Time dependence of the field energy densities surrounding sources: Application to scalar mesons near point sources and to electromagnetic fields near molecules

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The time dependence of the dressing-undressing process, i.e., the acquiring or losing by a source of a boson field intensity and hence of a field energy density in its neighborhood, is considered by examining some simple soluble models. First, the loss of the virtual field is followed in time when a point source is suddenly decoupled from a neutral scalar meson field. Second, an initially bare point source acquires a virtual meson cloud as the coupling is switched on. The third example is that of an initially bare molecule interacting with the vacuum of the electromagnetic field to acquire a virtual photon cloud. In all three cases the dressing-undressing is shown to take place within an expanding sphere of radius $r=ct$ centered at the source. At each point in space the energy density tends, for large times, to that of the ground state of the total system. Differences in the time dependence of the dressing between the massive scalar field and the massless electromagnetic field are discussed. The results are also briefly discussed in the light of Feinberg's ideas on the nature of half-dressed states in quantum field theory.

I. INTRODUCTION

A scalar boson field surrounding a fixed point source of coupling strength g has the well-known steady-state value

$$\langle \tilde{0} \mid \phi(\mathbf{r}) \mid \tilde{0} \rangle = g\frac{\exp(-\mu r)}{4\pi c^2 r} , \tag{1.1}$$

where the source is assumed fixed at the origin, $|\tilde{0}\rangle$ being the ground state of the complete source-field system, and where μ is the inverse Compton wave length of the scalar boson $\mu=mc/\hbar$. This implies that in the neighborhood of the source the boson field contributes to a field energy density, this can be evaluated from the known expression for the Hamiltonian density

$$\mathcal{H}_F(\mathbf{r})=\tfrac{1}{2}\{\pi^2(\mathbf{r})+c^2[\nabla\phi(\mathbf{r})]^2+\mu^2\phi^2(\mathbf{r})\} \tag{1.2}$$

and is

$$\langle \tilde{0} \mid \mathcal{H}_F(\mathbf{r}) \mid \tilde{0} \rangle = g^2\frac{\exp(-2\mu r)}{32\pi^2 c^2 r^4}(2\mu^2 r^2+2\mu r+1) . \tag{1.3}$$

$\pi^2(\mathbf{r})$ has zero expectation value in the ground state, so that the contributions to (1.3) arise only from the second and third terms in the field energy density (1.2).

In nonrelativistic quantum electrodynamics of the interaction of radiation with neutral atoms or molecules there is a well-defined energy density of the virtual electromagnetic fields in the neighborhood of the source. These fields and their concomitant energy densities have been considered in a series of recent papers.[1–4] The analogue to the scalar energy-density equation (1.2) is the sum of transverse electric and magnetic energy densities (1.4),

$$\mathcal{H}_F(\mathbf{r})=\mathcal{H}_{\text{elec}}(\mathbf{r})+\mathcal{H}_{\text{mag}}(\mathbf{r}) = \frac{1}{8\pi}[\mathbf{E}^{\perp 2}(\mathbf{r})+\mathbf{B}^2(\mathbf{r})] . \tag{1.4}$$

In particular we have shown[5] that at large distances from a molecule with static nonisotropic polarizability α_{mn} the virtual magnetic energy density is, to order e^2,

$$\langle \tilde{0} \mid \mathcal{H}_{\text{mag}}(\mathbf{r}) \mid \tilde{0} \rangle = -\frac{7}{32\pi^2}\hbar c\,\alpha_{mn}\frac{\delta_{mn}-\hat{r}_m\hat{r}_n}{r^7} \tag{1.5}$$

and that for an isotropic source with polarizability α the total virtual radiation energy density at large distances is

$$\langle \tilde{0} \mid \mathcal{H}_F(\mathbf{r}) \mid \tilde{0} \rangle = \frac{\hbar c\alpha}{\pi^2 r^7} . \tag{1.6}$$

A natural question arises, which has close connection with the more general problem of "half-dressed states" in quantum-field theory,[6] namely, how the virtual energy density is established in the space surrounding the source and how it changes when the source itself is subjected to changes in its dynamical state. Switching off a fixed source in a Klein-Gordon field such as (1.2) is known[7] to lead to the release of real mesons originally belonging to the virtual cloud, but processes of this sort do not seem to have been investigated from the point of view of the time development of the energy density surrounding the source. Although there have been detailed calculations of the electromagnetic field surrounding an atom during the emission of a real photon such as those by Bykov and Zadernovski[8] and by Power and Thirunamachandran[1] it would seem to be desirable to investigate also the time-dependent changes in the virtual radiation clouds due to sudden dynamical changes experienced by atomic or molecular sources.

As a preliminary to the electromagnetic field problem we first calculate, in Secs. II and III, the time developments of the energy density for a neutral boson field, i.e., Eq. (1.2), consequent upon well-defined sudden prescribed changes in a point source. The two models chosen are those where, for Sec. II, the source is completely removed or suddenly decoupled from the field and, for Sec. III, the building up of the virtual cloud around a meson source which is initially bare. We show that in the first case the meson cloud will eventually disappear, but the energy density at distance r from the source will remain until time $t=r/c$ has elapsed from the switching off of the source. In the second case we show that indeed the shape of the energy density tends to the ground-state shape given by Eq. (1.3) as $t\to\infty$, but for finite times the virtual cloud is absent at distances $r>ct$, in agreement with simple relativistic requirements. In Sec. IV we turn to the electromagnetic case and calculate the time-dependent dressing of a molecule. Both the transverse electric and the magnetic energy densities are computed for a molecule initially bare. The simplifying assumption that the coupling of the molecule to the radiation field is through the static polarizability is made throughout. This restricts the validity of the resulting expressions to large distances compared to optical wavelengths. There is no difficulty in principle to determine these energy densities at closer approach using the multipolar Hamiltonian but this would not change the basic time developments that occur. The steady-state energy densities given by Eq. (1.6) are the values obtained in the time-dependent case after a time r/c. In Sec. V we compare the electromagnetic case with the corresponding meson case, indicating the differences and similarities, and summarize our conclusions.

