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Nonresonant interaction of a three-level atom with cavity fields. I. General formalism and level occupation probabilities

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We present a general formalism of the theory of a three-level atom interacting with one-mode or two-mode cavity fields of arbitrary detunings. The dynamic behavior of the mean atomic level occupation probabilities are investigated numerically as functions of time and detuning parameters. A number of novel phenomena are discovered and discussed.

I. INTRODUCTION

There are very few quantum-mechanical models that are exactly soluble. The Jaynes-Cummings (JC) model[1] is one of them. It has been studied extensively because of its simple and realistic representation of the dipole interaction between an atom and the quantized radiation field.

When the cavity mode is initially prepared in a coherent state, the JC model is found to possess a number of interesting features. The Rabi oscillations for the principal dynamic variables such as atomic level occupations have a Gaussian decay envelope despite the loss free nature of the model.[2] Analytical expressions for the time dependence of the density matrix elements of the atom have been derived, and the coherent JC model has been shown to be related to the quantum theory of lasers[3] in an earlier review. It has also been found from the study of photon-number probabilities that the atom acts like a nonlinear filter on the coherent properties of the interacting field.[4] In addition, the model has a striking feature discovered in 1980—the decayed Gaussian envelope of Rabi oscillations revives spontaneously.[5] Thus it may be regarded as a useful model for the study of irreversibility and the long-time coherence properties of interacting fields. The quantum collapse and revival phenomena are recently studied in detail for the JC model with cavity damping[6] and for a system of two atoms interacting with a quantized radiation field.[7] It is found that the Rabi oscillation envelope is not Gaussian any more.

In short, the most important attraction of the JC model is that it exhibits, without perturbative expansions or statistical decorrelations, many quantum-mechanical effects in the case of strong or weak interaction, long- or short-time regime, on or off the resonance.

To investigate the physical phenomena associated with the two-photon process, it is necessary to extend the JC model to the three-level atom. The dynamical behavior of an atom (two level to three level) interacting with quantized cavity fields has been discussed in a recent review[8] in which one finds detailed discussions on quantum wave-packet collapse and revival phenomena, coherent trapping states, and experimental prospects. More recently, the interaction of a three-level atom with two cavity fields is investigated numerically.[9] The mean atomic occupation and mean photon numbers for different stimulating sources and equal detuning are discussed. Furthermore, the dependence of coherent properties of the stimulated fields on the stimulating sources has also been studied.[10] It is found that double stimulation will cause the field to approach its initial coherent state and that different stimulating sources have different effects.

However, discussions in existing literature have been limited to one-photon or two-photon resonance with equal detuning. It is therefore desirable to investigate the off-resonance behavior of the interaction. In this series of papers, we shall formulate the general problem of a three-level atom interacting with one- or two-mode cavity fields with arbitrary detunings. The theory developed here should also be useful in dealing with other three-level problems such as double resonance, two-mode lasers, and so on. We present in this first paper the general formalism which is then applied to investigate the time evolution as well as the detuning dependence of the mean atomic level occupation probabilities for two specific cases of interest. A number of novel phenomena such as symmetry, asymmetry, and antisymmetry of the probabilities with respect to the detuning parameter on or off the resonance of the other mode are exhibited. Studies on the photon-number probabilities, possibilities of squeezing and antibunching, coherent properties of the field, and the mean value of dipole operator are now underway and will be reported in the near future. As the experimental techniques are developing to realize a single atom in a cavity,[11] these studies are no longer limited to academic interest only.

II. FORMULATION OF THE THEORY

Consider a three-level atom interacting with the radiation field in a cavity. Let the three levels be denoted by $|a\rangle$, $|b\rangle$, and $|c\rangle$. If we limit our discussions to the electronic dipole transitions $|a\rangle$-$|b\rangle$ and $|a\rangle$-$|c\rangle$, then there are three distinct atomic level configurations as shown in Fig. 1. Following Ref. 8, these are called Λ, Ξ, and V types. The transition $|b\rangle$-$|c\rangle$ is forbidden in this scheme.

If only one mode with frequency Ω of the electromagnetic field participates, the Hamiltonian on the rotating-wave approximation can be written, in the interaction picture, as

$$H=\hbar(H_0+H') , \tag{1}$$

where

$$H_0=\sum_{\eta=a,b,c}\omega_\eta A_\eta^\dagger A_\eta+\Omega a^\dagger a \tag{2}$$

and the interaction H' for Λ type,

$$H'=\lambda_1 e^{-i\Delta_1 t} a A_a^\dagger A_b+\lambda_2 e^{-i\Delta_2 t} a A_a^\dagger A_c+\text{H.c.} , \tag{3a}$$

with $\Delta_1=\Omega-\omega_a+\omega_b$, $\Delta_2=\Omega-\omega_a+\omega_c$; for Ξ type,

$$H'=\lambda_1 e^{i\Delta_1 t} a A_b^\dagger A_a+\lambda_2 e^{-i\Delta_2 t} a A_a^\dagger A_c+\text{H.c.} , \tag{3b}$$

with $\Delta_1=-(\Omega-\omega_b+\omega_a)$, $\Delta_2=\Omega-\omega_a+\omega_c$; and for V type,

$$H'=\lambda_1 e^{i\Delta_1 t} a A_b^\dagger A_a+\lambda_2 e^{i\Delta_2 t} a A_c^\dagger A_a+\text{H.c.} , \tag{3c}$$

with $\Delta_1=-(\Omega-\omega_b+\omega_a)$, $\Delta_2=-(\Omega-\omega_c+\omega_a)$. In the above equations, the operator $A_\eta^\dagger(A_\eta)$ creates (annihilates) an atom in the state $|\eta\rangle$, and $a^\dagger(a)$ creates (annihilates) a photon in the mode under consideration. λ_1 and λ_2 are the usual coupling constants.[9]

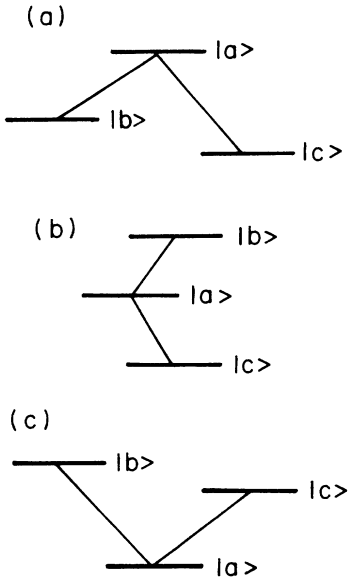

FIG. 1. Typical configurations of the energy levels of a three-level atom. (a) Λ type, (b) Ξ type, and (c) V type.

