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Modulational instability of two-transverse-dimensional surface polariton waves in nonlinear dielectric waveguides

J. V. Moloney
Department of Physics, Heriot-Watt University, Edinburgh EH14 4AS, United Kingdom
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It is predicted and confirmed numerically that the nonlinear stationary waves on the surface polariton branch of a thin-film dielectric waveguide are unstable to transverse spatial fluctuations in the dimension parallel to the confining layers. Bounded waves whose peaks are located within the linear thin-film guiding layer show behavior in qualitative agreement with that of earlier studies on the two-dimensional propagation problem.

An extensive theoretical literature has evolved since the late seventies on the subject of nonlinear stationary waves supported at optical interfaces and in layered dielectric media.[1] These latter structures are characterized by the fact that one or more of the dielectric layers shows either a positive or negative nonlinear optical response to an incident electromagnetic wave. Included in this class of problem are single interface surface polariton waves which propagate unattenuated along an interface separating two dielectric media with a small refractive index mismatch across the surface. These latter waves may, at least in principle, be generated as a consequence of the reflection of an incident Gaussian beam, at an angle close to that for total internal reflection, from a nonlinear interface. Trapped surface waves or transmitted self-focused channels may appear above a critical incident energy flux.[2] A recent theoretical analysis,[3] using an equivalent particle in a time-varying potential analogy, substantiates the numerical observations of Ref. 2 and leads to a global interpretation of the nonlinear wave dynamics. Of specific interest to us here are the nonlinear guided waves (NGW) which exist in a thin-film planar dielectric waveguide with the linear film, bounded by one or more nonlinear dielectric layers (cladding and substrate). Our analysis assumes a positive nonlinear (Kerr) coefficient.

An important consequence of the small refractive index mismatch between adjoining media is the occurrence of a power-dependent waveguide index β. Figure 1 illustrates this dependence for a thin-film dielectric planar waveguide with a linear guide and substrate and a nonlinear cladding. A variety of potentially useful all-optical devices has been proposed based on such power-dependent characteristics.[4] The curve in Fig. 1 represents the power versus β characteristic for a TE wave. Pictures such as this are generated by ignoring propagation effects and solving a nonlinear second-order ordinary differential equation (in x) for the electric field with matching of the field and its derivative across each interface. The stability of these waves can only be addressed by introducing an additional spatial dimension representing the propagating

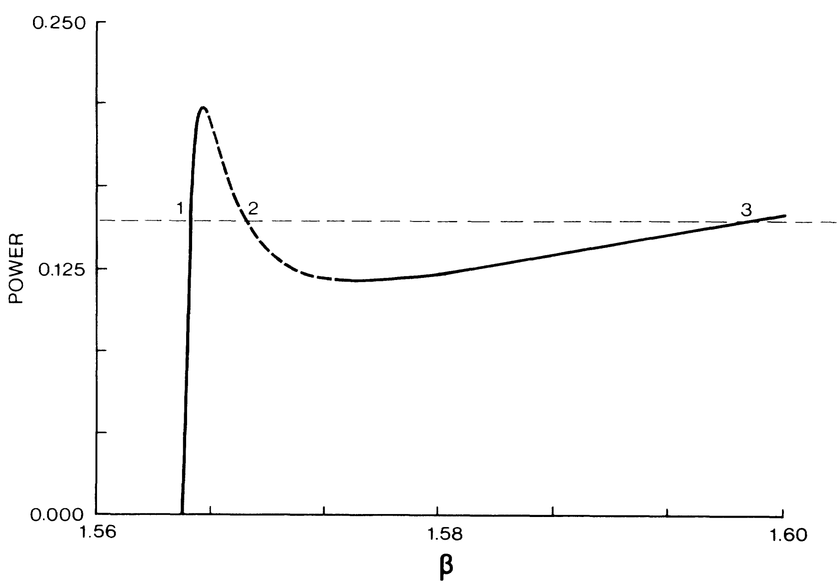

FIG. 1. Power vs waveguide index characteristics for a TE_0 NGW in an asymmetric thin-film planar waveguide. The solid curve represents stable and the dashed curve unstable NGW's in two dimensions (x,z). Parameters used in the present problem are $n_0=n_2=1.55$, $n_1=1.57$, $\alpha_0=0.01$, $\alpha_1=\alpha_2=0$, and the dimensionless guide width $2d/\lambda=8.4$.