II. BOSON SOURCE REMOVED AT $t=0$

In this section we follow the time dependence of the energy density (1.2) for the scalar meson field after the interaction with a source is suddenly switched off. Of the three separate terms the canonical momentum term can be evaluated in terms of Bessel functions. The remaining terms cannot be found in closed form but in terms of an integral of the Bessel function of zero order with a square root as argument.

In terms of the usual creation and annihilation operators the boson field amplitude is

$$\phi(\mathbf{r})=\sum_{\mathbf{k}}\left[\frac{\hbar}{2V\omega_k}\right]^{1/2}(a_{\mathbf{k}}e^{i\mathbf{k}\cdot\mathbf{r}}+a^{\dagger}_{\mathbf{k}}e^{-i\mathbf{k}\cdot\mathbf{r}}),\quad \omega_k=c(k^2+\mu^2)^{1/2},\tag{2.1}$$

where V is the quantization volume. The total Hamiltonian, before the source is removed, is

$$H=H_0+H',\quad H_0=\int\mathcal{H}_F(\mathbf{r})dV,\quad H'=-g\int\rho(\mathbf{r})\phi(\mathbf{r})dV,\tag{2.2}$$

where $\mathcal{H}_F$ is given by Eq. (1.3) and $\rho(\mathbf{r})$ is the source density. From (2.1) we obtain

$$H_0=\sum_{\mathbf{k}}\hbar\omega_k a^{\dagger}_{\mathbf{k}}a_{\mathbf{k}}+H_{\mathrm{ZPT}},\quad H'=-g\sum_{\mathbf{k}}\left[\frac{\hbar}{2V\omega_k}\right]^{1/2}(\rho^*_{\mathbf{k}}a_{\mathbf{k}}+\rho_{\mathbf{k}}a^{\dagger}_{\mathbf{k}}).\tag{2.3}$$

The zero-point terms (ZPT) will be ignored in all subsequent developments. In (2.3) $\rho_{\mathbf{k}}$ is the spatial Fourier transform of $\rho(\mathbf{r})$. As is well known[9] the ground state of total H (i.e., the physical or dressed ground state) is given by

$$|\tilde{0}\rangle=T|0\rangle,\quad T=e^{-\bar{n}/2}e^{\sum_{\mathbf{k}}\chi_{\mathbf{k}}a^{\dagger}_{\mathbf{k}}}e^{-\sum_{\mathbf{k}}\chi^*_{\mathbf{k}}a_{\mathbf{k}}},\tag{2.4}$$

where

$$\bar{n}=g^2\sum_{\mathbf{k}}\frac{|\rho_{\mathbf{k}}|^2}{2\hbar\omega_k^3V},\quad \chi_{\mathbf{k}}=g\frac{\rho_{\mathbf{k}}}{(2\omega_k^3V\hbar)^{1/2}},\tag{2.5}$$

and $|0\rangle$ is the ground state of H_0 (i.e., the bare ground state). Dressing the bare operators $a_{\mathbf{k}}$ is easily accomplished by the unitary operator T through

$$T^{-1}a_{\mathbf{k}}T=a_{\mathbf{k}}+\chi_{\mathbf{k}},\quad T^{-1}a^{\dagger}_{\mathbf{k}}T=a^{\dagger}_{\mathbf{k}}+\chi^*_{\mathbf{k}}.\tag{2.6}$$

The conditions proposed within the model considered are that the initial state, at $t=0$, is the dressed ground state $|\tilde{0}\rangle$ but that for $t>0$ g vanishes and the field evolves freely according to

$$e^{-iH_0t/\hbar}T|0\rangle.\tag{2.7}$$

Hence the quantum average of any operator Q at time t (>0) is

$$\langle Q\rangle_t=\langle 0|T^{-1}e^{iH_0t/\hbar}Qe^{-iH_0t/\hbar}T|0\rangle.\tag{2.8}$$

For $Q=\pi^2(\mathbf{r})$ and after some algebra (2.8) leads to

$$\langle\pi^2(\mathbf{r})\rangle_t=\frac{\hbar}{2V}\sum_{\mathbf{k},\mathbf{k}'}\sqrt{\omega_k\omega_{k'}}(\chi_{\mathbf{k}}\chi^*_{\mathbf{k}'}e^{i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{r}-i(\omega_k-\omega'_k)t}-\chi_{\mathbf{k}}\chi_{\mathbf{k}'}e^{i(\mathbf{k}+\mathbf{k}')\cdot\mathbf{r}-i(\omega_k+\omega_{k'})t}+\text{c.c.}),\tag{2.9}$$

which becomes in the usual limit of large V that turns the $\mathbf{k}$ sums to integrals,

$$\langle\pi^2(\mathbf{r})\rangle_t=\frac{1}{2}\frac{1}{(2\pi)^6}g^2\mathrm{Re}\int\frac{d^3\mathbf{k}\,d^3\mathbf{k}'}{\omega_k\omega_{k'}}(\rho_{\mathbf{k}}\rho^*_{\mathbf{k}'}e^{i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{r}-i(\omega_k-\omega_{k'})t}-\rho_{\mathbf{k}}\rho_{\mathbf{k}'}e^{i(\mathbf{k}+\mathbf{k}')\cdot\mathbf{r}-i(\omega_k+\omega_{k'})t})$$
$$=\frac{1}{(2\pi)^6}g^2\left[\mathrm{Im}\int\frac{1}{\omega_k}\rho_{\mathbf{k}}e^{i\mathbf{k}\cdot\mathbf{r}-i\omega_kt}d^3\mathbf{k}\right]^2.\tag{2.10}$$