In a similar fashion, we write the Hamiltonian for two-mode coupling in which mode 1 is coupled to the transition $|a\rangle$-$|b\rangle$ and mode 2 to $|a\rangle$-$|c\rangle$ as follows:

$$H_0=\sum_{\eta=a,b,c}\omega_\eta A_\eta^\dagger A_\eta+\sum_{i=1,2}\Omega_i a_i^\dagger a_i . \tag{4}$$

For Λ type,

$$H'=\lambda_1 e^{-i\Delta_1 t} a_1 A_a^\dagger A_b+\lambda_2 e^{-i\Delta_2 t} a_2 A_a^\dagger A_c+\text{H.c.} , \tag{5a}$$

with $\Delta_1=\Omega_1-\omega_a+\omega_b$, $\Delta_2=\Omega_2-\omega_a+\omega_c$; for Ξ type,

$$H'=\lambda_1 e^{i\Delta_1 t} a_1 A_b^\dagger A_a+\lambda_2 e^{-i\Delta_2 t} a_2 A_a^\dagger A_c+\text{H.c.} , \tag{5b}$$

with $\Delta_1=-(\Omega_1-\omega_b+\omega_c)$, $\Delta_2=\Omega_2-\omega_a+\omega_c$; for V type,

$$H'=\lambda_1 e^{i\Delta_1 t} a_1 A_b^\dagger A_a+\lambda_2 e^{i\Delta_2 t} a_2 A_c^\dagger A_a+\text{H.c.} , \tag{5c}$$

with $\Delta_1=(\Omega_1-\omega_b+\omega_a)$, $\Delta_2=-(\Omega_2-\omega_c+\omega_a)$. Here $a_i^\dagger(a_i)$ is the creation (annihilation) operator of a photon in mode i. For both cases of one-mode and two-mode field, the Schrödinger equation

$$i\frac{\partial}{\partial t}|\psi(t)\rangle=H'|\psi(t)\rangle \tag{6}$$

can be solved at the same time. Let us assume that the initial state is, for one mode,

$$|\psi(0)\rangle=|\eta,\xi\rangle=|\eta\rangle|\xi\rangle , \tag{7a}$$

and for two modes,

$$|\psi(0)\rangle=|\eta,\xi_1,\xi_2\rangle=|\eta\rangle|\xi_1\xi_2\rangle , \tag{7b}$$

where we have expanded the photon state into a set of Fock states, i.e., for one mode,

$$|\xi\rangle=\sum_n Q(n)|n\rangle \tag{8a}$$

and for two modes,

$$|\xi_1,\xi_2\rangle=\sum_{n_1,n_2}Q_1(n_1)Q_2(n_2)|n_1,n_2\rangle . \tag{8b}$$

At a time $t>0$, the state vector becomes[12,13] for one mode,

$$|\psi(t)\rangle=\sum_n Q(n)[A(n_a,t)|a,n_a\rangle+B(n_b,t)|b,n_b\rangle+C(n_c,t)|c,n_c\rangle] \tag{9a}$$

and for two modes,

$$\begin{aligned}|\psi(t)\rangle=\sum_{n_1,n_2}Q_1(n_1)Q_2(n_2)&\times[A(n_{1a},n_{2a},t)|a,n_{1a},n_{2a}\rangle\\&+B(n_{1b},n_{2b},t)|b_{n_{1b},n_{2b}}\rangle\\&+C(n_{1c},n_{2c},t)|c,n_{1c},n_{2c}\rangle] ,\end{aligned} \tag{9b}$$

where n_η is the photon number when the atom is in the level η, and $n_{i\eta}$ is the number referring to the mode i.

Using the interaction Hamiltonian (3) or (5) and the state vector (9), the Schrödinger equation (6) can be reduced by a standard procedure[13–15] to a set of coupled equations

$$\begin{aligned} i\dot{A} &= V_1 e^{-i\Delta_1 t} B + V_2 e^{-i\Delta_2 t} C \ , \\ i\dot{B} &= V_1 e^{i\Delta_1 t} A \ , \\ i\dot{C} &= V_2 e^{i\Delta_2 t} A \ , \end{aligned} \tag{10}$$

where A stands for the probability amplitude $A(n_a,t)$ or $A(n_{1a},n_{2a},t)$, etc. The V's are just constants depending upon the particular case and particular type of the atomic configuration.

To solve (10), we assume $C=e^{i\mu t}$. Substituting this C in (10) we find that μ satisfy the third-order equation,

$$\mu^3+x_1\mu^2+x_2\mu+x_3=0 \ , \tag{11}$$

where

$$\begin{aligned} x_1 &= \Delta_1-2\Delta_2 \ , \\ x_2 &= -[V_1^2+V_2^2+\Delta_2(\Delta_1-\Delta_2)] \ , \\ x_3 &= (\Delta_2-\Delta_1)V_2^2 \ . \end{aligned} \tag{12}$$

Equation (11) is similar to Eq. (3.2) in Ref. 14, Eq. (5) in Ref. 15, and Eq. (4) in Ref. 16. It is expected to have three different real roots.[15,17] The explicit expressions for these roots are

$$\mu_1=0 \ , \ \mu_{2,3}=\pm(V_1^2+V_2^2)^{1/2} \tag{13}$$

for one-photon resonance ($\Delta_1=\Delta_2=0$);

$$\mu_1=0 \ , \ \mu_{2,3}=\frac{\Delta}{2}\pm(V_1^2+V_2^2+\Delta^2/4)^{1/2} \tag{14}$$

for two-photon resonance ($\Delta_1=\Delta_2=\Delta$);

$$\mu_1=\frac{(\Delta_2-\Delta_1)V_2^2}{G^2-\Delta_2^2/4} \ , \ +\mu_{2,3}=\frac{\Delta_2}{2}\pm G\pm\frac{(\Delta_2-\Delta_1)V_1^2}{2G(\Delta_2/2\pm G)} \ ,$$

$$G=(V_1^2+V_2^2+\Delta_2^2/4)^{1/2} \tag{15}$$

for nearly two-photon resonance[14] ($|\Delta_1-\Delta_2| \ll \Delta_2$);

$$\begin{aligned} \mu_1 &= \frac{(\Delta_2-\Delta_1)V_2^2}{V_1^2+V_2^2} \\ \mu_{2,3} &= \frac{(2\Delta_2-\Delta_1)V_1^2+\Delta_2 V_2^2}{2(V_1^2+V_2^2)}\pm V_1^2+V_2^2 \end{aligned} \tag{16}$$

for small detunings or strong fields ($|\Delta_1|, |\Delta_2| \ll V_1^2+V_2^2$);

$$\mu_1=0 \ , \ \mu_2=\Delta_2-\Delta_1 \ , \ \mu_3=\Delta_2+V_1^2/\Delta_1+V_2^2/\Delta_2 \tag{17}$$

for large detunings and weak fields ($|\Delta_1|, |\Delta_2| \gg V_1, V_2$);

$$\mu_1=\Delta_2 \ ,$$

$$\mu_{2,3}=\tfrac{1}{2}(\Delta_2-\Delta_1)\pm[V_1^2+V_2^2+(\Delta_2-\Delta_1)^2/4]^{1/2} \tag{18}$$

for the special case[15] in which $\Delta_1/\Delta_2=-V_1^2/V_2^2$; and

$$\mu_1=-\tfrac{1}{3}x_1+\tfrac{2}{3}(x_1^2-3x_2)^{1/2}\cos\theta \ , \tag{19a}$$

$$\mu_2=-\tfrac{1}{3}x_1+\tfrac{2}{3}(x_1^2-3x_2)^{1/2}\cos(\theta+\tfrac{2}{3}\pi) \ , \tag{19b}$$

$$\mu_3=-\tfrac{1}{3}x_1+\tfrac{2}{3}(x_1^2-3x_2)^{1/2}\cos(\theta+\tfrac{4}{3}\pi) \ , \tag{19c}$$

$$\theta=\tfrac{1}{3}\cos^{-1}\left[\frac{9x_1x_2-2x_1^3-27x_3}{2(x_1^2-3x_2)^{3/2}}\right] \tag{19d}$$

in general. In any case, we can write $C=C_1e^{i\mu_1 t}+C_2e^{i\mu_2 t}+C_3e^{i\mu_3 t}$ and insert it in (10) to obtain the solution

$$A=-e^{-i\Delta_2 t}\sum_{i=1}^{3}U_i\mu_i e^{i\mu_i t} \ , \tag{20a}$$

$$B=\frac{1}{V_1}e^{i(\Delta_1-\Delta_2)t}\sum_{i=1}^{3}U_i(\mu_i^2-\Delta_2\mu_i-V_2^2)e^{i\mu_i t} \ , \tag{20b}$$

$$C=V_2\sum_{i=1}^{3}U_i e^{i\mu_i t} \ . \tag{20c}$$

The coefficients U_i depend on the level configuration and initial condition. Their explicit expressions are calculated and listed in Table I, which is very useful in various calculations.

