearized equations.[13] We note that $a(t)$ does not depend on $b_1^\dagger(0)$, $b_2(0)$, or $a^\dagger(0)$; this is a consequence of the form of the Hamiltonian.

Since the oscillators b_1, b_2, and a are independent at $t=0$, the variances of A_1 and A_2 are computed to be

$$[\Delta A_{1,2}(t)]^2=[\Delta\bar{B}_{1,2}(t)]^2+[\Delta\bar{C}_{1,2}(t)]^2\pm\tfrac{1}{4}\{\langle[F_3(t)a(0)\pm\text{H.c.}]^2\rangle-\langle F_3(t)a(0)\pm\text{H.c.}\rangle^2\}\ , \qquad (6)$$

where $\bar{B}_{1,2}$ and $\bar{C}_{1,2}$ are Hermitian operators explicitly dependent on time, $\bar{B}_1(t)=[G_1(t)b_1(0)+\text{H.c.}]/2$, $\bar{B}_2(t)=[G_1(t)b_1(0)-\text{H.c.}]/(2i)$, $\bar{C}_1(t)=[G_2(t)b_2^\dagger(0)+\text{H.c.}]/2$, $\bar{C}_2(t)=[G_2(t)b_2^\dagger(0)-\text{H.c.}]/(2i)$. The unprimed quantities $G_1(t)$, $G_2(t)$, and $F_3(t)$ are $G_1'(t)$, $G_2'(t)$, and $F_3'(t)$, multiplied, respectively, by $e^{i\theta}$. We now assume that the radiation field is initially in a coherent state, i.e., in an eigenstate of the annihilation operator a. This is what is realized in FEL experiments which start from noise (initial state of the radiation field is the vacuum), or from a given laser field. Then

$$[\Delta A_{1,2}(t)]^2=[\Delta\bar{B}_{1,2}(t)]^2+[\Delta\bar{C}_{1,2}(t)]^2+\tfrac{1}{4}\,|F_3(t)|^2\ . \qquad (7)$$

It is easy to show that the last term on the right-hand side of Eq. (6) cannot be made any smaller if the initial state of the radiation field is taken to be the number state, or to have a thermal distribution with the density operator[14]

$$\rho=\frac{1}{1+\langle n\rangle}\sum_j[\langle n\rangle/(1+\langle n\rangle)]^j|j\rangle\langle j|\ .$$

For the number state, the last term is multiplied by $2n+1$, whereas for the thermal distribution it is multiplied by $2\langle n\rangle+1$. We also remark that $[\Delta A_{1,2}(t)]^2$ given by Eq. (7) is manifestly positive definite (as it must be), and that $[\Delta A_{1,2}(0)]^2=\frac{1}{4}$, in agreement with the assumption that the radiation field is initially in a coherent state. Furthermore, Eq. (7) is valid for any initial state of the electrons.

We now examine the behavior of $|F_3(t)|^2$ as a function of parameters specifying the system, but prior to doing this we redefine some constants and determine the regimes of FEL operation. Since it is conventional to use ρ (Pierce's parameter) and δ (detuning parameter) as the relevant parameters of the FEL,[11] we expand α,β,D to order M^2/p_0^2 to get $(\beta/\alpha)^{2/3}=\rho$ and $D/(\alpha\rho)=\delta$. This also necessitates the change of t into a dimensionless variable $\tau=t(\alpha\beta^2)^{1/3}$. We use the instability condition $8\rho^3-\rho^2\delta^2+9\rho\delta-\delta^3+27/4\geq 0$,[12] and set $\rho=1\times 10^{-2}$, which is a typical experimental value of Pierce's parameter. We distinguish three regimes of FEL operation: $\delta<0$ (inverse FEL), $0<\delta<1.9$ (exponential gain regime), and $\delta>1.9$ (small gain regime). From the instability condition it follows that for $\delta<1.9$ the solutions are unstable.

From numerical studies of $F_3(\tau)$ (Fig. 1) we determine that in the exponential gain regime $|F_3(\tau)|^2$ is an exponentially increasing function of τ which has no minima at or below 1 (except at $t=0$). Hence, regardless of the initial state of the electrons, an FEL operating in the exponential gain regime cannot produce radiation in a squeezed state. For $\delta<0$, or $\delta>1.9$, on the other hand, $|F_3(\tau)|^2$ does have minima below 1. The most pronounced one, $|F_3(\tau)|^2\simeq 1\times 10^{-3}$, for example, occurs at $\tau=6.36$, $\delta=-4.6$. (The numerical simulations are done for $\tau\leq 10.0$ and $-5<\delta<5$.) Thus, in these regimes the squeezing of radiation depends on the initial state of the electron beam [first two terms in Eq. (7)], to which we now turn. For simplicity, from now on, we limit our con-