Similarly we find

$$\langle \phi^2(\mathbf{r}) \rangle_t = \frac{1}{(2\pi)^6} g^2 \left[\mathrm{Re} \int \frac{1}{\omega_k^2} \rho_k e^{i\mathbf{k}\cdot\mathbf{r} - i\omega_k t} d^3\mathbf{k} \right]^2 \tag{2.11}$$

and

$$\langle [\nabla\phi(\mathbf{r})]^2 \rangle_t = \frac{1}{(2\pi)^6} g^2 \left[\mathrm{Re}\, \nabla \int \frac{1}{\omega_k^2} \rho_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{r} - i\omega_k t} d^3\mathbf{k} \right]^2 . \tag{2.12}$$

For the case of a point source with $\rho(\mathbf{r}) = \delta^3(\mathbf{r})$, so that $\rho_{\mathbf{k}} = 1$, the field energy densities can all be expressed in terms of the two integrals

$$I_1 = \int \frac{d^3\mathbf{k}}{\omega_k} e^{i(\mathbf{k}\cdot\mathbf{r} - \omega_k t)}, \quad I_2 = \int \frac{d^3\mathbf{k}}{\omega_k^2} e^{i(\mathbf{k}\cdot\mathbf{r} - \omega_k t)} . \tag{2.13}$$

The angular integration is straightforward and if we define the function

$$F(t,r) = J_0[\mu c (t^2 - r^2/c^2)^{1/2}], \quad t > r/c$$
$$= 0, \quad t < r/c , \tag{2.14}$$

where J_0 is the zeroth-order Bessel function. We have

$$\mathrm{Im} I_1 = \frac{2\pi^2}{cr} \frac{\partial}{\partial r} F(t,r) ,$$

and

$$\mathrm{Re} I_2 = \frac{2\pi^2}{c^2 r} \frac{\partial}{\partial r} \left[-\frac{e^{-\mu r}}{\mu} + c \int_0^t F(t',r) dt' \right] .$$

Then the energy densities (2.10), (2.11), and (2.12) become

$$\langle \pi^2(\mathbf{r}) \rangle_t = \frac{1}{16\pi^2 c^2} g^2 \frac{1}{r^2} \left[\frac{\partial}{\partial r} F(t,r) \right]^2 , \tag{2.15}$$

$$\langle \phi^2(\mathbf{r}) \rangle_t = \frac{1}{16\pi^2 c^4} g^2 \frac{1}{r^2} \times \left[\frac{\partial}{\partial r} \left[\frac{e^{-\mu r}}{\mu} - c \int_0^t F(t',r) dt' \right] \right]^2 , \tag{2.16}$$

and

$$\langle [\nabla\phi(\mathbf{r})]^2 \rangle_t = \frac{1}{16\pi^2 c^4} g^2 \left[\frac{\partial}{\partial r} \frac{1}{r} \frac{\partial}{\partial r} \left[\frac{e^{-\mu r}}{\mu} - c \int_0^t F(t',r) dt' \right] \right]^2 . \tag{2.17}$$

The energy density due to the conjugate field $\pi(\mathbf{r})$ can be reduced further,

$$\langle \pi^2(\mathbf{r}) \rangle_t = \frac{1}{16\pi^2 c^4} g^2 \times \mu^2 \frac{\{J_1[\mu c (t^2 - r^2/c^2)^{1/2}]\}^2}{t^2 - r^2/c^2} , \quad t > r/c$$
$$= 0, \quad t < r/c . \tag{2.18}$$

Because $F(t',r)$ vanishes for $t' < r/c$ the integrals in (2.16) and (2.17) remain zero at points r as long as $r > ct$, where t is the minimum time required for any information to travel from the origin to r. Until the time, all the F terms vanish and the total energy density as calculated from the sum of (2.15), (2.16), and (2.17) coincides with the initial value given by Eq. (1.3). We see explicitly that the energy density at a point r remains unchanged even after decoupling has occurred between the source and the field until information of this event can reach point r. On the other hand, we can evaluate (see, for example, Gradshteyn and Ryznik[10]) the limiting values of the integral over t' for large t. We have

$$\int_0^\infty F(t',r) dt' = \int_{r/c}^\infty J_0[\mu c (t'^2 - r^2/c^2)^{1/2}] dt'$$
$$= \frac{e^{-\mu r}}{\mu c} . \tag{2.19}$$

Thus the $\langle \phi^2(\mathbf{r}) \rangle$ and $\langle [\nabla\phi(\mathbf{r})]^2 \rangle$ components of the energy density vanish for all r as $t \to \infty$. Also, from Eq. (2.18), the momentum term vanishes in the same limit. We may conclude that, as expected, the energy density vanishes at any distance r from the origin after sufficiently long times has elapsed, and that it remains equal to its initial value as long as $t < r/c$. It is to be noted that "sufficiently long time" means, due to the argument of the Bessel functions, $t \gg (r^2 + \hbar^2/m^2c^2)^{1/2}/c$.