The atomic level occupation probabilities are most easily calculated from the density matrix which is defined by

$$\rho(t)=|\psi(t)\rangle\langle\psi(t)| \ , \tag{21}$$

where the state vector is given by (9) with the probability amplitudes (20). The results are, for one mode,

$$P_a(t)=\sum_n P(n)\,|A(n_a,t)|^2 \ , \tag{22a}$$

and for two modes,

$$P_a(t)=\sum_{n_1,n_2}P(n_1,n_2)\,|A(n_{1a},n_{2a},t)|^2 \ ; \tag{22b}$$

for one mode,

$$P_b(t)=\sum_n P(n)\,|B(n_b,t)|^2 \ , \tag{23a}$$

and for two modes,

$$P_b(t)=\sum_{n_1,n_2}P(n_1,n_2)\,|B(n_{1b},n_{2b},t)|^2 \ ; \tag{23b}$$

for one mode,

$$P_c(t)=\sum_n P(n)\,|C(n_c,t)|^2 \ , \tag{24a}$$

and for two modes,

$$P_c(t)=\sum_{n_1,n_2}P(n_1,n_2)\,|C(n_{1c},n_{2c},t)|^2 \ , \tag{24b}$$

TABLE I. Coefficients for different atomic level structures and initial conditions.

Initial atomic state	U_i[a]	Mode	Level structure	V_1^2	V_2^2	A	B	C	Δ_1	Δ_2
	$U_1=-\frac{\mu_1+\Delta_{12}}{\mu_{12}\mu_{13}}$		Λ	$\lambda_1^2(n+1)$	$\lambda_2^2(n+1)$	$A(n,t)$	$B(n+1,t)$	$C(n+1,t)$	$\Omega-\omega_a+\omega_b$	$\Omega-\omega_a+\omega_c$
		1	Ξ	$\lambda_1^2 n$	$\lambda_2^2(n+1)$	$A(n,t)$	$B(n-1,t)$	$C(n+1,t)$	$-(\Omega-\omega_b+\omega_a)$	$\Omega-\omega_a+\omega_c$
			V	$\lambda_1^2 n$	$\lambda_2^2 n$	$A(n,t)$	$B(n-1,t)$	$C(n-1,t)$	$-(\Omega-\omega_b+\omega_a)$	$-(\Omega-\omega_c+\omega_a)$
$\|a\rangle$	$U_2=-\frac{\mu_2+\Delta_{12}}{\mu_{21}\mu_{23}}$									
			Λ	$\lambda_1^2(n_1+1)$	$\lambda_2^2(n_2+1)$	$A(n_1,n_2,t)$	$B(n_1+1,n_2,t)$	$C(n_1,n_2+1,t)$	$\Omega_1-\omega_a+\omega_b$	$\Omega_2-\omega_a+\omega_c$
		2	Ξ	$\lambda_1^2 n_1$	$\lambda_2^2(n_2+1)$	$A(n_1,n_2,t)$	$B(n_1-1,n_2,t)$	$C(n_1,n_2+1,t)$	$-(\Omega_1-\omega_b+\omega_a)$	$\Omega_2-\omega_a+\omega_c$
	$U_3=-\frac{\mu_3+\Delta_{12}}{\mu_{31}\mu_{32}}$		V	$\lambda_1^2 n_1$	$\lambda_2^2 n_2$	$A(n_1,n_2,t)$	$B(n_1-1,n_2,t)$	$C(n_1,n_2,-1,t)$	$-(\Omega_1-\omega_b+\omega_a)$	$-(\Omega_2-\omega_c+\omega_a)$
	$U_1=\frac{V_1}{\mu_{12}\mu_{13}}$		Λ	$\lambda_1^2 n$	$\lambda_2^2 n$	$A(n-1,t)$	$B(n,t)$	$C(n,t)$	$\Omega-\omega_a+\omega_b$	$\Omega-\omega_a+\omega_c$
		1	Ξ	$\lambda_1^2(n+1)$	$\lambda_2^2(n+2)$	$A(n+1,t)$	$B(n,t)$	$C(n+2,t)$	$-(\Omega-\omega_b+\omega_a)$	$\Omega-\omega_a+\omega_c$
			V	$\lambda_1^2(n+1)$	$\lambda_2^2(n+1)$	$A(n+1,t)$	$B(n,t)$	$C(n,t)$	$-(\Omega-\omega_b+\omega_a)$	$-(\Omega-\omega_c+\omega_a)$
$\|b\rangle$	$U_2=\frac{V_1}{\mu_{21}\mu_{23}}$									
			Λ	$\lambda_1^2 n_1$	$\lambda_2^2(n_2+1)$	$A(n_1-1,n_2,t)$	$B(n_1,n_2,t)$	$C(n_1-1,n_2+1,t)$	$\Omega_1-\omega_a+\omega_b$	$\Omega_2-\omega_a+\omega_c$
		2	Ξ	$\lambda_1^2(n_1+1)$	$\lambda_2^2(n_2+1)$	$A(n_1+1,n_2,t)$	$B(n_1,n_2,t)$	$C(n_1+1,n_2+1,t)$	$-(\Omega_1-\omega_b+\omega_a)$	$\Omega_2-\omega_a+\omega_c$
	$U_3=\frac{V_1}{\mu_{31}\mu_{32}}$		V	$\lambda_1^2(n_1+1)$	$\lambda_2^2 n_2$	$A(n_1+1,n_2,t)$	$B(n_1,n_2,t)$	$C(n_1+1,n_2-1,t)$	$-(\Omega_1-\omega_b+\omega_a)$	$-(\Omega_2-\omega_c+\omega_a)$
	$U_1=\frac{V_2^2+\mu_2\mu_3}{V_2\mu_{12}\mu_{13}}$		Λ	$\lambda_1^2 n$	$\lambda_2^2 n$	$A(n-1,t)$	$B(n,t)$	$C(n,t)$	$\Omega-\omega_a+\omega_b$	$\Omega-\omega_a+\omega_c$
		1	Ξ	$\lambda_1^2(n-1)$	$\lambda_2^2 n$	$A(n-1,t)$	$B(n-2,t)$	$C(n,t)$	$-(\Omega-\omega_b+\omega_a)$	$\Omega-\omega_a+\omega_c$
			V	$\lambda_1^2(n+1)$	$\lambda_2^2(n+1)$	$A(n+1,t)$	$B(n,t)$	$C(n,t)$	$-(\Omega-\omega_b+\omega_a)$	$-(\Omega-\omega_c+\omega_a)$
$\|c\rangle$	$U_2=\frac{V_2^2+\mu_1\mu_3}{V_2\mu_{21}\mu_{23}}$									
			Λ	$\lambda_1^2(n_1+1)$	$\lambda_2^2 n_2$	$A(n_1,n_2-1,t)$	$B(n_1+1,n_2-1,t)$	$C(n_1,n_2,t)$	$\Omega_1-\omega_a+\omega_b$	$\Omega_2-\omega_a+\omega_c$
		2	Ξ	$\lambda_1^2 n_1$	$\lambda_2^2 n_2$	$A(n_1,n_2-1,t)$	$B(n_1-1,n_2-1,t)$	$C(n_1,n_2,t)$	$-(\Omega_1-\omega_b+\omega_a)$	$\Omega_2-\omega_a+\omega_c$
	$U_3=\frac{V_2^2+\mu_1\mu_2}{V_2\mu_{31}\mu_{32}}$		V	$\lambda_1^2 n_1$	$\lambda_2^2(n_2+1)$	$A(n_1,n_2+1,t)$	$B(n_1-1,n_2+1,t)$	$C(n_1,n_2,t)$	$-(\Omega_1-\omega_b+\omega_a)$	$-(\Omega_2-\omega_c+\omega_a)$

