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Nonresonant interaction of a three-level atom with cavity fields. II. Coherent properties of the stimulated fields

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We study the interaction of a three-level atom with cavity fields of arbitrary detunings. We address ourselves in this paper to the question of how the time evolution of a coherent state depends upon the detunings. A number of interesting new phenomena contrary to the existing picture are found and discussed.

I. INTRODUCTION

In terms of a sequence of correlation functions for the field vectors, Glauber[1] first introduced the concept of coherence of the electromagnetic field. A coherent state is one in which the correlation function for all orders n can be expressed as a product of field values at $2n$ different space-time points. It is now well known that the electromagnetic field in a coherent state possesses a number of interesting properties.[2] For example, field components satisfy at all times the minimum uncertainty relations, and the photon number obeys Poisson distribution. The laser field may be regarded as one of the real systems that is almost in a pure coherent state.[3] As a consequence, a laser has very different physical properties from light coming out of ordinary sources. It is therefore of essential importance to discuss the coherence of electromagnetic fields in any serious investigation of interactions between radiation field and matter.

The coherent properties of stimulated radiation from a single two-level atom in a resonant cavity has been discussed by Cummings[4] who explored the first-order correlation function of the field. Meystre *et al.*[5] investigated the coherence properties of the resonance interactions of a single two-level atom with multimode cavity fields. They discussed the time variation of field coherence from characteristics of the photon-number distribution, and found that the radiation field, initially in a coherent state, gradually loses its coherence during its interaction with the atom as time increases. Thus, the atom acts like a nonlinear filter which screens out the coherence of the field. The conclusions in Refs. 4 and 5 are consistent.

The study of interactions of a three-level atom with two-mode cavity fields has recently been carried out by Li and Gong.[6] They first obtain the first-order correlation function of the electric field and phonon-number probability distribution which are then employed to investigate the coherent properties of the stimulated emission under various initial conditions. They conclude that double stimulation will cause the field of one mode to approach its initial coherent state. This effect is stronger by coherent stimulation than by thermal stimulation of the other mode.

In all the papers mentioned above, however, the discussions have been limited to the resonance case. In a previous paper which is the first of this series, Li, Gong, and Lin[7] have formulated the problem of nonresonant interaction of a single three-level atom with one- or two-mode cavity fields of arbitrary detunings. A number of interesting novel phenomena, especially in cases involving two-photon processes, have been revealed. In the present paper, we shall concentrate our attention on the coherence properties of the stimulated radiation field in the nonresonant interaction of cavity fields with the atom.

We calculate the first-order equal-time correlation function[4,5] $\langle E^- E^+ \rangle(t)$, and compare it with the product of mean values of the field $\langle E^- \rangle(t)\langle E^+ \rangle(t)$ as functions of time. Whenever the two curves coincide, we say that the electric field is in a coherent state, at least to the first order. It is found that the nonresonant results depend strongly upon the detunings. Under certain conditions, the following dramatic variations of the coherent state are discovered. The stimulated radiation field initially in a pure coherent state gradually evolves away from coherence as expected, but turns back to coherence at a later time. Such recurrence can occur periodically in some particular cases. Therefore, the conclusions of Refs. 4 and 5, in our opinion, are no longer true in general. They are not valid, at least for the case of a three-level atom interacting with the radiation field.

II. THEORY

The general formalism of a three-level atom interacting with cavity fields is given in I. Here we merely out-line what is essential for our present discussion of the coherence of radiation fields. We consider the two typical cases: one-mode Ξ type and two-mode Λ type. The relevant atomic level configurations are shown in Fig. 1.