III. BOSON SOURCE APPEARING AT $t = 0$

In this section the problem solved in Sec. II is essentially reversed. The source is assumed to be bare at $t = 0$ so that the initial state is $|0\rangle$ but for $t > 0$ this develops in time according to Schrödinger's equation with total H as the Hamiltonian. The state at time t is thus

$$e^{-iHt/\hbar} |0\rangle \tag{3.1}$$

with H given by Eq. (2.2). Using Eq. (2.6), it is easy to show that

$$H = T H_0 T^{-1} - A , \tag{3.2}$$

where A is the c number (3.3),

$$A = \frac{1}{2V} g^2 \sum_{\mathbf{k}} \frac{|\rho_{\mathbf{k}}|^2}{\omega_k^2} . \tag{3.3}$$

Hence (3.1) is

$$e^{iAt/\hbar} T e^{-iH_0 t/\hbar} T^{-1} |0\rangle \tag{3.4}$$

and the quantum average of any operator Q in this state is

$$\langle Q \rangle_t = \langle 0 | T e^{iH_0 t/\hbar} T^{-1} Q T e^{-iH_0 t/\hbar} T^{-1} | 0 \rangle . \tag{3.5}$$

In practice it is easiest to evaluate $T^{-1}QT$ first using (2.6) and proceed in the manner outlined in Sec. II. In particular, the kinetic energy density is

$$\langle \pi^2(\mathbf{r})\rangle_t = \frac{\hbar}{2V}\sum_{\mathbf{k},\mathbf{k}'}\sqrt{\omega_k\omega_{k'}}[\chi_k\chi_{k'}^*(1+e^{-i(\omega_k-\omega_{k'})t}-e^{-i\omega_k t}-e^{-i\omega_{k'}t})e^{i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{r}}$$
$$-\chi_k\chi_{k'}(1+e^{-i(\omega_k+\omega_{k'})t}-e^{-i\omega_k t}-e^{-i\omega_{k'}t})e^{i(\mathbf{k}+\mathbf{k}')\cdot\mathbf{r}}] , \quad (3.6)$$

which, for an infinite quantization volume and a point source, becomes

$$\langle \pi^2(\mathbf{r})\rangle_t = \frac{1}{2}\frac{1}{(2\pi)^6}g^2\,\mathrm{Re}\left[\left|\int \frac{1-e^{-i\omega_k t}}{\omega_k}e^{i\mathbf{k}\cdot\mathbf{r}}d^3\mathbf{k}\right|^2 - \left[\int \frac{1-e^{-i\omega_k t}}{\omega_k}e^{i\mathbf{k}\cdot\mathbf{r}}d^3\mathbf{k}\right]^2\right]$$
$$= \frac{1}{(2\pi)^6}g^2\left[\mathrm{Im}\int \frac{1-e^{i\omega_k t}}{\omega_k}e^{i\mathbf{k}\cdot\mathbf{r}}d^3\mathbf{k}\right]^2 . \quad (3.7)$$

We notice that Eqs. (3.7) and (2.10) differ only by the substitution of the time factor $(1-e^{-i\omega_k t})$ in place of $e^{-i\omega_k t}$. The same is true for the other two terms in the energy density $\langle \phi^2(\mathbf{r})\rangle_t$ and $\langle [\nabla\phi(\mathbf{r})]^2\rangle_t$. Consequently,

$$\langle \phi^2(\mathbf{r})\rangle_t = \frac{g^2}{(2\pi)^6}\left[\mathrm{Re}\int \frac{1}{\omega_k^2}(1-e^{-i\omega_k t})e^{i\mathbf{k}\cdot\mathbf{r}}d^3\mathbf{k}\right]^2 \quad (3.8)$$

and

$$\langle [\nabla\phi(\mathbf{r})]^2\rangle_t = \frac{g^2}{(2\pi)^6}\left[\mathrm{Re}\nabla\int \frac{1}{\omega_k^2}(1-e^{-i\omega_k t})e^{i\mathbf{k}\cdot\mathbf{r}}d^3\mathbf{k}\right]^2 . \quad (3.9)$$

Integration of the time-independent part of (3.7) yields

$$\int \frac{e^{i\mathbf{k}\cdot\mathbf{r}}}{\omega_k}d^3\mathbf{k} = \frac{4\pi}{rc}\int_0^\infty \frac{k\sin(kr)dk}{(k^2+\mu^2)^{1/2}} = -\frac{4\pi}{rc}\frac{\partial}{\partial r}K_0(\mu r) , \quad (3.10)$$

which, being real does not contribute to (3.7). Hence the conjugate momentum contribution to the energy density is the same as in Sec. II, i.e., that given by Eq. (2.15),

$$\langle \pi^2(\mathbf{r})\rangle_t = \frac{1}{16\pi^2c^2}g^2\frac{1}{r^2}\left[\frac{\partial F}{\partial r}(t,r)\right]^2 . \quad (3.11)$$

