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Proposed Aharonov-Casher effect: Another example of an Aharonov-Bohm effect arising from a classical lag

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In 1984 Aharonov and Casher proposed that a beam of neutral particles with magnetic dipole moments passing around opposite sides of a line charge will undergo a relative quantum phase shift. This is currently under experimental investigation. Aharonov and Casher claimed that the magnetic dipole particles should undergo the phase shift despite experiencing no classical force. They suggested that this new effect has a "duality" with the solenoid Aharonov-Bohm effect where charged particles passing around a magnetic solenoid experience a phase shift despite, it is claimed, experiencing no classical force. Here it is pointed out that a magnetic dipole particle passing a line charge does indeed experience a classical electromagnetic force in the usual electric-current model for a magnetic dipole. This force will produce a relative lag between dipoles passing on opposite sides of the line charge, and the classical lag then leads to a quantum phase shift in exact agreement with that calculated by Aharonov and Casher. Thus actually the proposed Aharonov-Casher effect has a transparent explanation as a classical lag effect. It is emphasized that the solenoid Aharonov-Bohm effect is currently the only phase-shift effect which cannot be explained in an obvious fashion as arising from a classical lag and it is again proposed that also this shift may actually involve a classical electromagnetic lag effect. A natural experimental consequence suggested by the lag point of view is the breakdown of the interference pattern when the lag becomes comparable to the wave-packet coherence length.

INTRODUCTION

In 1959 Aharonov and Bohm[1] suggested that charged particles passing around opposite sides of a long solenoid would suffer a quantum phase shift due to the enclosed magnetic flux even though the electrons move in a region free of electric and magnetic fields. This solenoid Aharonov-Bohm effect has been verified experimentally[2] and has formed a justification for suggesting nonlocal topological effects in quantum physics. The term "Aharonov-Bohm effect" has now been extended from the surprising phase shift for charged particles passing a solenoid to include many systems where a phase shift is not surprising. In these latter systems classical forces can account for the observed quantum interference shift in terms of a relative displacement of quantum wave packets traveling over different paths. Indeed all of the experimentally observed Aharonov-Bohm effects, (e.g., gravitational[3] and electrostatic[4]), except the solenoid effect, can be described in an obvious fashion as classical lag effects due to classical forces. In 1984 Aharonov and Casher suggested another Aharonov-Bohm effect[5] which they claimed did not involve classical forces. The purpose of the present article is to point out that the author believes this claim is in error. Classical forces account for the proposed Aharonov-Casher effect in an obvious fashion as a lag effect, provided one uses the usual electric-current model for the magnetic dipole and not a magnetic monopole model.[6]

The Aharonov-Casher proposal appears in Ref. 5. According to the authors the new effect has a "duality"[7] with the original solenoid Aharonov-Bohm proposal. In the original solenoid effect, charged particles passed around a neutral magnetic solenoid to give an interference effect; in the Aharonov-Casher effect, neutral particles with magnetic moments pass around a charged line to give interference effects. Aharonov and Casher write,[8] "In particular, a magnetic moment moving in the field of a straight homogeneous charged line feels no force and undergoes an A-B effect; the A-B phase is [in Eq. (13)]

$$S_{\mathrm{AB}} = -\oint e\,\mathbf{A}(\mathbf{r}-\mathbf{R})\cdot d\mathbf{R} = \mu\lambda$$

(Heaviside-Lorentz units with $\hbar = c = 1$) where λ is the charge per unit length on the line and μ the projection of the magnetic moment along the line."

This Aharonov-Casher effect is currently under experimental investigation by a collaboration[9] using a beam of neutrons and a Bonse-Hart perfect silicon crystal interferometer. In the course of describing his experiment recently, Professor Werner pointed out[10] that the magnetic dipole $\boldsymbol{\mu}$ passing the line charge has an interaction $\boldsymbol{\mu}\cdot\mathbf{B}$, where $\mathbf{B} \cong (\mathbf{v}/c)\times\mathbf{E}$ is the magnetic field seen in the instantaneous rest frame of the particle where the line charge λ is moving with velocity $\mathbf{v}$. Now the interaction looks like a classical energy; if there is a classical energy involved in the rest frame of the particle, then there must also be forces on the particles and these forces must be evident in any inertial frame. In other words, the claim of Aharonov and Casher that the passing magnetic dipole experiences no classical forces bears investigation. In this paper we will show that the proposed Aharonov-Casher effect can be described naturally as a relative lag effect caused by classical forces on the passing magnetic dipoles when the dipoles are described by the usual electric-current model.

