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State reduction and $|n\rangle$-state preparation in a high-Q micromaser

Joachim Krause
Center for Advanced Studies and Department of Physics and Astronomy, University of New Mexico, Albuquerque, New Mexico 87131

Marlan O. Scully
Center for Advanced Studies and Department of Physics and Astronomy, University of New Mexico, Albuquerque, Mew Mexico 87131
and Max-Planck-Institut für Quantenoptik, D-8046 Garching bei München, West Germany

Herbert Walther
Max-Planck-Institut für Quantenoptik, D-8046 Garching bei München, West Germany
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The long lifetime of photons in a high-Q micromaser cavity is used to generate a $|n\rangle$-state maser field by injecting atoms, one at a time. Every atom is probed for its excitation after leaving the cavity so that the precise number of photons in the field is known. We calculate the probability of obtaining n photons after m atoms have passed the cavity.

In the present paper we show, using state reduction logic, that a pure number state $|n\rangle$ (Ref. 1) is achievable with a micromaser operated at sufficiently low temperatures and negligible cavity losses in which the radiation has a very long cavity lifetime (up to the order of seconds).[2] Such cavities are actually now available. Under those conditions it is possible to obtain a state with no fluctuations in the photon number.[3]

The fact that a number state can be reached must be seen in connection with efforts to generate "squeezed-state" systems. These systems have received much study both theoretically and experimentally[4,5] because at present optical measurement has reached the point such that photon statistical shot noise is the limiting factor in many types of experiments. Examples include the passive laser gyroscope, the laser gravity wave detector, and optical heterodyne communication. In such experiments the error signal is determined by the uncertainty in the photon number, and the limit in sensitivity due to this uncertainty is known as the shot-noise limit.

While the generation of squeezed states involves generally nonlinear processes, the micromaser in our proposed experiment is a direct oscillator for the n-state radiation field. If very-high-Q optical cavities are available, one might envision performing a similar experiment with atoms whose two laser levels decay into two different metastable levels. After the atoms have left the cavity, it can be inferred from their final state whether they have emitted a photon or not.

Specifically, we envision producing pure number states in the following way (cf. Fig. 1): Atoms in their excited state are injected into the cavity. After they leave the cavity, they are probed by a static electric field which ionizes all atoms in their upper level. All the atoms that are not ionized have emitted a photon in the cavity. When these atoms are counted (via electron detection as in Fig. 1), the total number of photons in the maser field can be inferred.

We emphasize that state reduction and the connected ideas of measurement theory are essential to this logic. By the determination of the state of the outgoing atoms the photon number in the field is exactly known, i.e., the state of the field is reduced to a pure number state. Since we start the experiment with no radiation in the cavity, the field is always in a number state when an atom enters the cavity. By the interaction of the atom with the field, which is in a state $|\nu\rangle$, the field state will be changed to a superposition of states $|\nu\rangle$ and $|\nu+1\rangle$. Due to the measurement of the atomic state afterwards this superposition is reduced to one of the states $|\nu\rangle$ and $|\nu+1\rangle$, depending on the result of the measurement.

It is important to emphasize that a feature central to the present $|n\rangle$-state generation scheme is the long lifetime of the photons in the cavity. The decay time for photons is given by their frequency ν and the quality factor Q of the cavity:

$$\tau \sim Q/\nu \ .$$

For $\nu \sim 2\times10^{10}\ \mathrm{s}^{-1}$ and $Q \sim 5\times10^{10}$, which is now possi-

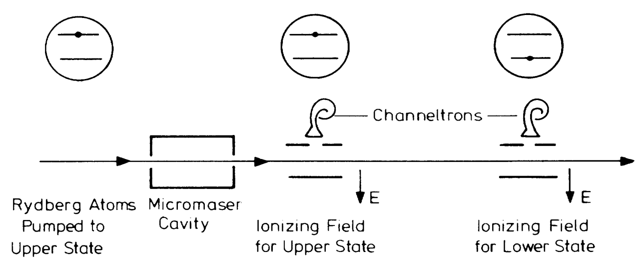

FIG. 1. Proposed experimental setup. Rydberg two-level atoms in their upper state are injected into the micromaser cavity. When they leave the cavity, the atoms pass an electric field which ionizes and deflects the excited atoms. The atoms in the lower level which have emitted a photon in the cavity are counted, and to check the total number the atoms in the upper level are counted as well.