However, the time-independent part of (3.8) is

$$\int \frac{e^{i\mathbf{k}\cdot\mathbf{r}}}{\omega_k^2}d^3\mathbf{k} = \frac{4\pi}{rc^2}\int_0^\infty \frac{k\sin(kr)}{k^2+\mu^2}dk = \frac{-2\pi^2}{rc^2}\frac{\partial}{\partial r}\left[\frac{e^{-\mu r}}{\mu}\right], \quad (3.12)$$

whereas the time-dependent part is the negative of (2.16). Consequently we obtain

$$\langle \phi^2(\mathbf{r})\rangle_t = \frac{1}{16\pi^2}\frac{g^2}{c^2}\frac{1}{r^2}\left[\frac{\partial}{\partial r}\int_0^t F(t',r)dt'\right]^2 . \quad (3.13)$$

Finally, proceeding in a similar way, we have

$$\langle [\nabla\phi(\mathbf{r})]^2\rangle_t = \frac{1}{16\pi^2}\frac{g^2}{c^2}\left[\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial}{\partial r}\int_0^t F(t',r)dt'\right]^2 . \quad (3.14)$$

The causal behavior of the energy density of the boson virtual cloud is exactly as expected. Since $\int_0^t F(t',r)dt'$ vanishes for any $r>ct$, the total energy density $\langle \mathcal{H}_F(\mathbf{r})\rangle_t$ is zero at these points, and we have a buildup of field energy at all points $r<ct$. On the basis of (2.18) and (2.19) the energy density tends asymptotically to the ground-state value equation (1.3). Thus the dressing of the initially bare source proceeds outwards with the speed of light but it is complete at any point in space only asymptotically—in the sense that it is essentially complete when $t \gg [(r^2+\hbar^2/m^2c^2)]^{1/2}/c$.

IV. TIME-DEPENDENT DRESSING OF A MOLECULE

In terms of photon annihilation and creation operators, the amplitudes of the electromagnetic field are

$$\mathbf{E}^\perp(\mathbf{r}) = i\sum_{\mathbf{k},j}\left[\frac{2\pi\hbar\omega_k}{V}\right]^{1/2}(\mathbf{e}_{\mathbf{k},j}a_{\mathbf{k},j}e^{i\mathbf{k}\cdot\mathbf{r}}-\mathbf{e}^*_{\mathbf{k},j}a^\dagger_{\mathbf{k},j}e^{-i\mathbf{k}\cdot\mathbf{r}}) , \quad \omega_k = ck$$
$$B(r) = i\sum_{\mathbf{k},j}\left[\frac{2\pi\hbar\omega_k}{V}\right]^{1/2}(\mathbf{b}_{\mathbf{k}j}a_{\mathbf{k}j}e^{i\mathbf{k}\cdot\mathbf{r}}-\mathbf{b}^*_{\mathbf{k},j}a^\dagger_{\mathbf{k},j}e^{-i\mathbf{k}\cdot\mathbf{r}}) , \quad (4.1)$$

where V is the quantization volume, $\mathbf{e}_{\mathbf{k},j}$ the unit vector defining the polarization, and $\mathbf{b}_{\mathbf{k},j}=\hat{\mathbf{k}}\times\mathbf{e}_{\mathbf{k}\cdot j}$. Within the multipolar-coupling formalism that we adopt here the matrix elements of $\mathbf{E}^\perp(\mathbf{r})$ correspond to those of the transverse Maxwell displacement field. For the field-molecule coupling we take the Craig-Power model,[11] whereby the molecule is taken as completely passive, and enters the interaction Hamiltonian only via the static polarizability tensor α_{mn}. Strictly speaking, this model can be considered realistic only to study the field in the radiation zone. However, we are interested within this paper in the model as a simplification (where the Maxwell fields can be easily found) of a more realistic but more complicated situation with a frequency-dependent polarizability. Thus we take

$$H = H_0 + H' ,$$
$$H_0 = \frac{1}{8\pi}\int \mathcal{H}_F(\mathbf{r})dV , \quad (4.2)$$
$$H' = -\tfrac{1}{2}\alpha_{mn}E_m^\perp(0)E_n^\perp(0) ,$$

where $\mathcal{H}_F$ is given by Eq. (1.4) and where the molecule is taken to be fixed at the origin. In terms of the expressions (4.1) we have

$$H_0=\sum_{\mathbf{k},j} a^\dagger_{\mathbf{k},j}a_{\mathbf{k},j}\hbar\omega_k \ ,$$

$$H'=\tfrac{1}{2}\alpha_{mn}\sum_{\substack{\mathbf{k}_1,j_1\\ \mathbf{k}_2,j_2}}\sqrt{\omega_{k_1}\omega_{k_2}}[(e_{\mathbf{k}_1,j_1})_m(e_{\mathbf{k}_2,j_2})_n a_{\mathbf{k}_1,j_1}a_{\mathbf{k}_2,j_2}-(e_{\mathbf{k}_1,j_1})_m(e^*_{\mathbf{k}_2,j_2})_n a_{\mathbf{k}_1,j_1}a^\dagger_{\mathbf{k}_2,j_2}+\text{H.c.}] \ , \tag{4.3}$$

and the transverse electric and magnetic densities that we investigate are (simplifying the notation $\mathbf{e}_1=e_{\mathbf{k}_1,j_1}$, $\mathbf{e}_2=e_{\mathbf{k}_2,j_2}$, $a_1=a_{\mathbf{k}_1,j_1}$, etc.)