(z) dimension along the waveguide. This leads to a nonlinear partial differential equation of the nonlinear Schrödinger (NLS) type with rather unusual boundary conditions due to the presence of the interfaces. The stability question is much more difficult and only one analytic prediction exists to date for the thin-film waveguide.[5] Stability can be established by numerically solving the NLS-type nonlinear evolution equation using analytically generated field shapes corresponding to different locations on the power versus β characteristic. The dashed line in Fig. 1 represents unstable waves determined by these methods.[6] We see that at a fixed energy flux (the horizontal line in Fig. 1) there may exist three nonlinear guided waves with one being unstable. The assumed form of the optical nonlinearity is cubic so that

$$n^2(x,|F|^2)=n_0^2+\alpha_0|F|^2,\ x<-d\ ,$$

$$n(x,|F|^2)=n_f^2,\ |x|<d\ ,$$

and

$$n(x,|F|^2)=n_0^2,\ x>d\ ,$$

and k_0 ($=2\pi/\lambda$) with λ the free space wavelength.

The instability referred to above refers to an instability to form where the NGW shape changes on propagation. In the present work we include the third space dimension (y), assuming, as in conventional waveguide analysis, that the wave front is planar in the y dimension. The curve of equilibrium NGW's shown in Fig. 1 is still applicable and the question that we now address is whether the resulting two-transverse dimensional profile is stable to weak transverse modulations along the planar wave front in the y dimension. Field shapes corresponding to fixed power at points 1, 2, and 3 in Fig. 1 are taken as initial data to the following nonlinear evolution equation:

$$2i\beta k_0\frac{\partial}{\partial z}F(\mathbf{r},z)+\nabla_T^2F(\mathbf{r},z)-k_0^2[\beta^2-n(x,|F|^2)]F(\mathbf{r},z)=0\ , \tag{1}$$

a modified NLS equation: transverse spatial coordinate $\mathbf{r}=x\hat{\mathbf{i}}+y\hat{\mathbf{j}}$, where $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ are unit vectors. Equation (1) has an identical structure to the earlier one-transverse-dimensional modified NLS equation employed to study instability for form, except that the two-transverse-dimensional Laplacian operator ∇_T^2 replaces $\partial^2/\partial x^2$ and the initial data $F(\mathbf{r},0)$ consists of a one-dimensional NGW in x embedded in a two-dimensional (x,y) space; a small amplitude perturbation of wave number K (uniform in x), of the form $\delta F(\mathbf{r},0)=\epsilon\cos Ky$ with $\epsilon=0.001$, is added to the initial NGW envelope $F(\mathbf{r},0)$ in each case. The NGW field shapes corresponding to the labels 1, 2, and 3 in Fig. 1 have the following distinctive features. The wave at label 1, stable to propagation in 2D (x,z), is localized mostly in the guiding linear film with tails extending into the surrounding cladding and substrate layers. This should behave essentially as a linear TE_0 wave of the guide. At label 2 the peak is displaced towards the nonlinear cladding but most of the energy of the wave still remains in the film. In 2D this wave is unstable and oscillates back and forth in the film. The surface polariton wave at label 3 is most interesting, being stable in 2D, with its peak lying in the nonlinear cladding. Given that this latter wave can be viewed as a perturbed spatial soliton of the 1D NLS equation (x,z) we anticipate that it most likely should be unstable to transverse fluctuations in the y dimension. One of the few established instability results for the 2D NLS equation (x,y,z) is that of a 1D soliton embedded in a 2D (x,y) space.[7]