ble to reach, we obtain $\tau \sim 2$ s. The fluxes of atoms used in these experiments is of the order of 1000 s^{-1}.[2] Loss of radiation is a statistical process. This can be seen by investigating, for example, a beam splitter. For such a device we assume the probability for a photon to propagate in one direction is η.[6] When light in an $|n\rangle$ state is shined onto the beam splitter, the field behind the beam splitter will no longer be a pure number state. Instead, its state will be

$$|\psi\rangle = \sum_{m=0}^{n} \begin{pmatrix} n \\ m \end{pmatrix} \eta^m (1-\eta)^{n-m} |m\rangle \ ,$$

which is a binomial superposition of number states. Therefore, we must have zero losses in order to maintain pure number states via the state reduction argument. In the experiment, thermal photons in the cavity have to be suppressed. This is achieved by cooling down the cavity to a temperature close to absolute zero. The mean number of thermal photons for $\nu = 2\times 10^{10}\ s^{-1}$ is $\langle n_{th}\rangle = 0.016$ at $T=0.25$ K and $\langle n_{th}\rangle = 3.3\times 10^{-5}$ at $T=0.1$ K. These temperatures can be reached in a ^{3}He cryostat which is now available for our maser. Thermal photons do not only induce statistical decay but result in a superposition of number states from the beginning of the experiment.

Now, with zero cavity losses, we will maintain a number state since no radiation will be lost from the cavity in the present experiment. However, we will have only *a priori* probabilities as to which number state we actually generate in the present state reduction scheme. These *a priori* probabilities should not be confused with photon statistical distributions. For example, if one considers a coherent state of the radiation field, then every laser or every system being considered would be in an indefinite superposition of number states. Whereas in our experiment, every system is in a specific number state; however, we do not know prior to the experiment which state that will be. Therefore we have to perform the experiment repeatedly with a constant total number of atoms, thus generating a large number of different number states. The distribution of the photon numbers will be given by the *a priori* probability distribution, which we are going to calculate. It should be emphasized that the number of atoms leaving the cavity in the lower state is equal to the number of photons in the cavity only for a lossless cavity.

It should be mentioned here that if there is a photon field other than a pure number state initially in the cavity (e.g., a thermal radiation field), one does not obtain a number state but a superposition of number states. If the atoms leaving the cavity in the lower state of the maser transition are counted, the statistical distribution of these numbers differs from the case where no radiation was initially present in the cavity. If, for example, every emitted photon decays before the next atom enters the cavity, so that the field is in a steady state, every atom interacts with the same radiation field, and the number of atoms in the lower state follows a biominal distribution.

We now turn to the calculation of the probability $P_n(m)$ of having n photons in the field after m atoms have passed the cavity. To obtain this probability, we derive a recursion relation. When we assume that the field is the state $|\nu\rangle$ with ν photons, the time-development operator $U(\tau)$ for the interaction of one two-level atom with the field is given by[7]

$$U(\tau) = \begin{pmatrix} \cos(\sqrt{\nu+1}g\tau) & i\sin(\sqrt{\nu+1}g\tau) \\ -i\sin(\sqrt{\nu+1}g\tau) & \cos(\sqrt{\nu+1}g\tau) \end{pmatrix} \ , \quad (1)$$

where g is the coupling constant for the interaction and τ its duration. We start with the combined density operator for the atom in the upper level $|a\rangle$ and the field $\rho = |a,\nu\rangle\langle a,\nu|$. After the interaction time τ we have

$$\rho(\tau) = U(\tau)|a,\nu\rangle\langle a,\nu|U^\dagger(\tau) = \cos^2(\sqrt{\nu+1}g\tau)|a,\nu\rangle\langle a,\nu| + \sin^2(\sqrt{\nu+1}g\tau)|b,\nu+1\rangle\langle b,\nu+1|$$
$$+ i\sin(\sqrt{\nu+1}g\tau)\cos(\sqrt{\nu+1}g\tau)[|b,\nu+1\rangle\langle a,\nu| - |a,\nu\rangle\langle b,\nu+1|] \ , \quad (2)$$

where $|b\rangle$ denotes the lower atomic level. The state of the radiation field is now determined via state reduction. That is if we determine that the atom is in the upper state $|a\rangle$, then the density matrix (2) is reduced to the state

$$\rho(\tau) = \cos^2(\sqrt{\nu+1}g\tau)|a,\nu\rangle\langle a,\nu| \ , \quad (3a)$$

and if the atom is found to be in the state $|b\rangle$ the system density matrix is given by

$$\rho(\tau) = \sin^2(\sqrt{\nu+1}g\tau)|b,\nu+1\rangle\langle b,\nu+1| \ . \quad (3b)$$

From this we find that the probability for the field to remain in the state $|\nu\rangle$ is $c(\nu) \equiv \cos^2(\sqrt{\nu+1}g\tau)$ and the probability for a transition to the state $|\nu+1\rangle$ is $s(\nu) \equiv \sin^2(\sqrt{\nu+1}g\tau)$.