The Hamiltonian is, in the interaction picture,

$$H=\hbar(H_0+H_1)\ , \tag{1}$$

where, for one-mode Ξ type,

$$H_0=\sum_{\eta=a,b,c}\omega_\eta A_\eta^\dagger A_\eta+\Omega a^\dagger a\ , \tag{2}$$

$$H_1=\lambda_1 e^{i\Delta_1 t}aA_b^\dagger A_a+\lambda_2 e^{-i\Delta_2 t}aA_a^\dagger A_c+\text{H.c.}\ , \tag{3}$$

$$\Delta_1=-(\Omega-\omega_b+\omega_a),\ \Delta_2=\Omega-\omega_a+\omega_c\ ;$$

and for two-mode Λ type,

$$H_0=\sum_{\eta=a,b,c}\omega_\eta A_\eta^\dagger A_\eta+\sum_{i=1,2}\Omega_i a_i^\dagger a_i\ , \tag{4}$$

$$H_1=\lambda_1 e^{-i\Delta_1 t}a_1A_a^\dagger A_b+\lambda_2 e^{-i\Delta_2 t}a_2A_a^\dagger A_c+\text{H.c.}\ , \tag{5}$$

$$\Delta_1=\Omega_1-\omega_a+\omega_b,\ \Delta_2=\Omega_2-\omega_a+\omega_c\ .$$

The operators in the Hamiltonian are defined as follows. $A_\eta^\dagger$ creates an atom in the state $|\eta\rangle$, $a^\dagger$ creates a photon, λ_i are the usual coupling constants, and Δ_i are the detuning parameters.

As has been shown in I, the Schrödinger equation can be solved by the state vector for one mode,

$$|\psi(t)\rangle=\sum_n Q(n)[A(n_a,t)|a,n_a\rangle+B(n_b,t)|b,n_b\rangle+C(n_c,t)|c,n_c\rangle]\ , \tag{6a}$$

or for two modes,

$$|\psi(t)\rangle=\sum_{n_1,n_2}Q_1(n_1)Q_2(n_2)\times[A(n_{1a},n_{2a},t)|a,n_{1a},n_{2a}\rangle+B(n_{1b},n_{2b},t)|b_{n_{1b}},n_{2b}\rangle+C(n_{1c},n_{2c}t)|c,n_{1c},n_{2c}\rangle]\ , \tag{6b}$$

with the corresponding initial conditions for one mode,

$$|\psi(0)\rangle=|\eta,\xi\rangle=|\eta\rangle\sum_n Q(n)|n\rangle\ , \tag{7a}$$

and for two modes,

$$|\psi(0)\rangle=|\eta,\xi_1,\xi_2\rangle=|\eta\rangle\sum_{n_1,n_2}Q_1(n_1)Q_2(n_2)|n_1,n_2\rangle\ , \tag{7b}$$

where n_η is the photon number when the atom is in the level η, and $n_{i\eta}$ is the photon number referring to the mode i. The probability amplitudes in (6) are

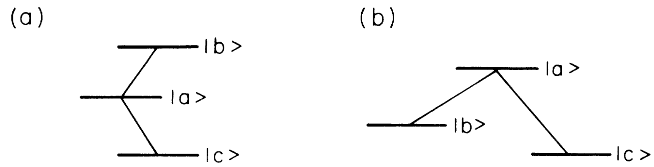

FIG. 1. Atomic energy level configuration for (a) Ξ type, (b) Λ type.

$$A=-e^{i\Delta_2 t}\sum_{i=1}^{3}U_i\mu_i e^{i\mu_i t}\ , \tag{8a}$$

$$B=\frac{1}{V_1}e^{i(\Delta_1-\Delta_2)t}\sum_{i=1}^{3}U_i(\mu_i^2-\Delta_2\mu_i-V_2^2)e^{i\mu_i t}\ , \tag{8b}$$

$$C=V_2\sum_{i=1}U_i e^{i\mu_i t}\ , \tag{8c}$$

where

$$\mu_1=-\tfrac{1}{3}x_1+\tfrac{2}{3}(x_1^2-3x_2)^{1/2}\cos\theta\ , \tag{9a}$$

$$\mu_2=-\tfrac{1}{3}x_1+\tfrac{2}{3}(x_1^2-3x_2)^{1/2}\cos(\theta+\tfrac{2}{3}\pi)\ , \tag{9b}$$