$$\begin{aligned}\mathcal{H}_{\text{elec}}&=\frac{1}{8\pi}\mathbf{E}^{\perp 2}(\mathbf{r})\\ &=-\frac{\hbar}{4V}\sum_{\substack{\mathbf{k}_1,j_1\\ \mathbf{k}_2,j_2}}\sqrt{\omega_{k_1}\omega_{k_2}}(\mathbf{e}_1\cdot\mathbf{e}_2 a_1 a_2 e^{i(\mathbf{k}_1+\mathbf{k}_2)\cdot\mathbf{r}}\\ &\qquad -\mathbf{e}_1\cdot\mathbf{e}_2^* a_1 a_2^\dagger e^{i(\mathbf{k}_1-\mathbf{k}_2)\cdot\mathbf{r}}\\ &\qquad +\text{H.c.}) \ ,\end{aligned} \tag{4.4}$$

$$\begin{aligned}\mathcal{H}_{\text{mag}}&=\frac{1}{8\pi}\mathbf{B}^{2}(\mathbf{r})\\ &=-\frac{\hbar}{4V}\sum_{\substack{\mathbf{k}_1 j_1\\ \mathbf{k}_2,j_2}}\sqrt{\omega_{\mathbf{k}_1}\omega_{k_2}}(\mathbf{b}_1\cdot\mathbf{b}_2 a_1 a_2 e^{i(\mathbf{k}_1+\mathbf{k}_2)\cdot\mathbf{r}}\\ &\qquad -\mathbf{b}_1\cdot\mathbf{b}_2^* a_1 a_2^\dagger e^{i(\mathbf{k}_1-\mathbf{k}_2)\cdot\mathbf{r}}\\ &\qquad +\text{H.c.}) \ .\end{aligned} \tag{4.5}$$

For the initial ($t=0$) state of the system we take the bare vacuum $|0\rangle$. This state develops, for $t>0$, according to

$$\exp(-iHt/\hbar)\,|\,0\rangle \ , \tag{4.6}$$

where H is given by Eq. (4.2). The energy densities within the photon cloud are given, as functions of time, as

$$\begin{aligned}\langle\mathcal{H}^{(\mathbf{r})}_{\text{elec-mag}}\rangle_t&=\langle 0\,|\,e^{iHt/\hbar}\mathcal{H}_{\text{elec-mag}}(\mathbf{r})e^{-iHt/\hbar}\,|\,0\rangle\\ &=\langle 0\,|\,\mathcal{H}_{\text{elec-mag}}(\mathbf{r},t)\,|\,0\rangle \ .\end{aligned} \tag{4.7}$$

The interaction Hamiltonian (4.2) is more complicated than its analogue in the meson case and we resort to approximate methods. We carry out the calculation in the Heisenberg representation and obtain approximations to order e^2, i.e., one power of the polarizability α_{mn}, for $a(t)$ in (4.4) and (4.5) from which we can evaluate (4.7).

The Heisenberg equation for $a_1(t)$ is easily shown to be

$$\begin{aligned}i\hbar\dot a_1=\hbar\omega_1 a_1-\tfrac{1}{2}\alpha_{mn}\frac{2\pi\hbar}{V}\\ \times\Bigg[\sum_2\sqrt{\omega_1\omega_2}[\mathbf{e}_{2m}\mathbf{e}^*_{1n}+\mathbf{e}^*_{1m}\mathbf{e}_{2n}]a_2\\ -\sum_2\sqrt{\omega_1\omega_2}[\mathbf{e}^*_{2m}\mathbf{e}^*_{1n}+\mathbf{e}^*_{1m}\mathbf{e}^*_{2n}]a_2^\dagger\Bigg] \ .\end{aligned} \tag{4.8}$$

It is sufficient for our purposes to solve Eq. (4.8) to first order in α_{mn}. We find

$$\begin{aligned}a_1(t)=a_1(0)e^{-i\omega_1 t}+\tfrac{1}{2}\alpha_{mn}\frac{2\pi\hbar}{V}\sum_2\sqrt{\omega_1\omega_2}[(\mathbf{e}_{2m}\mathbf{e}^*_{1n}+\mathbf{e}^*_{1m}\mathbf{e}_{2n})e^{-i\omega_1 t}F_{k_1k_2}(t)a_2(0)\\ -(\mathbf{e}^*_{2m}\mathbf{e}^*_{1n}+\mathbf{e}^*_{1m}\mathbf{e}^*_{2n})e^{-i\omega_1 t}G_{k_1k_2}(t)a_2^\dagger(0)] \ ,\end{aligned} \tag{4.9}$$

where

$$F_{k_1k_2}(t)=\frac{e^{i(\omega_1-\omega_2)t}-1}{\omega_1-\omega_2}, \quad G_{k_1k_2}(t)=\frac{e^{i(\omega_1+\omega_2)t}+1}{\omega_1+\omega_2} \ . \tag{4.10}$$

If we use the symmetry relation $F_{k_1k_2}(t)=-F^*_{k_2k_1}(t)$ and neglect the zero-point terms and terms $O(\alpha^2_{mn})$, it is easy to obtain

$$\begin{aligned}\langle 0\,|\,a_1(t)a_2(t)\,|\,0\rangle&=\langle 0\,|\,a_1^\dagger(t)a_2^\dagger(t)\,|\,0\rangle=-\tfrac{1}{2}\alpha_{mn}\frac{2\pi}{V}\sqrt{\omega_1\omega_2}G_{k_1k_2}(t)e^{-i(\omega_1+\omega_2)t}[\mathbf{e}^*_{1m}\mathbf{e}^*_{2n}+\mathbf{e}^*_{2m}\mathbf{e}^*_{1n}] \ ,\\ \langle 0\,|\,a_1^\dagger(t)a_2(t)\,|\,0\rangle&=\langle 0\,|\,a_1(t)a_2^\dagger(t)\,|\,0\rangle=0 \ .\end{aligned} \tag{4.11}$$