[a] $\mu_{ij}=\mu_i-\mu_j$ $(i,j=1,2,3)$; $\Delta_{12}=\Delta_1-\Delta_2$.

where

$$P(n)=|Q(n)|^2$$

and

$$P(n_1,n_2)=|Q_1((n_1)|^2|Q_2(n_2)|^2$$

are the initial photon distributions[8] and hence are directly related to the initial state of the field. Examples are, for the Fock state,

$$P(n)=\delta_{\bar{n}n} , \tag{25a}$$

for the coherent state,

$$P(n)=e^{-\bar{n}}\bar{n}^n/n! , \tag{25b}$$

for the thermal state

$$P(n)=\bar{n}^n/(\bar{n}+1)^{n+1} , \tag{25c}$$

and for the squeezed state,

$$P(n)=(n!\cosh\gamma)^{-1}(\beta/2)(\cosh\gamma)^{2n}Y^{-2n}H_n^2(Y) \times\exp[-\beta^2(1-\tanh\gamma)] , \tag{25d}$$

where $\bar{n}$ is the mean photon number in the initial state. In the squeezed state[18] $|\alpha,\gamma\rangle$, we have defined $Y=\beta/(2\cosh\gamma\sinh\gamma)^{1/2}$ and $\beta=\alpha e^{\gamma}$. It is a simple matter to verify that

$$P_a(t)+P_b(t)+P_c(t)=1 . \tag{26}$$

III. ATOMIC LEVEL OCCUPATION PROBABILITIES

We shall now discuss the atomic level occupation probabilities with coherent stimulating source for two typical cases, namely, one-mode Ξ type and two-mode Λ type.

For one-mode Ξ type, the atom starts in the state $|b\rangle$ and the initial photon distribution is given by (25b). From Table I we find

$$U_1=V_1/\mu_{12}\mu_{13} ,$$
$$U_2=V_1/\mu_{21}\mu_{23} , \tag{27}$$
$$U_3=V_1/\mu_{31}\mu_{32} ,$$

and $V_1^2=\lambda_1^2(n+1)$, $V_2^2=\lambda_2^2(n+2)$. Substituting these V's in (27) in which we take (19) and (12) for the μ's, we find after somewhat lengthy but straightforward calculation the atomic occupation probabilities

$$P_a(t)=2\sum_{n=0}^{\infty}\{U_1U_2\mu_1\mu_2[\cos(\mu_1-\mu_2)t-1] +U_1U_3\mu_1\mu_3[\cos(\mu_1-\mu_3)t-1] +U_2U_3\mu_2\mu_3[\cos(\mu_2-\mu_3)t-1]\}P(n) , \tag{28a}$$

$$P_c(t)=2\sum_{n=0}^{\infty}\{U_1U_2[\cos(\mu_1-\mu_2)t-1] +U_1U_3[\cos(\mu_1-\mu_3)t-1] +U_2U_3[\cos(\mu_2-\mu_3)t-1]\}V_2^2P(n) , \tag{28b}$$

$$P_b(t)=1-P_a(t)-P_c(t) . \tag{28c}$$

Here we have made use of the initial conditions $P_a(0)=P_c(0)=0$ and $P_b(0)=1$.

The mean photon number can be obtained directly from these results. With the initial photon number $\bar{n}$, it is given by

$$\langle n(t)\rangle=\bar{n}+P_a(t)+2P_c(t) . \tag{29}$$

The factor 2 in the last term is a result of the two-photon transition $|b\rangle\rightarrow|a\rangle$ and $|a\rangle\rightarrow|c\rangle$. It is evident, when $t>0$, that the field intensity will be enhanced by the one- and two-photon stimulated emission.

The probabilities given by Eqs. (28) are calculated numerically. For simplicity, we assume $\lambda_1=\lambda_2=\lambda$. The detuning parameters Δ_i $(i=1,2)$ are measured in the unit of λ, and the time t is in $1/\lambda$. The initial mean

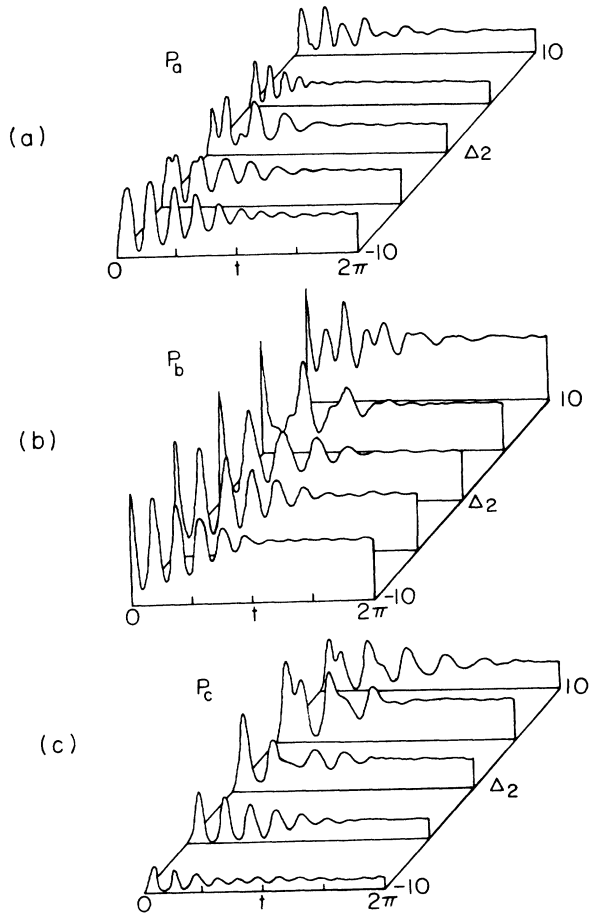

FIG. 2. Short-time behavior of the atomic level occupation probabilities for $\Delta_1=5$. (a) $P_a(t)$, (b) $P_b(t)$, and (c) $P_c(t)$.

photon number is arbitrarily taken to be $\bar{n}=30$. The results are shown in Figs. 2–5. For a fixed value of Δ_1, the probabilities are plotted as functions of t for five different values of Δ_2. Figure 2 depicts the variation in short-time regime and Fig. 3 shows the variation in long-time regime for $\Delta_1=5$. It is observed that the occurrence of collapse and revival[5,6] depends upon the detunings. In the case of two-photon resonance, or when $\Delta_1=\Delta_2=5$, both the first collapse and first revival occur in the shortest time. This time becomes longer as $|\Delta_1-\Delta_2|$ increases. On the other hand, the effects of detunings on the oscillation frequency and on the time of revivals after the first one do not seem to be regular. This means that the dependence on the detunings of the