Equation (1) was integrated for each NGW (1,2,3) in turn over 200 wavelengths employing periodic boundary conditions in the y dimension. The relevant physical parameter values are given in the figure captions. Wave 1, as expected, was stable over the full propagation length. The NGW on the unstable branch of Fig. 1 (label 2) showed the same qualitative behavior as in 2D. Figure 2 shows the initial x and y cross sections of this NGW at $z=0$; the constant wavefront in the y cross section corre-

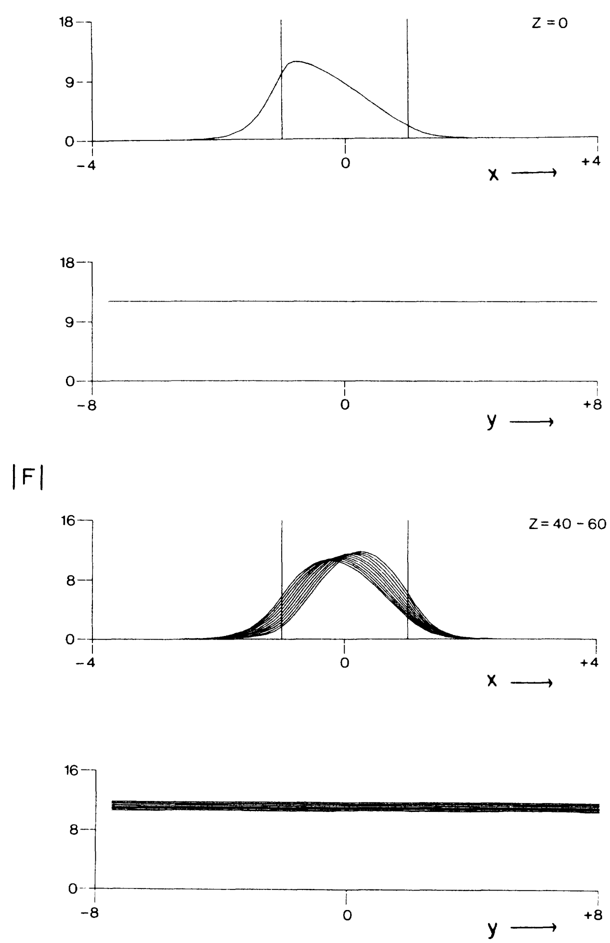

FIG. 2. Cross sections of the two-transverse-dimensional profile corresponding to label 2 in Fig. 1 in x and y at $z=0$ (initial data) and evolving in z between $z=40$ and 60. Both x and y axes are in units of d/λ, the dimensionless guide half-width.