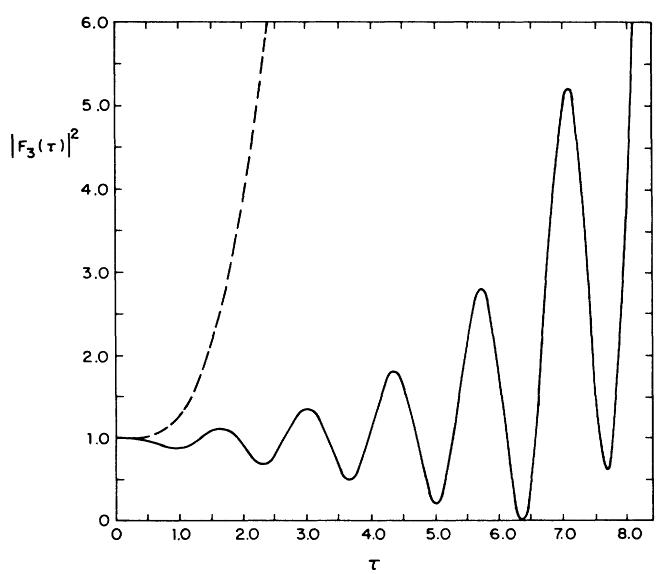

FIG. 1. $|F_3(\tau)|^2$ vs τ for $\delta=-4.6$ (solid line) and $\delta=1.8$ (dashed line). $\rho=1.0\times 10^{-2}$.

siderations to squeezing of the Hermitian component A_1. The analysis for the operator A_2 is identical, with θ replaced by $\theta-\pi/2$.

Formally, the most favorable initial electron state can be derived as follows: let $|F_3(\tau)|^2$ have a minimum at $\tau=\tau_0$. Then, if the initial electron state is an eignstate of operators $\bar{B}_1(\tau_0)$ and $\bar{C}_1(\tau_0)$, the variances of $\bar{B}_1$ and $\bar{C}_1$ evaluated at $\tau=\tau_0$ are zero, and $[\Delta A_1(\tau_0)]^2 = \frac{1}{4}|F_3(\tau_0)|^2<1$. For convenience, we introduce the following Hermitian operators:

$$u_{1,2}=[G_{1,2}(\tau_0)x(0)+G^*_{1,2}(\tau_0)x^\dagger(0)] ,$$

and

$$v_{1,2}=\pm i[G_{2,1}(\tau_0)y(0)-G^*_{2,1}(\tau_0)y^\dagger(0)] ,$$

which satisfy the commutation relations $[u_1,u_2]=[v_1,v_2]=0$, and $[u_i,v_j]=i\delta_{ij}\,\mathrm{Im}\Lambda(i,j=1,2)$, where $\Lambda=2G_1(\tau_0)G_2^*(\tau_0)$ (independent of θ). In terms of these operators, in the $\{v_1,v_2\}$ representation, the eigenvalue equations for $\bar{B}_1(\tau_0)$ and $\bar{C}_1(\tau_0)$ take the form

$$\left[i\,\mathrm{Im}\Lambda\frac{\partial}{\partial v_1}-v_2\right]\phi=\bar{b}_1\phi , \tag{8}$$

$$\left[i\,\mathrm{Im}\Lambda\frac{\partial}{\partial v_2}-v_1\right]\phi=\bar{c}_1\phi , \tag{9}$$

where $\bar{b}_1$ and $\bar{c}_1$ are the eigenvalues of $\bar{B}_1(\tau^0)$ and $\bar{C}_1(\tau_0)$, respectively, multiplied by $2\sqrt{2}$. From Eqs. (8) and (9) we get the eigenfunction

$$\phi=(2\pi)^{-1}\exp[-i(\mathrm{Im}\Lambda)^{-1}(\bar{b}_1+v_2)(\bar{c}_1+v_1)] . \tag{10}$$

It is easy to show that ϕ allows for the required normalizations, $\langle\bar{b}_1',\bar{c}_1'|\bar{b}_1,\bar{c}_1\rangle=\delta(\bar{b}_1'-\bar{b}_1)\delta(\bar{c}_1'-\bar{c}_1)$ and $\langle v_1',v_2'|v_1,v_2\rangle=\delta(v_1'-v_1)\delta(v_2'-v_2)$. The solution to Eqs. (8) and (9) can also be obtained by a unitary transformation $U=\exp(iu_1u_2/\mathrm{Im}\Lambda)$ from the eigenstate of operators v_1 and v_2 with eigenvalues $-\bar{c}_1$ and $-\bar{b}_1$, respectively, or by a unitary transformation $V=\exp(-iv_1v_2/\mathrm{Im}\Lambda)$ from the eigenstate of operators u_1 and u_2 with eigenvalues $\bar{b}_1$ and $\bar{c}_1$, respectively.