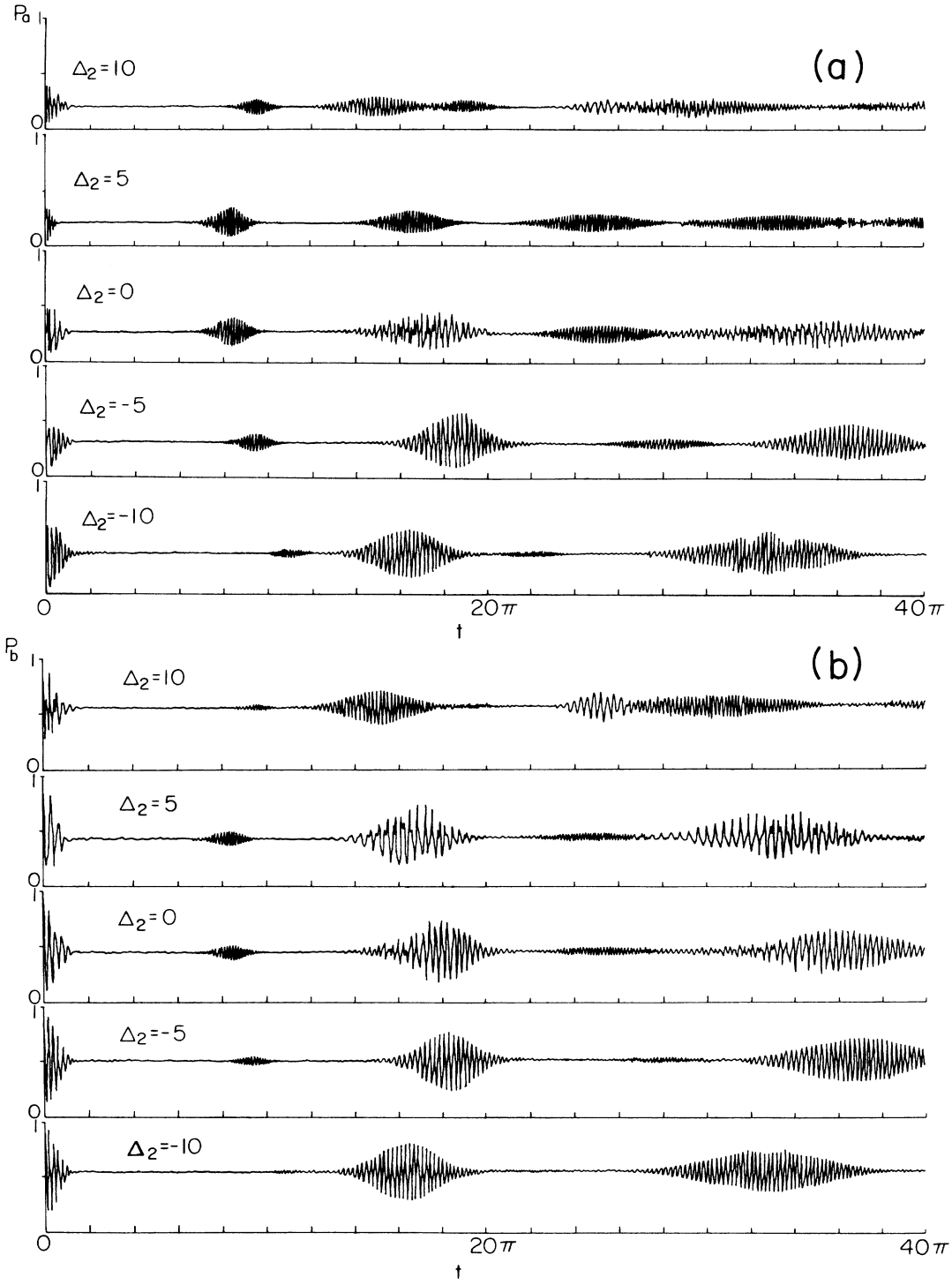

FIG. 3. Long-time behavior of atomic level occupation probabilities for $\Delta_1=5$ and several choices of Δ_2. (a) $P_a(t)$, (b) $P_b(t)$, (c) $P_c(t)$.

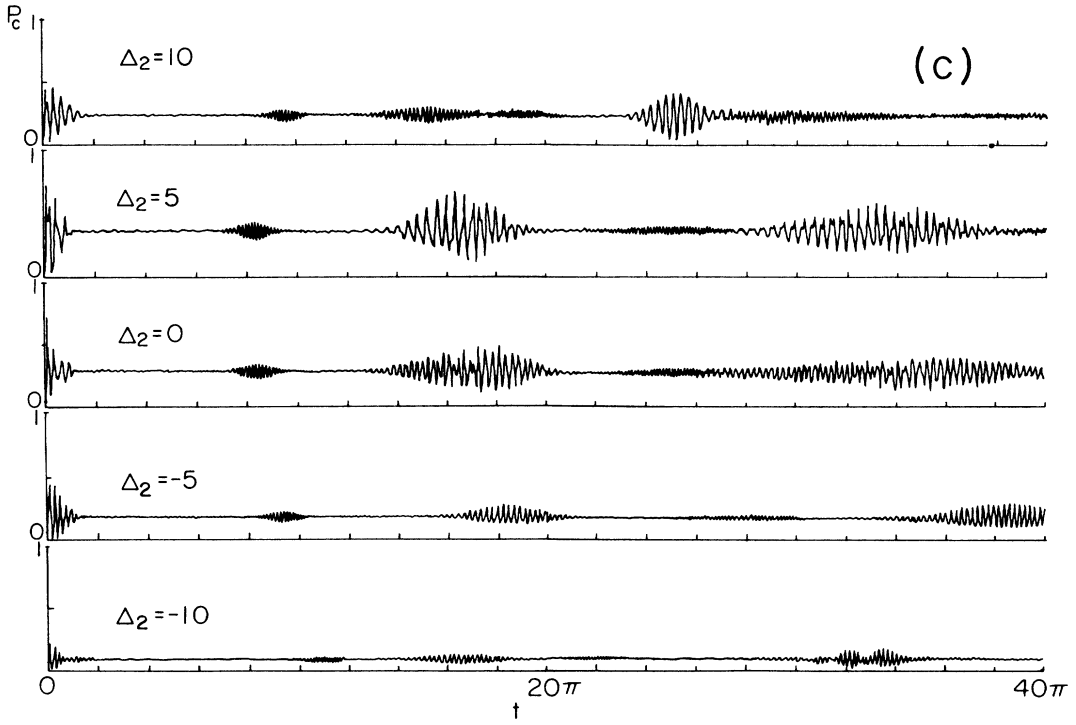

FIG. 3. (*Continued*).

coherence can be rather complicated, and further study is necessary before more definite conclusions can be made.

During the time period between the collapse and the revival, all the occupation probabilities are nearly constants and their values depend strongly upon the detunings. This may reflect the "competition" among the three probabilities.[10] It is also seen from these figures that the probabilities take their extremum values at the two-photon resonance $\Delta_1=\Delta_2=5$. While P_a and P_b are both at minimum as $\Delta_1=\Delta_2=5$, P_c is at its maximum. This interesting phenomenon is resulted from the two-photon transition. In addition to the existing transition $|a\rangle\rightarrow|c\rangle$, there is now strong two-photon transition $|b\rangle\rightarrow|c\rangle$. Thus the probability of finding the atom in $|c\rangle$ is greatly enhanced and at the same time P_a and p_b are at their minima. When the detunings are far from two-photon resonance, the amplitude of P_c decreases. This may also be attributed to the two-photon process because there is practically no two-photon transition in this case. Since the larger value of $|\Delta_1-\Delta_2|$ implies that Δ_2 is also far from one-photon resonance, there is only weak correlation between P_c and the field. Consequently, the coherence effect is weak and P_c oscillates with small amplitude.