$$\mu_3=-\tfrac{1}{3}x_1+\tfrac{2}{3}(x_1^2-3x_2)^{1/2}\cos(\theta+\tfrac{4}{3}\pi)\ , \tag{9c}$$

$$\theta=\tfrac{1}{3}\cos^{-1}\left[\frac{9x_1x_2-2x_1^3-27x_3}{2(x_1^2-3x_2)^{3/2}}\right]\ , \tag{9d}$$

and

$$x_1=\Delta_1-2\Delta_2\ , \tag{10a}$$

$$x_2=-[V_1^2+V_2^2+\Delta_2(\Delta_1-\Delta_2)]\ , \tag{10b}$$

$$x_3=(\Delta_2-\Delta_1)V_2^2\ . \tag{10c}$$

The probability amplitudes for one- and two-mode cases take the same expressions (8) and depend on the photon number in different modes through the coupling strength parameters V_1 and V_2. The explicit forms of these parameters are listed in Table I of I for various cases.

The atomic level occupation probabilities can be found directly from (6) and (8). They are, for one mode,

$$P_a(t)=\sum_n P(n)|A(n_a,t)|^2\ , \tag{11a}$$

for two modes,

$$P_a(t)=\sum_{n_1,n_2}P(n_1,n_2)|A(n_{1a},n_{2a},t)|^2\ ; \tag{11b}$$

for one mode,

$$P_c(t)=\sum_n P(n)|C(n_c,t)|^2\ , \tag{12a}$$

for two modes,

$$P_c(t)=\sum_{n_1,n_2}P(n_1,n_2)|C(n_{1c},n_{2c},t)|^2\ ; \tag{12b}$$

where

$$P(n) = |Q(n)|^2$$

and

$$P(n_1, n_2) = |Q_1(n_1)|^2 |Q_2(n_2)|^2$$

are the initial photon distributions for one- and two-mode cases, respectively.

III. COHERENCE OF STIMULATED FIELDS

As is well known, the electric field operator E can always be separated into positive and negative frequency parts. The positive frequency part E^+ has the property of annihilation operator and the negative frequency part E^- has the property of the creation operator. The two parts are Hermitian conjugate to each other. We shall investigate the time evolution of the quantities $\langle E^- E^+ \rangle$ and $\langle E^- \rangle \langle E^+ \rangle$ for two typical cases in the following.

A. One-mode Ξ type

We first define

$$E = E^+ + E^- = \epsilon(a + a^\dagger) , \tag{13}$$

where $E^+ = \epsilon a$, $E^- = \epsilon a^\dagger$, and ϵ is a c number with the dimension of the electric field. In the interaction picture, the first-order correlation function is given by

$$\begin{aligned} \langle E^- E^+ \rangle(t) &= \mathrm{tr}[\rho E^- E^+] , \\ &= \epsilon^2 \mathrm{tr}[\rho a^\dagger a] , \\ &= \epsilon^2[\bar{n} + P_a(t) + 2P_c(t)] , \end{aligned} \tag{14}$$

where the density matrix

$$\rho(t) = |\psi(t)\rangle\langle\psi(t)| , \tag{15}$$

and we have assumed that initially ($t = 0$) the atom is in the state $|b\rangle$ while the photon is in the coherent state

$$P(n) = e^{-\bar{n}} \bar{n}^n / n! \tag{16}$$

The mean value of the negative frequency part is

$$\langle E^-(t) \rangle = \mathrm{tr}[\rho E^-(t)] = \epsilon e^{i\Omega t} \alpha^* S(t) , \tag{17}$$

where α is the complex amplitude of the initial coherent field, and the function $S(t)$ is defined by

$$S(t) = \sum_{n=0}^{\infty} \left[B(n,t) B^*(n+1,t) + \left[\frac{n+2}{n+1} \right]^{1/2} A(n+1,t) A^*(n+2,t) + \left[\frac{n+3}{n+1} \right]^{1/2} C(n+2,t) C^*(n+3,t) \right] P(n) . \tag{18a}$$