To compute ϕ in terms of single-electron momenta, we write $\bar{B}_1(\tau_0)$ and $\bar{C}_1(\tau_0)$ as sums over single-electron variables. Then the equivalents of Eqs. (8) and (9) are

$$f\left[\bar{c}_1-K\sum_j\delta p_j\mathrm{Im}\tau_j\right]=iK'\sum_j\mathrm{Re}\tau_j\left[\partial f/\partial p_j\right]$$

and

$$f\left[\bar{b}_1+K\sum_j\delta p_j\mathrm{Im}\mu_j\right]=iK'\sum_j\mathrm{Re}\mu_j(\partial f/\partial p_j) ,$$

with the solution

$$\phi=(2\pi)^{-1}\exp\left[-i(\mathrm{Im}\Lambda)^{-1}\left[\bar{b}_1+K\sum_j\delta p_j\mathrm{Im}\mu_j\right]\times\left[\bar{c}_1-K\sum_j\delta p_j\mathrm{Im}\tau_j\right]\right] .$$

Here

$$K=2\{p_0/[(p_0^2+M^2)N(k+k_w)]\}^{1/2} ,$$

$$K'=2[(p_0^2+M^2)(k+k_w)/(p_0N)]^{1/2} ,$$

$$\mu_j=e^{-i\phi_{0j}}G_1(\tau_0),\quad \tau_j=e^{-i\phi_{0j}}G_2(\tau_0) .$$

This expression for ϕ gives the normalization

$$\langle v_1,v_2|\delta p_1\ldots\delta p_N\rangle=\delta\left[v_1+K\sum_j\delta p_j\mathrm{Im}\tau_j\right]\times\delta\left[v_2-K\sum_j\delta p_j\mathrm{Im}\mu_j\right] .$$

We find, however, two objections to the physical realizability of state ϕ. First, even though a momentum eigenstate can be approached experimentally, a Hamiltonian whose interaction part in the interaction picture would give the unitary transformation U would have to contain terms proportional to $\delta p_j\delta p_{j'}\tau^2$. This holds irrespective of the value of θ, as is manifest from the fact that the form of Eq. (10) is independent of the phase of G_1 or G_2. It is not clear that a Hamiltonian which gives this transformation can be realized experimentally. Second, the solution ϕ cannot be written as a product of single electron wave functions, which seems to be a reasonable requirement for all electron beams produced by accelerators prior to FEL interaction. In those beams the overlap of electron wave packets and the interaction between electrons can be treated as small perturbations. Hence in view of the difficulties of physical interpretation and possible experimental realization of the eigenstates of $\bar{B}_1(\tau_0)$ and $\bar{C}_1(\tau_0)$, we turn to initial states which have natural physical interpretations, and we investigate whether they can produce squeezing in the allowed regimes.

We begin by writing $(\Delta\bar{B}_1)^2+(\Delta\bar{C}_1)^2$ in terms of single electron variables δz_j and δp_j $[\delta z_j=\delta\phi_j(0)/(k+k_w)]$. This can be done by expressing either u_1,u_2,v_1,v_2, or $b_1,b_1^\dagger,b_2,b_2^\dagger$ as appropriate sums over δz_j and δp_j. The result is

$$\begin{aligned}(\Delta\bar{B}_1)^2+(\Delta\bar{C}_1)^2=\tfrac{1}{8}\sum_{j,j'}[&K'^2(\mathrm{Re}\mu_j\mathrm{Re}\mu_{j'}+\mathrm{Re}\tau_j\mathrm{Re}\tau_{j'})(\langle z_jz_{j'}\rangle-\langle z_j\rangle\langle z_{j'}\rangle)\\&+K^2(\mathrm{Im}\mu_j\mathrm{Im}\mu_{j'}+\mathrm{Im}\tau_j\mathrm{Im}\tau_{j'})(\langle p_jp_{j'}\rangle-\langle p_j\rangle\langle p_{j'}\rangle)\\&-KK'(\mathrm{Re}\mu_j\mathrm{Im}\mu_{j'}-\mathrm{Re}\tau_j\mathrm{Im}\tau_{j'})(\langle z_jp_{j'}+p_{j'}z_j\rangle-2\langle z_j\rangle\langle p_{j'}\rangle)] .\end{aligned} \tag{11}$$