CLASSICAL MAGNETIC DIPOLES PASSING A LINE CHARGE

In order to understand the Aharonov-Casher proposal as a classical lag effect, we will calculate the classical electromagnetic forces[11] on a neutral particle of mass m which has a magnetic dipole moment $\boldsymbol{\mu}$ in its instantaneous rest frame. Now in the electric-current model a magnetic dipole must be described in terms of a second-rank Lorentz tensor $M^{\mu\nu}$; in a frame in which the particle is moving with a velocity $\mathbf{v}$ the magnetic dipole $\boldsymbol{\mu}$ acquires an electric dipole moment $\mathbf{p}$ given by[12]

$$\mathbf{p}\cong(\mathbf{v}/c)\times\boldsymbol{\mu} \tag{1}$$

in the nonrelativistic approximation. This electric dipole will then interact with the electrostatic field $\mathbf{E}$ of the line charge according to the force law,

$$\mathbf{F}=(\mathbf{p}\cdot\nabla)\mathbf{E}\ . \tag{2}$$

We will consider a particle of initial velocity $\mathbf{v}_0=v_0\hat{\mathbf{j}}$ moving parallel to the y axis in the xy plane which passes an infinite line of charge λ per unit length along the z axis. The passing particle moves initially along the line $x=d$, $z=0$, or along $x=-d$, $z=0$, corresponding to passing on opposite sides of the line charge. We will assume that the interaction between the magnetic dipole and the line charge is small so that any departure from uniform motion can be regarded as a small correction to the rectilinear motion. For convenience we will take the magnetic dipole $\boldsymbol{\mu}$ as oriented along the z axis, $\boldsymbol{\mu}=\mu\hat{\mathbf{k}}$. Then the electric dipole moment $\mathbf{p}$ in Eq. (1) is parallel to the x axis,

$$\mathbf{p}\cong\hat{\mathbf{i}}v_0\mu/c\ , \tag{3}$$

and the force $\mathbf{F}$ in Eq. (2) becomes

$$\mathbf{F}=(v_0\mu/c)(\partial\mathbf{E}/\partial x)\ , \tag{4}$$

where in Gaussian units the electric field of the line charge is

$$\mathbf{E}=\frac{2\lambda(\hat{\mathbf{i}}x+\hat{\mathbf{j}}y)}{x^2+y^2}\ . \tag{5}$$

The change in velocity $\mathbf{v}-\mathbf{v}_0=\Delta\mathbf{v}$ of the magnetic dipole particle can be found from its acceleration $\mathbf{a}=\mathbf{F}/m$ with $\mathbf{F}$ as in (4) and (5)

$$\Delta\mathbf{v}=\frac{1}{m}\int_{-\infty}^{t}dt'\frac{v_0\mu}{c}\frac{2\lambda[\hat{\mathbf{i}}(y^2-d^2)\mp\hat{\mathbf{j}}2dy]}{(d^2+y^2)^2}\ . \tag{6}$$

Here we have substituted $x=\pm d$, corresponding to the dipole passing on opposite sides of the line charge. Now in the approximation of nearly uniform motion for the magnetic dipole, we may write $dt'=dy'/v_0$ and may integrate from $y'=-\infty$ to $y'=y(t)$ to obtain

$$\Delta\mathbf{v}(t)\cong\frac{2\mu\lambda}{mc}\int_{-\infty}^{y}d\mathbf{y}'\left[\frac{\hat{\mathbf{i}}(y'^2-d^2)}{(d^2+y'^2)^2}\mp\frac{\hat{\mathbf{j}}2dy'}{(d^2+y'^2)^2}\right]$$
$$=\frac{2\mu\lambda}{mc}\left[\frac{-\hat{\mathbf{i}}y}{d^2+y^2}\pm\frac{\hat{\mathbf{j}}d}{d^2+y^2}\right]\ . \tag{7}$$

Next the relative displacement $\Delta\mathbf{r}$ of the particle from the position for uniform motion is $\Delta\mathbf{r}(t)=\int_{-\infty}^{t}dt'\Delta\mathbf{v}(t')$, and when the particle is far past the line charge this is

$$\Delta\mathbf{r}\cong\int_{-\infty}^{\infty}\frac{d\mathbf{y}}{\mathbf{v}_0}\frac{2\mu\lambda}{mc}\left[\frac{-\hat{\mathbf{i}}y}{d^2+y^2}\pm\frac{\hat{\mathbf{j}}d}{d^2+y^2}\right]$$
$$=\pm\hat{\mathbf{j}}\frac{2\pi\mu\lambda}{mcv_0}\ . \tag{8}$$