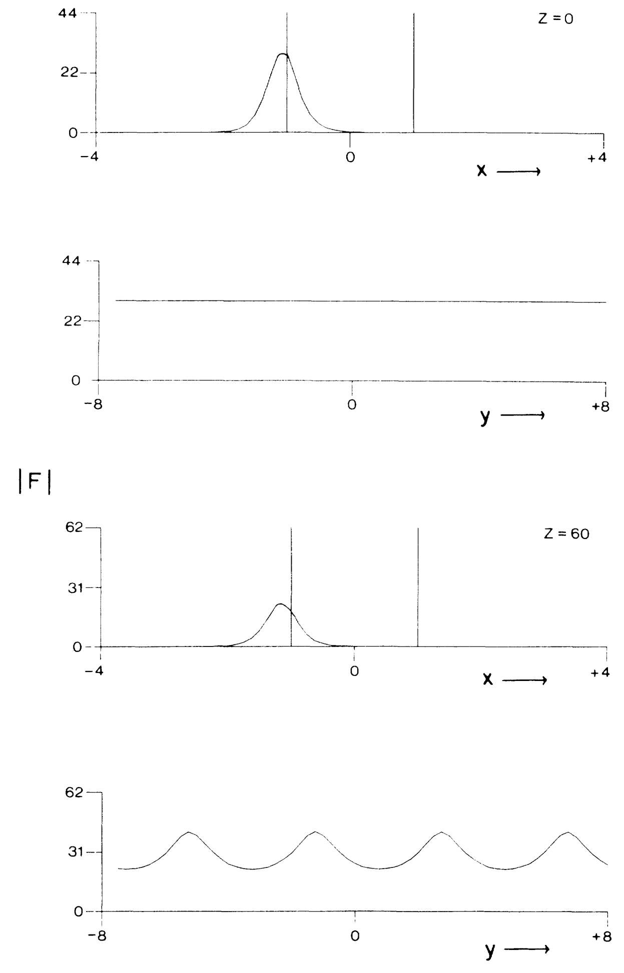

FIG. 3. Similar to Fig. 2 for the surface polariton wave at label 3 in Fig. 1.

sponds to the peak amplitude of the NGW. Oscillations of the wave in the film are shown from $z = 40$–60 wavelengths, showing that the entire wave sloshes back and forth in the guiding film. The sequence of flat lines in the y cross section represents the local x-envelope amplitude with respect to a fixed reference point. The behavior of the surface polariton wave at point 3 is quite different. Shown in Fig. 3 are x and y cross sections of the initial field and the corresponding sections after propagating 60 wavelengths. The NGW, while remaining fixed in the x dimension, develops a deep modulation along the originally planar wave front. The modulation deepens rapidly signaling the appearance of filaments associated with blowup in the cubic 2D NLS equation. Figure 4 shows some pictures of the development of this modulational instability on the two-transverse-dimensional spatial profile.

The wavelength of the modulational instability lies within the instability band predicted for the case of a 1D soliton embedded in two space dimensions in Ref. 7. Quantitative predictions cannot be made, however, as a significant portion of the NGW lies in the linear film making a comparison with the work in Ref. 7 difficult. Results from Ref. 7 can be used to anticipate general trends such as the dependence of the instability wavelength and the growth rate on the peak amplitude. The maximum unstable wave number K scales as the square root of the peak amplitude and the growth rate as the product of the square root of the peak amplitude and K. Our numerical results show similar trends, suggesting that the slowest instability growth may be achieved for the smallest possible refractive index mismatch. Finally, a comment is in order on the nature of the optical nonlinearity. The cubic case considered here shows the well-known blow-up singularity of the 2D NLS equation. Other power-law or saturable nonlinearities should tend to stabilize the appropriate NGW against modulational instability growth. Too strong a saturation, however, rules out power versus β characteristics like those shown in Fig. 1.

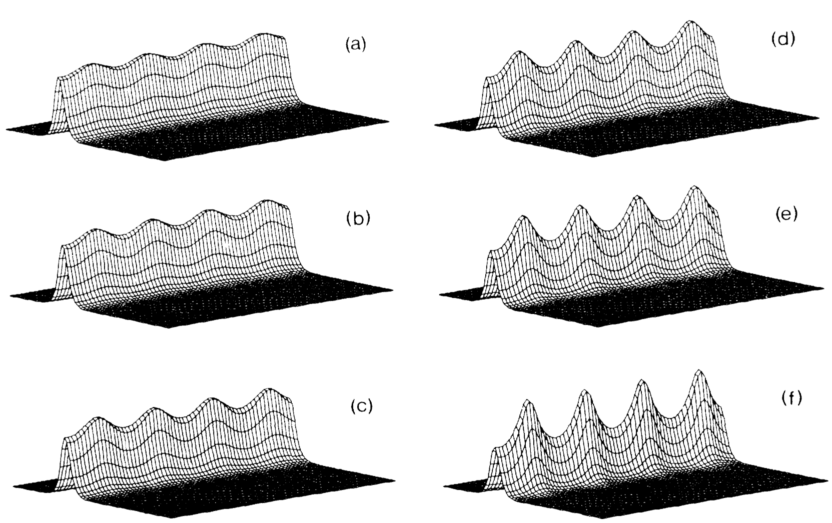

FIG. 4. Progressive development of the modulational instability on the initially planar wave front in the y dimension. Shown are the two-transverse-dimensional profiles (x,y) between $z = 50\lambda$ and 60λ at increments of $\Delta z = 2\lambda$.

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