We now introduce assumption 1: The electrons are initially statistically independent, i.e., the initial state is a product of single electron states. Using the definitions $z_j'=z_j-\langle z_j\rangle$ and $p_j'=p_j-\langle p_j\rangle$, and assumption 1, the expression above yields

$$\begin{aligned}(\Delta\bar{B}_1)^2+(\Delta\bar{C}_1)^2=\tfrac{1}{8}\sum_j\{&\langle K'^2[(\mathrm{Re}\mu_j)^2+(\mathrm{Re}\tau_j)^2]z_j'^2\\&+K^2[(\mathrm{Im}\mu_j)^2+(\mathrm{Im}\tau_j)^2]p_j'^2\rangle+KK'(\mathrm{Re}\tau_j\mathrm{Im}\tau_j-\mathrm{Re}\mu_j\mathrm{Im}\mu_j)\langle z_j'p_j'+p_j'z_j'\rangle\}\end{aligned} \tag{12}$$

In order to proceed further we introduce assumption 2: the wave packets of all electrons have the same width in position and also in momentum. This assumption can be less restrictive: all electrons can be divided into groups consisting of a large number of particles, and all particles within the same group can be described by wave packets with the same width in position and momentum. We note that it is not important that the shapes of the wave packets be the same for all electrons, nor is it necessary to consider the distribution of the centers of wave packets in position and momentum. Since in most experimental manipulations of electron beams one deals with large aggregates of particles, rather than individual particles, we expect the less restrictive form of the assumption above to hold for the preparation of almost all electron beams used in FEL experiments. Therefore, we use assumption 2 to simplify Eq. (12). Adhering to the linearization procedure $\sum_j e^{in\phi_{0j}}=0$, $n=\pm 1, \pm 2, \ldots$ we obtain for the first two terms $\frac{1}{16}N(|G_1|^2+|G_2|^2)\langle K'^2 z'^2+K^2 p'^2\rangle$ (no index on z' or p'). But the quantity in angular brackets is exactly the expectation value of the Hamiltonian for a harmonic oscillator with angular frequency $2KK'$. It is therefore subject to a lower limit of KK'. To consider the third term we note that in the absence of momentum-dependent potentials, all particles satisfy

$$\frac{d}{dt}(\langle z_j^2\rangle-\langle z_j\rangle^2)=\frac{1}{m}(\langle z_j p_j+p_j z_j\rangle-\langle z_j\rangle\langle p_j\rangle)$$

$$=\frac{1}{m}\langle z_j' p_j'+p_j' z_j'\rangle .$$

Hence, the third term of Eq. (12) is $-KK'm(d/dt)\langle z'^2\rangle \times \sum_j \mathrm{Re}\mu_j \mathrm{Im}\mu_j$, which is zero. Therefore, under assumption 2,

$$(\Delta\bar{B}_1)^2+(\Delta\bar{C}_1)^2 \geq \frac{1}{16}(|G_1|^2+|G_2|^2)NKK' ,$$

or, using the definitions of K and K',

$$(\Delta\bar{B}_1)^2+(\Delta\bar{C}_1)^2 \geq \frac{1}{4}(|G_1|^2+|G_2|^2) . \quad (13)$$

There is another way to obtain inequality (13) starting from Eq. (12). Our linearization procedure is valid for small departures from equilibrium which corresponds to a uniform distribution of particles in phase, i.e., $\sum_j e^{in\phi_{0j}}=0$. Then, in the domain of validity of our theory, the terms in the sum of Eq. (12) which are multiplied by a factor of $e^{\pm 2i\phi_{0j}}$ contribute much less than terms without such a factor, and can therefore be neglected. The remaining terms give

$$(\Delta\bar{B}_1)^2+(\Delta\bar{C}_1)^2 \simeq \frac{1}{16}(|G_1|^2+|G_2|^2) \times \sum_j \langle K'^2 z_j'^2+K^2 p_j'^2\rangle ,$$

which has the same lower limit as that given by inequality (13).

Using Eq. (7) and inequality (13) we now have for the variance of A_1,

$$[\Delta A_1(t)]^2 \geq \frac{1}{4}[|G_1(t)|^2+|G_2(t)|^2+|F_3(t)|^2] .$$

We note that the functions G_1, G_2, and F_3 are not independent. From the requirement that $[a(t),a^\dagger(t)]=1$ at all times, we get $|G_1(t)|^2-|G_2(t)|^2+|F_3(t)|^2=1$. We thus obtain the lower limit on the variance of A_1,

$$[\Delta A_1(t)]^2 \geq \frac{1}{4}+\frac{1}{2}|G_2(t)|^2 . \quad (14)$$

Since the inequality is independent of θ, an identical result holds for $[\Delta A_2(t)]^2$. Therefore, to the extent of validity of linear theory and assumptions 1 and 2, a free-electron laser cannot produce radiation in a squeezed state.

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