Thus we find that there is a relative displacement in the direction of motion which is in opposite directions for particles passing on opposite sides of the line charge λ. The relative displacement Δy for particles which pass on opposite sides of the line charge is thus double the result of (8),

$$\Delta y=\frac{4\pi\mu\lambda}{mcv_0}\ . \tag{9}$$

In the analysis above we have taken the frame of the line charge λ as the observer's inertial frame. We can also go to the inertial frame at rest with respect to the uniformly moving magnetic dipole. In this frame, the electric line charge is moving with velocity $\mathbf{v}_\lambda=-v_0\hat{\mathbf{j}}$ and produces a magnetic field

$$\mathbf{B}\cong(-\mathbf{v}_0/c)\times\mathbf{E}\ , \tag{10}$$

where $\mathbf{E}$ is the field of the line charge in its own rest frame given in (5). Then the force on the magnetic dipole $\boldsymbol{\mu}$ is[13]

$$\mathbf{F}=\nabla(\boldsymbol{\mu}\cdot\mathbf{B})\ . \tag{11}$$

For our situation $\boldsymbol{\mu}=\mu\hat{\mathbf{k}}$ and $\mathbf{B}$ in Eqs. (5) and (10) is

$$\mathbf{B}=\frac{\hat{\mathbf{k}}v_0 2\lambda}{c}\frac{x}{x^2+y^2}\ , \tag{12}$$

giving, from Eq. (11),

$$\mathbf{F}=\left[\hat{\mathbf{i}}\frac{\partial}{\partial x}+\hat{\mathbf{j}}\frac{\partial}{\partial y}\right]\left[\frac{\mu v_0 2\lambda}{c}\frac{x}{x^2+y^2}\right]$$
$$=\frac{\mu v_0 2\lambda}{c}\frac{[\hat{\mathbf{i}}(y^2-x^2)-\hat{\mathbf{j}}2xy]}{(x^2+y^2)^2}\ . \tag{13}$$

This is exactly the force which we calculated earlier from Eqs. (4) and (5), and used in Eq. (6). The force in (13) will lead to the same relative displacement as found earlier in the other inertial frame.

AHARONOV-CASHER PHASE SHIFT

At this point we must go to a quantum interpretation in order to make contact with the phase shift S_{AB} computed by Aharonov and Casher. We think of two wave packets of de Broglie wavelength $\lambda_{\rm deB}=h/mv_0$. Then the relative displacement Δy in (9) introduces a relative phase

$$\frac{2\pi\Delta y}{\lambda_{\rm deB}}=2\pi\left[\frac{4\pi\mu\lambda}{mcv_0}\right]\frac{mv_0}{h}=\frac{4\pi\mu\lambda}{\hbar c}\ . \tag{14}$$

This agrees exactly with the result of Aharonov and Casher when we recognize that the 4π appears from our use of Gaussian rather than Heaviside units.

DISCUSSION OF AHARONOV-BOHM EFFECTS

The solenoid Aharonov-Bohm effect has fascinated physicists since 1959, and the idea of nonlocal topological effects in quantum theory has diffused widely into the theoretical literature. However, the topological viewpoint can be viewed with scepticism when it is realized that all but one of the experimentally observed effects can actually be accounted for by classical forces which lead to relative particle lags. Only the solenoid Aharonov-Bohm effect is claimed to involve no velocity changes for the particles passing along different paths and this claim is disputed.[14]

In 1984 Aharonov and Casher suggested an "Aharonov-Bohm effect" which they claimed did not involve classical forces; hence it could not be viewed as arising from a classical lag effect. In this paper we have shown that the Aharonov-Casher claim is in error. Actually their proposed effect has a transparent explanation as a classical lag effect using the usual electric-current model for a magnetic dipole. Thus the solenoid Aharonov-Bohm effect still stands alone in not allowing a *transparent* lag explanation.