To see how the probabilities depend on the detuning parameters at a given time, we plot in Fig. 4 the variation with Δ_2 at different stages of time for $\Delta_1=0$. The same plot is made in Fig. 5 for $\Delta_1=5$. In both cases, it is observed that the oscillatory dependence of the probabilities on Δ_2 will appear, then disappear, and then appear again. This is because the probabilities also have oscillatory time dependence. We are not able to explain why rapid oscillations occur in these figures. We note, however, that the probabilities oscillate with large amplitudes, indicating strong correlation, when $t=8\pi$ (in the first revival region) and $t=15\pi$ (in the second revival region), while the oscillation amplitudes are small, indicating weak correlation at $t=10\pi$ (in the collapse region).

Another feature we observe from Figs. 4 and 5 is that all the curves in Fig. 4 ($\Delta_1=0$, on resonance) are symmetric with respect to $\Delta_2=0$, and those in Fig. 5 are all asymmetric ($\Delta_1=5$, off resonance) with respect to $\Delta_2=0$. We believe that this is due to the joint effect on the two-photon transition and ac Stark shift of the atomic levels. This asymmetry which is sometimes known as dispersion[17] can also be observed in Figs. 2 and 3.

We now turn our attention to the second case, two-mode Λ type. Here the initial atomic state is $|a\rangle$ and the initial photon distribution is again a coherent state, namely,

$$P(n_1,n_2)=e^{-(\bar{n}_1+\bar{n}_2)}\bar{n}_1^{n_1}\bar{n}_2^{n_2}/n_1!n_1!\ , \tag{30}$$

where $\bar{n}_1(\bar{n}_2)$ is the initial mean photon number for mode 1(2). As what we have done previously, we first find from Table I

$$\begin{aligned} U_1&=-(\mu_1+\Delta_{12})/\mu_{12}\mu_{13}\ , \\ U_2&=-(\mu_2+\Delta_{12})/\mu_{21}\mu_{23}\ , \\ U_3&=-(\mu_3+\Delta_{12})/\mu_{31}\mu_{32}\ , \end{aligned} \tag{31}$$

and

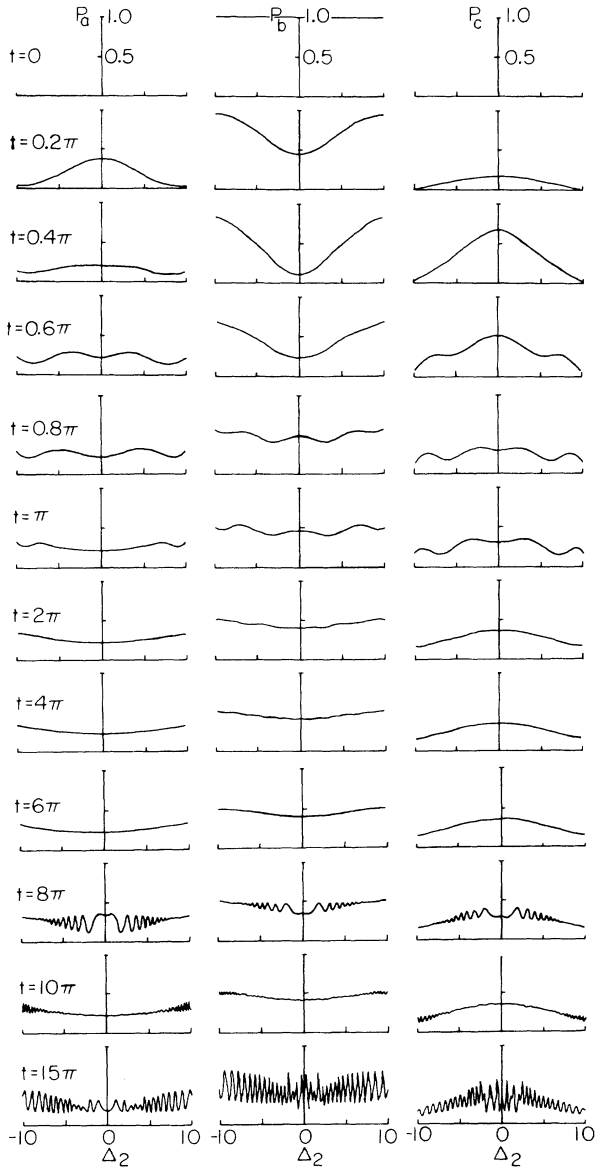

FIG. 4. Atomic level occupation probabilities vs Δ_2 for $\Delta_1=0$ at different times.

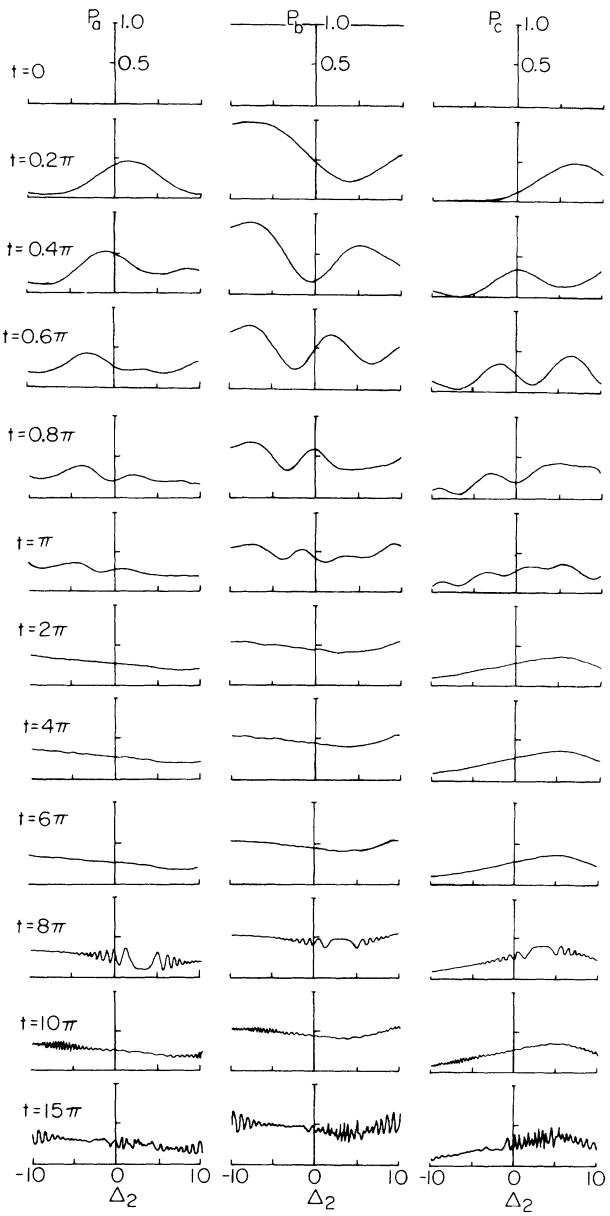

FIG. 5. Atomic level occupation probabilities vs Δ_2 for $\Delta_1=5$ at different times.

$$V_1^2=\lambda_1^2(n_1+1)\ ,$$
$$V_2^2=\lambda_2^2(n_2+1)\ . \tag{32}$$

Substituting (32) in (12) and then (19), we can find the occupation probabilities from the two-mode equations in (22)–(24). The calculation is tedious but straightforward and the results are

$$P_a(t)=1+2\sum_{n_1,n_2}\{U_1U_2\mu_1\mu_2[\cos(\mu_1-\mu_2)t-1]+U_1U_3\mu_1\mu_3[\cos(\mu_1-\mu_3)t-1]+U_2U_3\mu_2\mu_3[\cos(\mu_2-\mu_3)t-1]\}\times P(n_1,n_2)\ , \tag{33a}$$

$$P_c(t)=2\sum_{n_1,n_2}\{U_1U_2[\cos(\mu_1-\mu_2)t-1]+U_1U_3[\cos(\mu_1-\mu_3)t-1]+U_2U_3[\cos(\mu_2-\mu_3)t-1]\}P(n_1,n_2)\ , \tag{33b}$$

$$P_b(t)=1-P_a(t)-P_c(t)\ , \tag{33c}$$

where we have made use of the initial conditions $P_a(0)=1$ and $P_b(0)=P_c(0)=0$.