The polarization sums in (4.4) and (4.5) can be affected using

$$\sum_{j_1j_2}(\mathbf{e}^*_{1m}\mathbf{e}^*_{2n}+\mathbf{e}^*_{2m}\mathbf{e}^*_{1n})\mathbf{e}_{1l}\mathbf{e}_{2l}=2\delta_{mn}-2\hat k_{1m}\hat k_{1n}-2\hat k_{2m}\hat k_{2n}+\hat{\mathbf{k}}_1\cdot\hat{\mathbf{k}}_2(\hat k_{1m}\hat k_{2n}+\hat k_{2m}\hat k_{1n}) \tag{4.12}$$

and

$$\sum_{j_1j_2}(\mathbf{e}^*_{1m}\mathbf{e}^*_{2n}+\mathbf{e}^*_{2m}\mathbf{e}^*_{1n})\mathbf{b}_{1l}\mathbf{b}_{2l}=\epsilon_{lkm}\epsilon_{lhn}(\hat k_{1k}\hat k_{2h}+\hat k_{1h}\hat k_{2k}) \ . \tag{4.13}$$

This leads to

$$\langle \mathbf{E}^{\perp 2}(\mathbf{r})\rangle_t = \frac{2\pi^2\hbar}{V^2}\alpha_{mn}\sum \omega_1\omega_2[2\delta_{mn} - 2\hat{k}_{1m}\hat{k}_{1n} - 2\hat{k}_{2m}\hat{k}_{2n} + \hat{\mathbf{k}}_1\cdot\hat{\mathbf{k}}_2(\hat{k}_{1m}\hat{k}_{2m} + \hat{k}_{2m}\hat{k}_{1n})] \times [G_{k_1k_2}(t)e^{-i(\omega_1+\omega_2)t}e^{i(\mathbf{k}_1+\mathbf{k}_2)\cdot\mathbf{r}} + \text{c.c.}] \tag{4.14}$$

and

$$\langle \mathbf{B}^2(\mathbf{r})\rangle_t = \frac{2\pi^2\hbar}{V^2}\alpha_{mn}\sum \omega_1\omega_2\epsilon_{lkm}\epsilon_{lhn}(\hat{k}_{1k}\hat{k}_{2h} + \hat{k}_{1h}\hat{k}_{2k})[G_{k_1k_2}(t)e^{-i(\omega_1+\omega_2)t}e^{i(\mathbf{k}_1+\mathbf{k}_2)\cdot\mathbf{r}} + \text{c.c.}] , \tag{4.15}$$

which can be transformed into the following integrals (4.16) and (4.17):

$$\langle \mathbf{E}^{\perp 2}(\mathbf{r})\rangle_t = \frac{2\pi^2\hbar c}{(2\pi)^6}\alpha_{mn}\lim_{\mathbf{r}'\to\mathbf{r}}[2\delta_{mn}\nabla^2\nabla'^2 - 2\nabla^2\nabla'_m\nabla'_n - 2\nabla'^2\nabla_m\nabla_n + (\nabla\cdot\nabla')(\nabla_m\nabla'_n + \nabla'_m\nabla_n)] \times \left[\int \frac{e^{i(\mathbf{k}\cdot\mathbf{r}+\mathbf{k}'\cdot\mathbf{r}')}}{k+k'}(1-e^{-i(k+k')ct})\frac{d^3k\,d^3k'}{kk'} + \text{c.c.}\right] \tag{4.16}$$

and

$$\langle \mathbf{B}^2(\mathbf{r})\rangle_t = \frac{2\pi^2\hbar c}{(2\pi)^6}\alpha_{mn}\lim_{\mathbf{r}'\to\mathbf{r}}(\nabla_m\nabla'_n + \nabla'_m\nabla_n - 2\delta_{mn}\nabla\cdot\nabla')\left[\int \frac{e^{i(\mathbf{k}\cdot\mathbf{r}+\mathbf{k}'\cdot\mathbf{r}')}}{k+k'}(1-e^{-i(k+k')ct})d^3k\,d^3k' + \text{c.c.}\right] , \tag{4.17}$$

where ∇ acts on $\mathbf{r}$ and ∇' acts on r'.

If $J(\mathbf{r},\mathbf{r}',t)$ is defined by Eq. (4.18),

$$J(\mathbf{r},\mathbf{r}',t) = \int \frac{e^{i(\mathbf{k}\cdot\mathbf{r}+\mathbf{k}'\cdot\mathbf{r}')}e^{-i(k+k')ct}}{k+k'}\frac{d^3k\,d^3k'}{kk'} + \text{c.c.} , \tag{4.18}$$

the term in small parentheses in Eq. (4.16) is $[J(\mathbf{r},\mathbf{r}',0) - J(\mathbf{r},\mathbf{r}',t)]$. Now

$$\frac{\partial J}{\partial t} = -icK(\mathbf{r},t)\cdot K(\mathbf{r}',t) + \text{c.c.} , \tag{4.19}$$

where

$$K(\mathbf{r},t) = \int e^{i\mathbf{k}\cdot\mathbf{r}}e^{-ikct}\frac{d^3k}{k} = \frac{2\pi}{r}\left[\frac{1}{r-ct} + \frac{1}{r+ct}\right]\frac{-2\pi^2}{r}i\delta(r-ct) . \tag{4.20}$$