The positive frequency part is simply the Hermitian conjugate of (17). Thus,

$$\langle E^+ \rangle(t) = \epsilon e^{-i\Omega t} \alpha S^*(t) . \tag{18b}$$

Combining these expressions we find directly

$$\langle E^- \rangle(t) \langle E^+ \rangle(t) = \epsilon^2 \bar{n} |S|^2(t) . \tag{19}$$

Equations (14) and (19) are calculated numerically as functions of t for different $\bar{n}$ and different detunings. For the initial mean photon number $\bar{n} = 10$ and $\lambda_1 = \lambda_2 = \lambda$, the results calculated for different detunings are plotted in Figs. 2–4. The dashed lines represent $\langle E^- E^+ \rangle(t)/\epsilon^2$ and the solid lines represent $\langle E^- \rangle(t) \langle E^+ \rangle(t)/\epsilon^2$. Whenever the two curves coincide, the field is in the coherent state. It is observed from these figures that the time evolution of the state of the field can be drastically different when the detuning parameters change. We

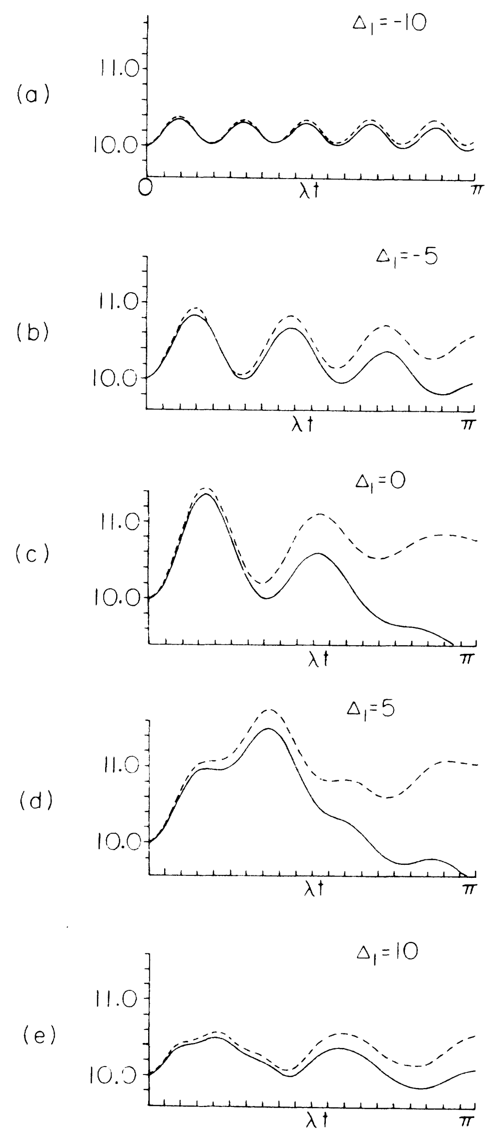

FIG. 2. Time evolution of coherence state of the field. The dashed line represents $\langle E^- E^+ \rangle/\epsilon^2$, and the solid line represents $\langle E^- \rangle\langle E^+ \rangle/\epsilon^2$. The initial mean photon number $\bar{n} = 10$ and $\Delta_2 = 5$.