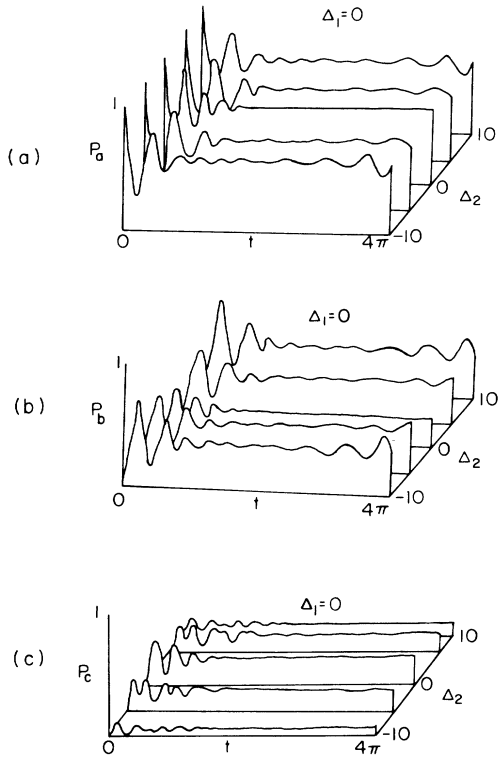

FIG. 6. Short-time behavior of (a) $P_a(t)$, (b) $P_b(t)$, (c) $P_c(t)$ for $\Delta_1=0$.

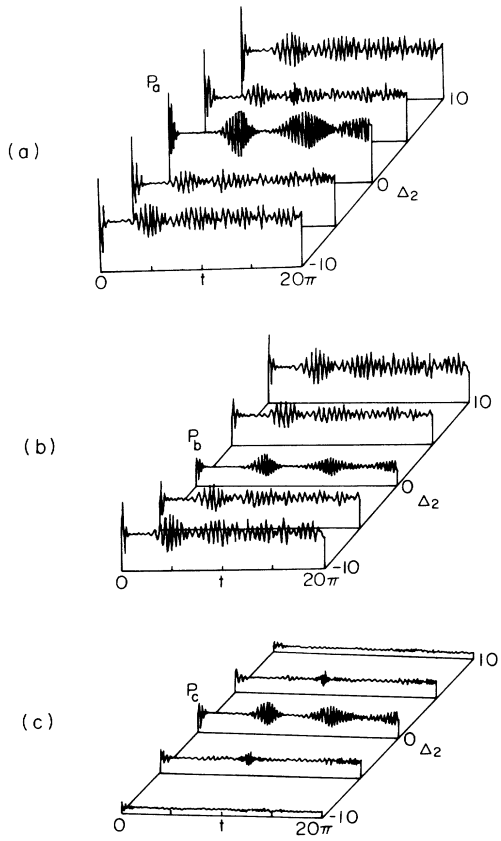

FIG. 7. Long-time behavior of (a) $P_a(t)$, (b) $P_b(t)$, (c) $P_c(t)$ for $\Delta_1=0$.

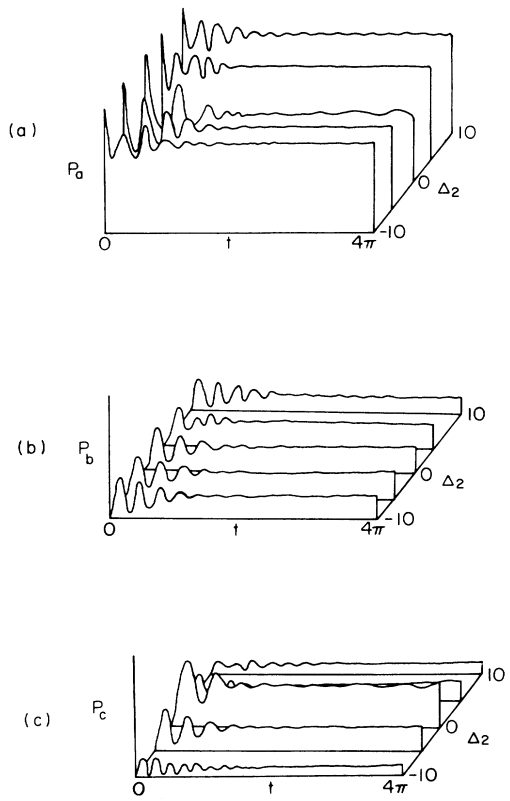

FIG. 8. Short-time behavior of (a) $P_a(t)$, (b) $P_b(t)$, (c) $P_c(t)$ for $\Delta_1=5$.

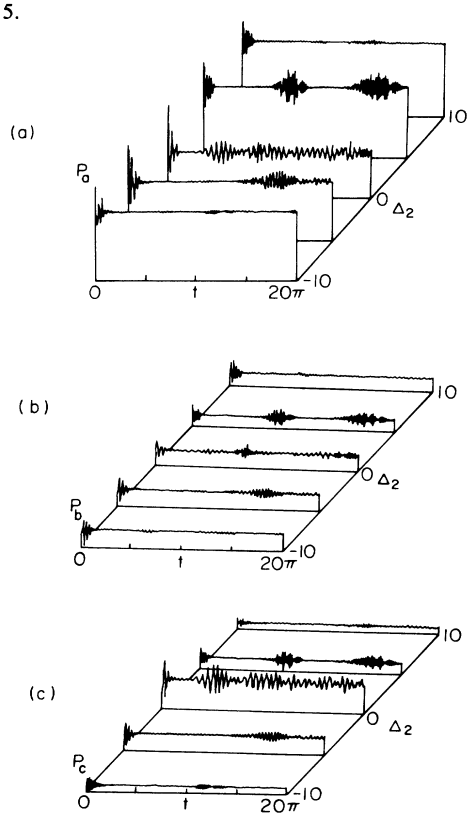

FIG. 9. Long-time behavior of (a) $P_a(t)$, (b) $P_b(t)$, (c) $P_c(t)$ for $\Delta_1=5$.

With the initial mean photon numbers $\bar{n}_1$ and $\bar{n}_2$, we have the mean photon numbers for the two modes

$$\langle n_1(t)\rangle = \bar{n}_1 + P_b(t) , \tag{34a}$$

$$\langle n_2(t)\rangle = \bar{n}_2 + P_c(t) . \tag{34b}$$

It seems that only one-photon transition is possible in this case.

Equations (33) are calculated numerically for the occupation probabilities. For simplicity and definiteness, we set $\lambda_1=\lambda_2=\lambda$ and $\bar{n}_1=\bar{n}_2=5$. The detuning parameters are measured in the unit of λ, and the time is in $1/\lambda$. The time variation of the probabilities are plotted for selected Δ_2 values in Figs. 6 and 7 when mode one is on resonance, that is, when $\Delta_1=0$. Figures 8 and 9 show the same plots when mode one is off resonance, $\Delta_1=5$ in this case. It is observed in Figs. 6 and 7 that as $|\Delta_2|$ increases, the collapse time increases, the revival time decreases, and the probabilities oscillate with lower frequencies. There is no such regularity when mode 1 is off the resonance as can be seen in Figs. 8 and 9. Therefore the dependence of coherence on detunings can be very complicated. The complex situation is understandable if one looks at Eqs. (33). Each of the occupation probabilities contains three sets of beat frequency, namely, $\mu_1-\mu_2$, $\mu_2-\mu_3$, and $\mu_1-\mu_3$. They determine, in turn, three series of coherent oscillations of which every one has its own collapse time, revival time, and oscillation frequency. Furthermore, the eigenvalues μ_1, μ_2, and μ_3 are all complicated functions of Δ_1 and Δ_2. Therefore, the dependence of occupation probabilities on the Δ's cannot be analyzed in any simple manner.