Thus, except at $t = r/c$ and r'/c, $\partial J/\partial t$ is zero and, since

$$J(\mathbf{r},\mathbf{r}',\infty) = 0 \quad\text{and}\quad J(\mathbf{r},\mathbf{r}',0) = \frac{16\pi^3}{rr'}\frac{1}{(r+r')} ,$$

we have

$$J(\mathbf{r},\mathbf{r}',t) = \frac{16\pi^3}{rr'}\frac{1}{(r+r')} \quad \text{for } t = 0 \text{ to minimum of } r/c,\ r'/c$$
$$= 0 \quad \text{for } t = \text{maximum of } r/c,\ r'/c \text{ to } \infty . \tag{4.21}$$

It follows that, except for distribution singularities at $t = r/c$,

$$\langle \mathbf{E}^{\perp 2}(\mathbf{r})\rangle_t = \frac{\hbar c}{2\pi}\alpha_{mn}\lim_{\mathbf{r}'\to\mathbf{r}}[2\delta_{mn}\nabla^2\nabla'^2 - 2\nabla^2\nabla'_m\nabla'_n - 2\nabla'^2\nabla_m\nabla_n + (\nabla\cdot\nabla')(\nabla_m\nabla'_n + \nabla'_m\nabla_n)] \times \frac{1}{rr'(r+r')}\cdot[1-\Theta(r-ct)] . \tag{4.22}$$

The fourth-order derivatives, together with the limiting process, are elementary though tedious and we find, again for all times distinct from r/c when the fields are highly singular, that

$$\langle \mathbf{E}^{\perp 2}(\mathbf{r})\rangle_t = \frac{\hbar c\alpha_{mn}}{4\pi r^7}(13\delta_{mn} + 7\hat{r}_m\hat{r}_n)[1-\Theta(r-ct)] . \tag{4.23}$$

The analogous equations for $\langle \mathbf{B}^2(\mathbf{r})\rangle$ are

$$\langle \mathbf{B}^2(\mathbf{r})\rangle_t = -\frac{\hbar c}{\pi}\alpha_{mn}\lim_{\mathbf{r}'\to\mathbf{r}}(\nabla_m\nabla'_n + \nabla'_m\nabla_n - \delta_{mn}\nabla\cdot\nabla') \times \frac{1}{rr'(r+r')^3}[1-\Theta(r-ct)] \tag{4.24}$$

and

$$\langle \mathbf{B}^2(\mathbf{r})\rangle_t = -\frac{7\hbar c}{4\pi}\frac{\alpha_{mn}}{r^7}(\delta_{mn}-\hat{r}_m\hat{r}_n)[1-\Theta(r-ct)] \ . \quad (4.25)$$

The energy density of the radiation field at the point $\mathbf{r}$, namely, $\langle \mathcal{H}_F\rangle_t$ is easily obtained by dividing the sum of the transverse electric and magnetic terms (4.23) and (4.24) by 8π,

$$\langle \mathcal{H}_F\rangle_t = \frac{\hbar c}{16\pi^2}\alpha_{mn}\frac{3\delta_{mn}+7\hat{r}_m\hat{r}_n}{r^7}[1-\Theta(r-ct)] \ . \quad (4.26)$$

In the limit as $t\to\infty$ (4.25) yields the static expression given by Eq. (1.5) and for $t=0$ it vanishes everywhere as expected. The energy density (4.26) at a given point $\mathbf{r}$ is zero as long as $t<r/c$, and after time r/c attains immediately the ground-state value. For an isotropic polarization this ground-state expectation value for the energy density is that given by Eq. (1.6) for the isotropic-source case.

V. SUMMARY AND CONCLUSIONS

We have considered the problem of the time dependence of the cloud of virtual particles surrounding a source of a field. This time dependence sets in when the strength of the coupling between the source and the field changes with time. We have discussed this problem by studying three simple examples: a "static" point source whose interaction with a scalar meson-meson field is suddenly cut off (source removed at $t=0$), the same static source suddenly starting its interaction with the meson field (source appearing at $t=0$), and a molecule in its ground state suddenly starting to interact with the electromagnetic field. In all three cases the energy density of the virtual cloud dressing the source (made up of virtual mesons in the first two cases and of virtual photons in the third) has a time-dependent shape as a function of the distance r from the source and tends to the normal ground-state distribution for the final total Hamiltonian, which in the first case is the bare vacuum, in the second is the ground-state Yukawa-like distribution, and in the third is the ground-state far-field Casimir, van der Waals-like distribution. The time dependence of the virtual cloud in all cases is characterized by an expanding sphere of radius $r=ct$ centered on the position of the source, within which the cloud is time dependent, and outside which is constant. In the case of the massive scalar field the virtual distribution of energy density within the sphere readjusts to its asymptotic value in a time of the order of $(\hbar/mc^2)$ which can be thought of as the maximum lifetime of a virtual meson. The corresponding readjustment of the virtual photon cloud is instantaneous.

The above results seem to confirm in a qualitative way Feinberg's ideas about the nature of the so-called "half-dressed states" in quantum-field theory,[6] particularly in connection with the characteristic times involved in the establishment of an equilibrium structure in the field surrounding the source, at least in the sense that this regeneration time is independent of the source-field coupling strength in all the cases considered. Moreover, the present work might provide a quantitative basis for a discussion of the detection of the virtual field energies during the regeneration time, thus extending the direct measurements of the asymptotic energies in the electromagnetic case through the Casimir effect.

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