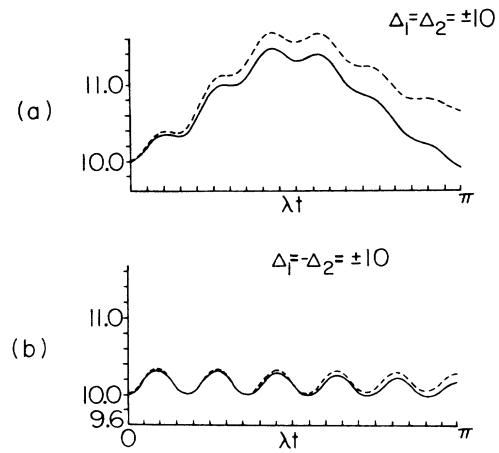

FIG. 3. Same as Fig. 2 except $\Delta_2 = \pm 10$.

note in general that the farther away the two detuning parameters are from the resonance conditions, the more coherent the state of fields becomes. Figure 4(a) shows the worst situation in which $\Delta_1 = \Delta_2 = 0$, when both the one-photon and two-photon resonance conditions are satisfied at the same time. This is because the coupling between the atom and cavity fields become strongest at the resonances. Thus, both the one-photon and two-photon processes are most active and the interactions quickly alter the statistical properties of the fields. Consequently, the field moves away from its initial coherent state almost immediately as t increases. Similarly, we see from Figs. 2(d) and 3(a) that the coherence is as bad since the condition of two-photon resonance is still satisfied in these two cases even though they are away from the one-photon resonance. When Δ_1 or Δ_2 becomes nonzero and they are not equal, the resonance conditions are no longer satisfied, and the atomic interaction with the fields weakens. Hence the coherence situation improves as can be seen by comparing Figs. 2(c) or 4(b) with 4(a). The field in these cases can remain approximately in the initial coherent state for a short while without changing its statistical property significantly.

As the difference between the two detuning parameters increases further, the interaction is far from the one-photon and two-photon resonance. As a consequence, the radiation field can remain in the initial coherent state for a longer time. This is clearly illustrated by comparing the following pairs of figures: 2(a) and 2(e), 2(b) and 2(d), 3(a) and 3(b).

To see how the two detunings influence the field coherence separately, we reduce greatly the intensity of the stimulated field by choosing the initial mean photon number $\bar{n} = 1$. The results are plotted in Fig. 5. Evidently, the effect of Δ_1 on the coherence is much more remarkable than that of Δ_2. This is because the one-photon transition $|b\rangle$-$|a\rangle$ depends only on Δ_1, but the one-photon transition $|a\rangle$-$|c\rangle$ and two-photon transition $|b\rangle$-$|c\rangle$ depend on both Δ_1 and Δ_2.

Finally, we come to the most interesting phenomenon, the recurrence of coherence state of the radiation field in the presence of an atom. In Figs. 2(a), 2(b), 3(b), and

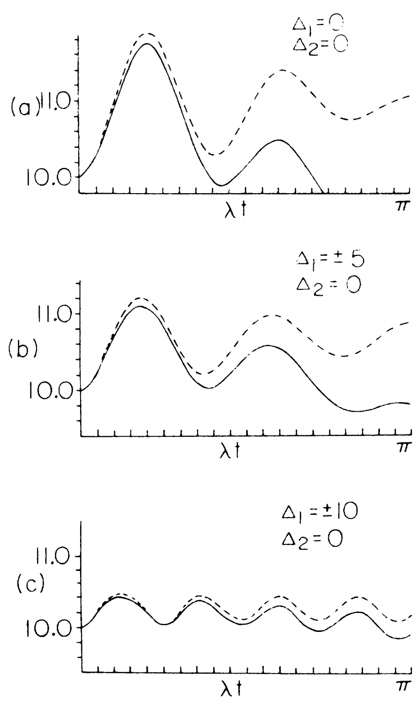

FIG. 4. Same as Fig. 2 except $\Delta_2 = 0$.