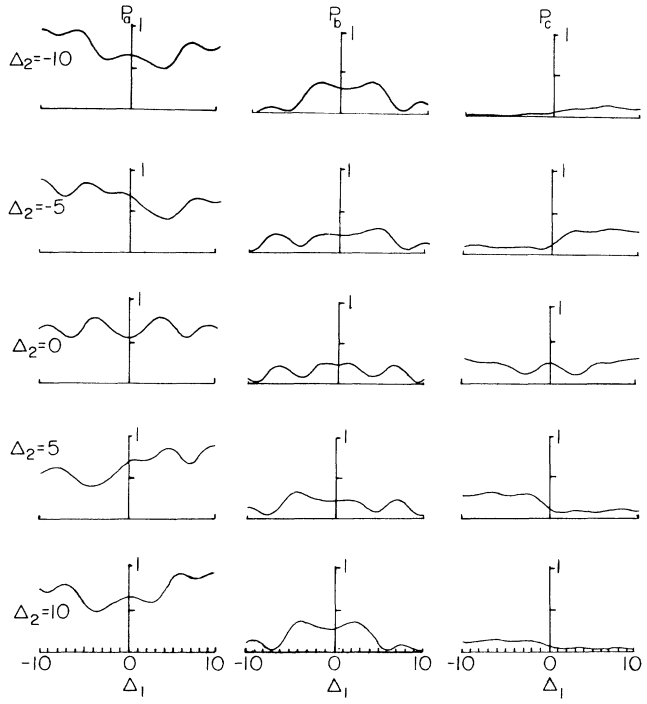

FIG. 10. Atomic level occupation probabilities vs Δ_1 for various choices of Δ_2 at $t=0.5$.

It is also observed from these figures that in the case of two-photon resonance or $\Delta_1=\Delta_2$, the collapse time is shortest, the revival time longest, and the vibration frequency highest. The probabilities P_b and P_c oscillate in phase and both of them are out of phase with P_a which remains large. Consequently it is more probable to find the atom in $|a\rangle$ than in other states and hence the two-photon transition is suppressed by the one-photon transitions $|a\rangle - |b\rangle$ and $|a\rangle - |c\rangle$. Now these one-photon transitions have the same coupling constant ($\lambda_1=\lambda_2$), and the stimulated fields have the same strength ($\bar{n}_1=\bar{n}_2$). When the detunings are also the same ($\Delta_1=\Delta_2$), the two one-photon transitions are under identical conditions, and therefore have the same properties. Apparently these conditions are just right for the collapse time, revival time, and oscillation frequency to be at their extrema. Whether or not this is a mere coincidence remains unknown, and more careful investigation is required to understand the physical picture.

Moreover, we notice that as $|\Delta_2|$ increases, the amplitude as well as the mean value of P_c decreases. This indicates that the more mode 2 deviates from the resonance, the weaker its coupling with the atom results. On the other hand, the coupling of mode 1 with the atom is strengthened because both the mean value and

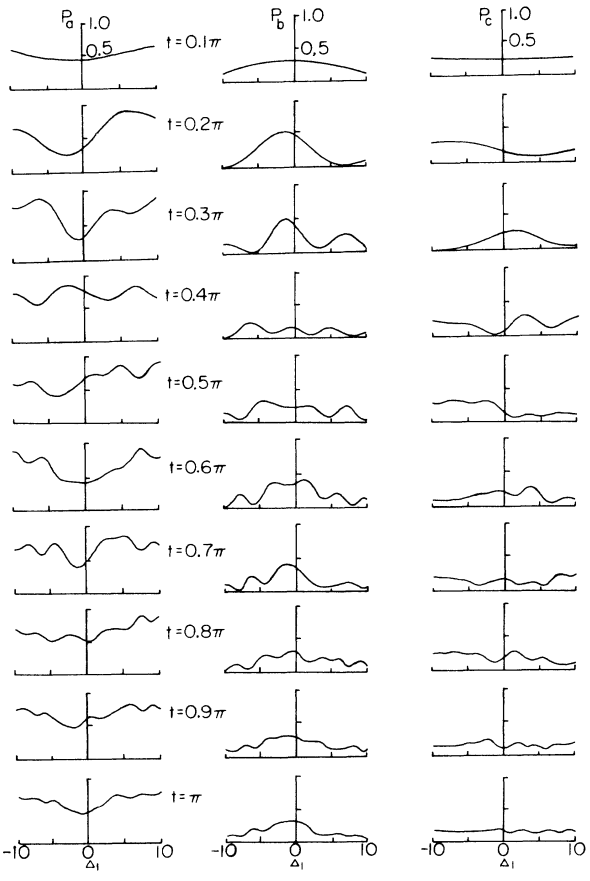

FIG. 11. Atomic level occupation probabilities vs Δ_1 for various choices of time with $\Delta_2=5$.

amplitude of P_b increase with increasing $|\Delta_2|$. This reflects the competition between the atomic occupation probabilities. From Figs. 9(b) and 9(c), we see that the mean value of P_b is greater for $\Delta_2=-5$ than for $\Delta_2=5$. From the competition point of view, we expect the mean value of P_c for $\Delta_2=-5$ to be smaller than that for $\Delta_2=5$. Actually, the opposite is true, the mean value of P_c is also greater when $\Delta_2=-5$. This may be attributed to the ac Stark effect.[19] When Δ_1 and Δ_2 have opposite signs different modes appear to mutually support each other[20] provided that the field strengths are within a certain range of variation.

The occupation probabilities at $t=0.5\pi$ are plotted in Fig. 10 as functions of Δ_1 for various choices of Δ_2. Evidently the curves for $\Delta_2=0$ are symmetric with respect to $\Delta_1=0$. On the other hand, each of the probabilities is antisymmetric with respect to $\Delta_2=0$. Such novel phenomena exist at any arbitrary time. The probabilities are also plotted in Fig. 11 for a fixed $\Delta_2=5$ but different times. The asymmetry or dispersion phenomena become remarkable for $t>0.1\pi$. In addition, it is also clear that the probabilities show oscillatory dependence on Δ_1 and the oscillation frequencies become alternatively high and low as t increases. We are not able to offer explanation for these interesting phenomena at this stage but merely point out that similar situations have been reported in Refs. 16, 20, and 21.

IV. SUMMARY

We have established the general formalism for a three-level atom interacting with one- or two-mode fields of arbitrary detunings. The formalism applies to semiclassical as well as quantum-mechanical treatment of the problem. The atomic level occupation probabilities and mean photon numbers are obtained for two typical cases as functions of the detuning parameters. For the special case of two-photon resonance, $\Delta_1=\Delta_2$, our results reduce to those in the existing literature.[8,9] A large amount of numerical work has been carried out and a number of interesting novel phenomena are discovered and discussed. Off-resonance behavior of such properties as coherence, squeezing, and antibunching as well as photon-number distribution will be reported elsewhere.

ACKNOWLEDGMENT

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