It should be emphasized just how anomalous is the solenoid Aharonov-Bohm effect. All the other Aharonov-Bohm effect systems involve changes of particle velocities due to classical forces. According to the orthodox interpretation of the solenoid Aharonov-Bohm effect, there is no velocity change as the particles pass the solenoid, but this has never been verified experimentally. Furthermore, the solenoid Aharonov-Bohm effect is anomalous, as currently described, in declaring that the phase shift can never be broken down by increasing the flux in the solenoid. On the other hand, all of the Aharonov-Bohm effect systems based upon classical lag effects should show a breakdown in the interference pattern when the relative lag between the particles becomes comparable to the coherence length of the particles. This is a natural experimental consequence of the lag point of view which is obscured in the topological point of view. Indeed a breakdown in the interference pattern for neutron beams by the introduction of a retarding potential in one beam of a neutron interferometer has been reported.[15] It corresponds to a breakdown when the relative lag between particles on the different paths reaches a coherence length $\Delta y \sim \hbar/\Delta p_y$. Does one observe a breakdown of the interference pattern for large solenoid fluxes in the solenoid Aharonov-Bohm effect? Such a breakdown does not appear in the usual quantum description. If such a breakdown is observed, then the usual no-lag interpretation of the solenoid Aharonov-Bohm effect is untenable.

Note added in proof. J. D. Jackson has shown that all known intrinsic magnetic moments (of electron, muon, proton, neutron, nuclei) are caused, to very high precision, by circulating electron currents and not by magnetic charges [CERN Report No. 77-17, Theory Division, 1977 (unpublished)]. I wish to thank Professor Jackson for sending me a copy of his CERN report.

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[1]Y. Aharonov and D. Bohm, Phys. Rev. **115**, 485 (1959).

[2]See the review by S. Olarius and J. Jovitzu Popescu, Rev. Mod. Phys. **57**, 339 (1985).

[3]R. Colella, A. W. Overhauser, and S. A. Werner, Phys. Rev. Lett. **34**, 1472 (1974).

[4]S. Matteucci and G. Pozzi, Phys. Rev. Lett. **54**, 2469 (1985).

[5]Y. Aharonov and A. Casher, Phys. Rev. Lett. **53**, 319 (1984).

[6]Magnetic dipoles can be modeled within classical physics by electric-current loops or by magnetic charges of opposite sign separated in space. The electric-current loop description, the one usually used in physics, involves classical forces on a magnetic dipole passing a line charge. The separated magnetic charge description, with $\nabla\cdot\mathbf{B}\neq 0$ inside the magnetic dipole, involves no forces on a magnetic dipole passing a line charge. In the present article the usual electric-current loop description is used for the magnetic dipole. Presumably the *ad hoc* Lagrangian introduced by Aharonov and Casher in Ref. 5 involves a magnetic dipole fitting the magnetic charge model. A detailed discussion of the differences between the two magnetic dipole models will appear in Am. J. Phys. (to be published).

[7]See Ref. 5, p. 320. This duality is made more explicit by A. G. Klein, Physica (Utrecht) **137B**, 230 (1986). Actually the appropriate duality with the no-force situation claimed in the Aharonov-Casher paper (Ref. 5) involves a "solenoid" made of magnetic dipoles which are formed of magnetic monopole charges. This is not the situation of the experimental observations for the Aharonov-Bohm effect using microsolenoids.

[8]See Ref. 5, p. 320.

[9]This collaboration consists of S. A. Werner, H. Kaiser, and M. Arif at the University of Missouri, and A. G. Klein, A. Commino, and G. I. Opat at the University of Melbourne.

[10]This observation also appears in Klein's article in Ref. 7.

[11]The forces on the magnetic dipole in the usual electric-current model can be calculated in fully Lorentz-covariant

notation using $K_\alpha = \frac{1}{2} M^{\beta\gamma} \partial_\alpha \mathcal{F}_{\beta\gamma}$. Here K_α is the four-vector force on the dipole $M^{\beta\gamma}$, where $M^{\beta\gamma}$ reduces to a magnetic dipole $\boldsymbol{\mu}$ and vanishing electric dipole $\mathbf{p}=0$ in the instantaneous particle rest frame, and $\mathcal{F}_{\beta\gamma}$ is the electromagnetic field tensor. See H. C. Corben, *Classical and Quantum Theories of Spinning Particles* (Holden-Day, San Francisco, 1968), p. 69, Eq. (6.18).

[12]See J. D. Jackson, *Classical Electrodynamics,* 1st ed. (Wiley, New York, 1962), p. 389, Problem 11.9.

[13]See J. D. Jackson, *Classical Electrodynamics,* 2nd ed. (Wiley, New York, 1975), p. 185, Eq. (5.69).

[14]See T. H. Boyer, Phys. Rev. D **8**, 1680 (1973).

[15]K. Kaiser, S. A. Werner, and E. A. George, Phys. Rev. Lett. **50**, 560 (1983).