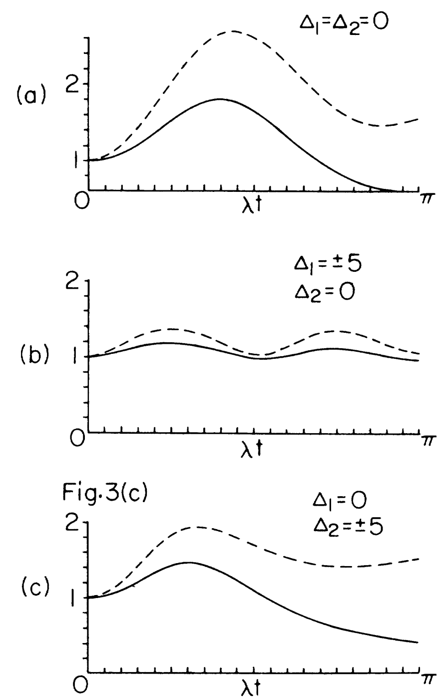

FIG. 5. Same as Fig. 2 except $\bar{n} = 1$ and Δ_1, Δ_2 are as specified.

4(c) we find that two curves separate apart at first as t increases and then approach each other to produce a coherent state again. Such recurrence may even occur periodically as in Figs. 2(a) and 3(b). This means that for a short period of time, the evolution of the state appears to be quasireversible,[8] and hence is in contrast with what is generally believed in the literature.[4,5] The atom does not act like a nonlinear filter, and its interaction with the fields does not destroy the coherence right away even though the coherence will eventually be lost in the long run. It may also be of interest to point out that the mean number of photons at coherence is almost the same as the initial value, namely, $\langle n(t)\rangle = \langle E^- E^+\rangle/\epsilon^2 \cong \bar{n}$. In other words, the probability of finding the atom in the state $|a\rangle$ or $|c\rangle$ is small. It has indeed returned to its original state $|b\rangle$.

B. Two-mode Λ type

Here we define the electric field operator for mode i as

$$E_i = E_i^+ + E_i^- = \epsilon_i(a_i + a_i^\dagger), \quad i=1,2 \tag{20}$$

where $E_i^\dagger = \epsilon_i a_i$ and $E_i^- = \epsilon_i a_i^\dagger$. Since the two modes are completely equivalent in a Λ-type transition, it is sufficient to consider one of them. Thus, we shall confine our discussions to mode 2. Following Ref. 7, we assume the initial condition that the atom is in $|a\rangle$ and the photon is in the coherent state,

$$P(n_1,n_2) = e^{-(\bar{n}_1+\bar{n}_2)} \bar{n}_1^{n_1} \bar{n}_2^{n_2} / n_1! n_2! \ . \tag{21}$$

The first-order correlation function is then

$$\begin{aligned}\langle E_2^- E_2^+\rangle(t) &= \mathrm{tr}[\rho E_2^- E_2^+] \\ &= \epsilon_2^2\, tr[\rho a_2^\dagger a_2] \\ &= \epsilon_2^2[\bar{n}_2 + P_c(t)] \ ,\end{aligned} \tag{22}$$

where the density matrix is obtained by plugging (6b) in (15) and the probability $P_c(t)$ is given by (12b).

The mean value of the negative-frequency part of the field is found in a similar fashion as in A,

$$\langle E_2^-\rangle(t) = \epsilon_2 e^{i\Omega_2 t} \alpha_2^* S_2(t) \ , \tag{23}$$

where α_2 is the initial complex amplitude of the mode 2 radiation field, and the function $S_2(t)$ is defined by

$$\begin{aligned} S_2(t) = \sum_{n_1 n_2} \Bigg[& A(n_1,n_2,t) A^*(n_1,n_2+1,t) \\ & + B(n_1+1,n_2,t) B^*(n_1+1,n_2+1,t) \\ & + \left[\frac{n_2+2}{n_2+1}\right]^{1/2} C(n_1,n_2+1,t) \\ & \times C^*(n_1,n_2+2,t) \Bigg] P(n_1,n_2) \ . \end{aligned} \tag{24}$$