When $m-1$ atoms have passed, the field is in a state $|n\rangle$ with the probability $P_n(m-1)$ and in the state $|n-1\rangle$ with a probability $P_{n-1}(m-1)$. The probability for the field to be in the state $|n\rangle$ after m atoms have been in the cavity is then simply

$$P_n(m) = c(n)P_n(m-1) + s(n-1)P_{n-1}(m-1) \ . \quad (4)$$

We assume that the field is initially in the vacuum state $|0\rangle$, i.e., $P_0(0)=1$. Then we have for one atom $P_0(1)=c(0)$ and $P_1(1)=s(0)$, for two atoms

$$P_0(2) = c(0)P_0(1) = [c(0)]^2 \ ,$$
$$P_1(2) = c(1)P_1(1) + s(0)P_0(1) = s(0)[c(0)+c(1)] \ ,$$
$$P_2(2) = s(1)P_1(1) = s(0)s(1) \ ;$$

and for three atoms

$$P_0(3) = c(0)P_0(2) = [c(0)]^3 \ ,$$
$$P_1(3) = c(1)P_1(2) + s(0)P_0(2) = s(0)\{[c(0)]^2 + c(0)c(1) + [c(1)]^2\} \ ,$$
$$P_2(3) = c(2)P_2(2) + s(1)P_1(2) = s(0)s(1)[c(0)+c(1)+c(2)] \ ,$$
$$P_3(3) = s(2)P_2(2) = s(0)s(1)s(2) \ .$$

Obviously $P_n(m)=0$ for $n<0$ or $n>m$. In view of the above we can derive the general expression

$$P_n(m)=\prod_{i=0}^{n-1}s(i)\sum_{\substack{i_j=0\\(i_{m-1}\le\cdots\le i_n)}}^{n}\prod_{j=n}^{m-1}c(i_j)\ , \tag{5}$$

where we define

$$\sum_{\substack{i_j=0\\(i_{m-1}\le\cdots\le i_n)}}^{n}\prod_{j=n}^{m-1}c(i_j)\equiv\begin{cases}\displaystyle\sum_{i_n=0}^{n}\sum_{i_{n+1}=0}^{i_n}\cdots\sum_{i_{m-1}=0}^{i_{m-2}}\prod_{j=n}^{m-1}c(i_j), & \text{for } m>n\ ,\\ 1, & \text{for } m=n\ ,\\ 0, & \text{for } m<n \text{ or } n<0\ .\end{cases}$$

With this definition and that of the product symbol $\prod_{i=0}^{-1}s(i)=1$, we obtain the correct result for $m=0$, i.e., the initial condition for the experiment $P_0(0)=1$. For $m=1$ we get

$$P_0(1)=\prod_{i=0}^{-1}s(i)\sum_{i_0=0}^{0}\prod_{j=0}^{0}c(i_j)=c(0),\quad P_1(1)=\prod_{i=0}^{0}s(i)1=s(0)\ ,$$

and $P_n(1)=0$ for $n\neq 0,1$.

Assuming that Eq. (5) is correct for $m-1$, we have with Eq. (4)

$$\begin{aligned}P_n(m)&=c(n)\prod_{i=0}^{n-1}s(i)\sum_{\substack{i_j=0\\(i_{m-2}\le\cdots\le i_n)}}^{n}\prod_{j=n}^{m-2}c(i_j)+s(n-1)\prod_{i=0}^{n-2}s(i)\sum_{\substack{i_j=0\\(i_{m-2}\le\cdots\le i_{n-1})}}^{n-1}\prod_{j=n-1}^{m-1}c(i_j)\\
&=\prod_{i=0}^{n-1}s(i)c(n)\sum_{\substack{i_j=0\\(i_{m-1}\le\cdots\le 1_{n+1})}}^{n}\prod_{j=n+1}^{m-1}c(i_j)+\prod_{i=0}^{n-1}s(i)\sum_{\substack{i_j=0\\(i_{m-1}\le\cdots\le i_n)}}^{n-1}\prod_{j=n}^{m-1}c(i_j)\\
&=\prod_{i=0}^{n-1}s(i)\left(c(n)\sum_{i_{n+1}=0}^{n}\sum_{i_{n+2}=0}^{i_{n+1}}\cdots\sum_{i_{m-1}=0}^{i_{m-2}}\prod_{j=n+1}^{m-1}c(i_j)+\sum_{i_n=0}^{n-1}c(i_n)\sum_{i_{n+1}=0}^{i_n}\cdots\sum_{i_{m-1}=0}^{i_{m-2}}\prod_{j=n+1}^{m-1}c(i_j)\right)\\
&=\prod_{i=0}^{n-1}s(i)\left(\sum_{i_n=0}^{n}c(i_n)\sum_{i_{n+1}=0}^{i_n}\cdots\sum_{i_{m-1}=0}^{i_{m-2}}\prod_{j=n+1}^{m-1}c(i_j)\right)\\
&=\prod_{i=0}^{n-1}s(i)\sum_{\substack{i_j=0\\(i_{m-1}\le\cdots\le i_n)}}^{n}\prod_{j=n}^{m-1}c(i_j)\ ,\end{aligned}$$

which proves Eq. (5).