Since the positive-frequency part is simply the Hermitian conjugate of (23), we find

$$\langle E_2^-\rangle(t)\langle E_2^+\rangle(t) = |\langle E^-\rangle(t)|^2 = \epsilon_2^2 n_2 |S_2(t)|^2 \ . \tag{25}$$

The functions in (22) and (25) are again calculated numerically for $\lambda_1=\lambda_2=\lambda$. The initial photon numbers are arbitrarily taken to be $\bar{n}_1=\bar{n}_2=4$. Large amount of calculations have been performed for various detuning parameters and some of the results are presented in Figs. 6–8. It should be emphasized that in these figures, the ordinate has been greatly amplified. The results are therefore more accurately represented and conclusions more convincing.

As $t>0$, the cavity fields interact with the atom immediately. For a very short period of time, however, the radiation field remains in its initial coherent state. The two curves coincide for $\lambda t < 0.02\pi$. What is more interesting is that this is true for all cases of arbitrary detunings we have employed as illustrated in the figures.

When $t > 0.02\pi$, the dependence of coherence on detuning parameters becomes obvious. For example, we see from Fig. 6 that for fixed $\Delta_1=0$, the state of the mode-2 field becomes more coherent as Δ_2 increases. It is also seen in Figs. 6(b) and 6(c) that the two curves keep close to each other for a long time. Let us imagine that for larger Δ_2, so that the atomic interaction with the mode-2 field is weak, the radiation field in the cavity may be considered as the superposition of a coherent field established initially and an incoherent field resulted from the nonlinear coupling of the atom and field. Since the latter is weak for large Δ_2, we can write

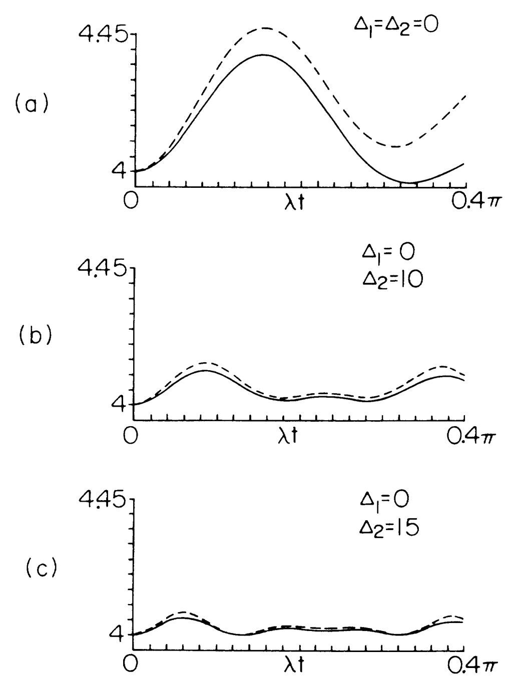

FIG. 6. Evolution of coherence of the mode-2 field. The dashed line represents $\langle E_2^- E_2^+\rangle/\epsilon_2^2$, and the solid line represents $\langle E_2^-\rangle\langle E_2^+\rangle/\epsilon_2^2$. The initial mean photon numbers are $\bar{n}_1=\bar{n}_2=4$, and $\Delta_1=0$.

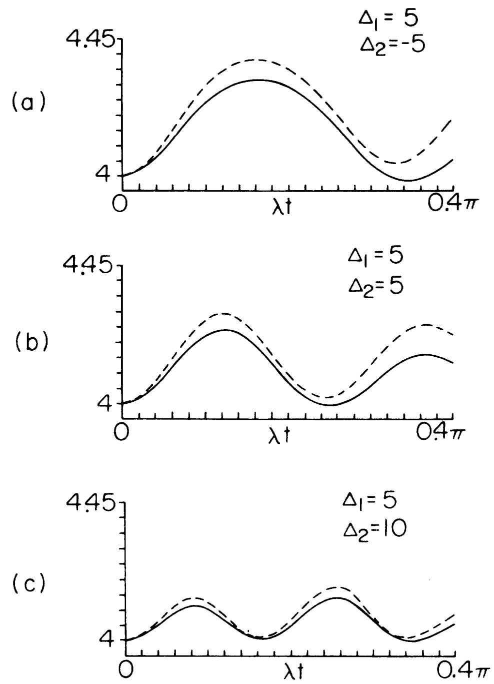

FIG. 7. Same as Fig. 6 expect $\Delta_1 = 5$.

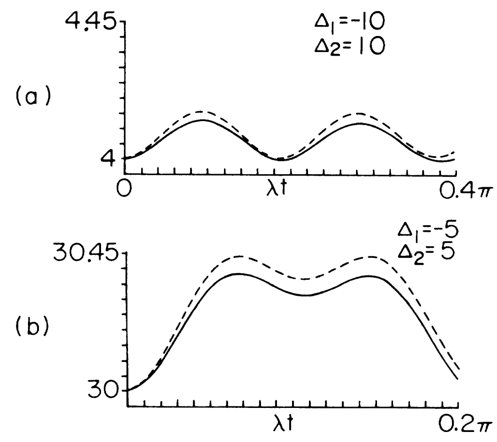

FIG. 8. Same as Fig. 6 except $\bar{n}_1 = \bar{n}_2 = 30$ and Δ_1, Δ_2 are as specified.

$\langle E_2^- E_2^+ \rangle = \langle E_2^- \rangle \langle E_2^+ \rangle + \epsilon$, where ϵ is a small quantity and is roughly $\epsilon \sim |\Delta_1/\Delta_2|$. Thus the state may be called "quasicoherent" if ϵ is small enough. As Δ_1 increases, the state moves away from coherence. Therefore the effects of Δ_1 and Δ_2 tend to offset each other. A comparison between Figs. 6(b) and 7(c) or between 6(b) and 8(a) illustrates this behavior.

In addition, we also find as in the one-mode case that recurrence of coherence appears when $|\Delta_1 - \Delta_2|$ becomes large. This phenomenon is clearly shown in Figs. 6(c), 7(c), and 8(a). Once more, the recurrence occurs at times when the mean photon number $\langle n_2(t) \rangle = \langle E_2^- E_2^+ \rangle / \epsilon_2^2 \cong \bar{n}_2$.

Finally, we compare Figs. 7(a) and 7(b) and find that two-photon transitions do not have significant influence on the coherence property of the fields in Λ-type transitions. This agrees with the analysis of Ref. 9, in which it is found that no obvious change in coherence results from making $\Delta_1 = \Delta_2$.

IV. CONCLUSIONS

We have investigated the coherence of the radiation field which interacts with a three-level atom in a cavity. One- and two-mode fields of arbitrary detuning parameters are considered. Large amounts of data are obtained by numerical calculation of two typical types of transition. In contrast with what has been believed thus far, that the atom acts like a nonlinear filter, our calculations show that different detunings result in qualitatively different behavior of the time evolution of the originally coherent state. In fact, when the difference between the two detuning parameters is large enough, the state goes back to the coherence state in which the mean photon number is roughly the same as the initial value. Such recurrence phenomenon can be found in both one- and two-mode cases. It indicates that the atom-field interaction may be regarded as quasireversible. The nearly symmetric appearance of curves in Fig. 8 may be regarded as another evidence of quasireversible nature. Therefore we conclude that the nonlinear-filter point of view is no longer valid. It may even be possible to keep the field in the coherent state for a long time if one chooses sufficiently large Δ_2 while keeping $\Delta_1 = 0$ or vise versa for two-mode cases and if one chooses nonzero Δ_1 and Δ_2 with sufficiently large differences $|\Delta_1 - \Delta_2|$ for one-mode cases.

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