The probability distribution can be evaluated numerically for different values of $g\tau$ as a function of the number of passing atoms. We show results for up to 1000 atoms. Obviously, the probability $P_n(m)$ is very strongly dependent on the value of $g\tau$. This parameter can be varied experimentally by changing the velocity of the atomic beam. When $g\tau<1$, then a peak in the photon distribution develops and moves towards higher photon numbers as the number of passing atoms grows (cf. Fig. 2). This peak halts its drift and becomes very narrow when a photon number $n_0\lesssim(\pi/g\tau)^2$ is reached because then the probability $s(n_0)$ to add one photon to the field becomes very small. In theory, $s(n_0)$ could become exactly 0, so that the probability distribution will be a δ function in the steady state, a case discussed by Filipowicz, Javanainen, and Meystre.[1] In any experiment, however, the velocity distribution of the atomic beam is never that sharply defined.

Therefore the height of the peak in the probability distribution diminishes as more atoms are injected, and a new peak develops in front of the next barrier at about $(2\pi/g\tau)^2$. Thus the realization of a number state is coupled to the detection via the outgoing atoms.

Atomic velocity itself is not a complicative factor in the present scheme of n-state preparation. It leads to a change in the probability of emitting a photon, but only the fact whether a photon has been emitted or not is important for the experiment. The basic notion of atomic observation leading to field information is in force regardless of complicating influences such as atomic motion. If the experiment is performed repeatedly, however, the statistics of the obtained photon numbers in the number states will be influenced.

In conclusion, a pure number state is generated in a lossless maser cavity and determined via atomic interception and state reduction.[8] The *a priori* probability of being in the number state $|n\rangle$ after m photons have passed through the cavity is given by $P_n(m)$. By performing the experiment very often for a given m, the probability distribution $P_n(m)$ can be measured, and it is sensitive to the photon statistics in the cavity.

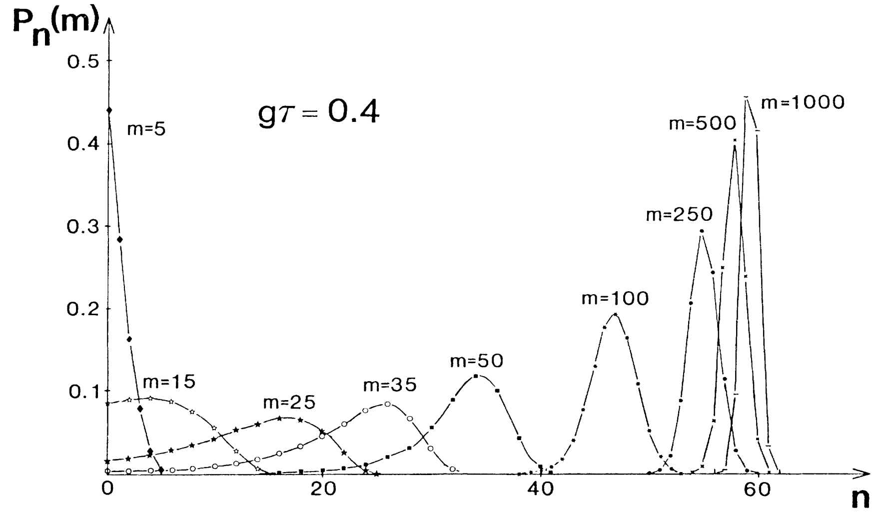

FIG. 2. Probability of obtaining n photons in the cavity after m atoms have passed. The curves are calculated for $g\tau = 0.4$.

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[1]In the recent work of P. Filipowicz, J. Javanainen, and P. Meystre [J. Opt. Soc. Am. B **3**, 906 (1986)] $|n\rangle$-state generation in a micromaser was considered from a very different point of view. See also their paper, in *Coherence, Cooperation and Fluctuations,* edited by F. Haake, L. Narducci, and D. Walls (Cambridge Univ. Press, Cambridge, 1986), p. 206, and the work of M. Kitagawa and Y. Yamamoto, Phys. Rev. A **34**, 3974 (1986).

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[8]In the present paper we note that the maser field is always in a number state $|n\rangle$. In a recent work P. Meystre [Opt. Lett. **12**, 669 (1987)] has shown that the situation is more complex when beginning from a thermal photon distribution.