Absorption of β rays in matter

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Energy spectra of transmitted β rays are compared with spectra generated by a Monte Carlo procedure. Reasons for the similarities between the experimental results of β and γ interactions are explored. In addition, the energy distribution of the backscattered β rays and variation of absorbed energy as a function of absorber thicknesses are presented.

Even though the fundamental nature of the interaction of β rays in matter is different from that of γ rays, the intensity of transmitted β rays decreases almost exponentially with thickness of the absorber. This exponential decrease in intensity of β rays is always described as "accidental," resulting from the continuous energy distribution of β rays and from their absorption and scattering.[1,2] To understand further the absorption of β rays, we measured the energy distribution of the transmitted β rays and compared the distribution with a calculated energy distribution using the Monte Carlo technique. The shape of the energy spectrum obtained experimentally and theoretically for various absorber thicknesses is almost similar to the original β spectrum (with no absorber). Thus in this respect also β rays and γ rays behave similarly.

During the passage of γ rays through matter, a single interaction removes a γ ray from the beam. The intensity of the transmitted beam in such cases decreases exponentially and the shape of the energy spectrum of the transmitted γ rays is identical to that of the incident beam. Charged particles, on the other hand, are not removed from the beam in a single collision. They undergo several interactions before they are stopped or they come out of the absorber. Heavy particles like α particles rarely suffer large-angle scattering. The number of transmitted α particles, therefore, stays constant till the end of their range. The energy spectrum, however, shifts to the low-energy side with increasing full width at half maximum.[3] Monoenergic electrons suffer large-angle scattering more often than α particles. Consequently, intensity decreases almost linearly with thickness.[4] β rays, as mentioned earlier, have an exponential decrease in intensity with absorber thickness. Our attempt here is to clarify the similarities in the experimental results of γ and β interactions with matter.

EXPERIMENTAL RESULTS

β rays from Tl^{204} with a maximum range of 1400 μm in silicon are used for this study. The detector is a surface-barrier silicon detector with a depletion width of 1500 μm. The depletion width is greater than the range of the β rays. In the experiment, the source was kept at a distance of 1 cm from the detector. An aluminum collimator was used to obtain a parallel beam of β rays of 1 cm^2 cross-sectional area. The distance between the source and the absorber was 0.5 cm.

Figure 1 shows the energy spectra for various absorber thicknesses. From a casual observation, the shape of the spectra of the transmitted β rays appears to be similar to the β spectrum with no absorber. There is, however, a significant difference; the mean energy of the transmitted β rays first increases and then decreases (Table I). A semilog plot of the number of transmitted β rays as a function of thickness in Fig. 2 shows an almost linear decrease indicating the well-known exponential decrease in intensity.

MONTE CARLO PROCEDURE

The Monte Carlo procedure is based upon the following assumptions: (i) the absorber is a slab of uniformly distributed material of given atomic number, (ii) the dominant electron energy-loss mechanism in the energy range of interest (40–800 keV) is ionization, and (iii) elastic collisions of electrons with atomic nuclei are the source of all large-angle scattering.

Ionization collisions are not considered individually. Instead, we use the relativistic form of Bethe's ionization loss equation,[5]

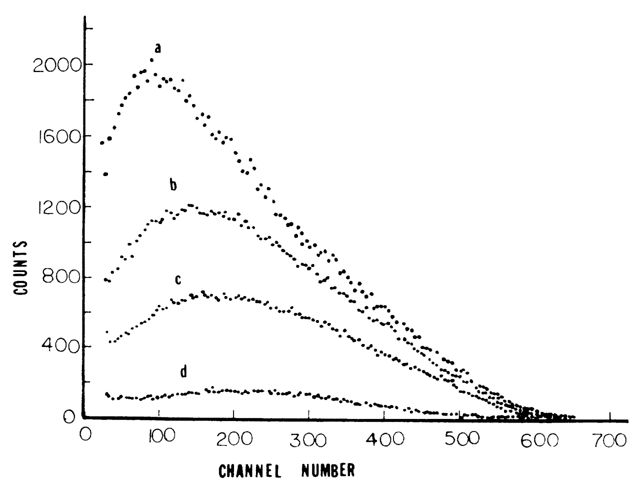

FIG. 1. Energy distribution of transmitted β particles for 0, 24.8, 75.9, and 283.7 μm thick absorbers.

$$\frac{dt}{ds}=\frac{e^4N_0Z}{8\pi\epsilon_0^2m_0v^2}\left[\ln\left[\frac{m_0v^2T}{2I^2(1-\beta^2)}\right] -[2(1-\beta^2)^{1/2}-1+\beta^2]\ln 2 +1-\beta^2+\tfrac{1}{8}[1-(1-\beta^2)^{1/2}]^2\right], \quad (1)$$

where dt/ds is the energy loss per unit path length; e, the electron charge; N_0, Avogadro's number; Z, atomic number of absorber; ϵ_0, permittivity of vacuum; m_0, electron mass; v, electron velocity; T, electron kinetic energy; I, the geometric mean of the excitation and ionization potentials of the absorber atoms; and $\beta=v/c$.

We use a shielded form of the Rutherford scattering differential cross section to describe the electron scatters. Assuming a simple exponential decrease of the Coulomb field of the nucleus caused by its atomic electrons, the following differential cross section is obtained:[6]

$$\frac{d\sigma}{d\Omega}=\frac{Z(Z+1)e^4}{p^2v^2}\frac{1}{(1-\cos\theta+2\alpha)^2}, \quad (2)$$

where $\alpha=0.25(1.12\lambda_0\hbar/p)^2$; $\lambda_0=Z^{1/3}/0.885a_0$, a_0, the Bohr radius; v, the electron velocity; and p, the electron momentum.

Equation (2) leads to a finite total cross section

$$\sigma_T=\int\left[\frac{d\sigma}{d\Omega}\right]d\Omega=\frac{\pi}{\alpha(1+\alpha)}\frac{Z(Z+1)e^4}{p^2v^2}. \quad (3)$$

Assuming that the electron beam is exponentially attenuated, then the mean free path λ is given by

$$\lambda=\frac{A}{\rho N_0\sigma_T}, \quad (4)$$

where A is the atomic weight of the absorber.

The energy loss and scattering process for an electron is then simulated as follows. Determine the distance traveled to the next scatter by means of a logarithmic distribution of path lengths based upon the mean free path λ. Evaluate the ionization energy loss during the traversal using Eq. (1) and recalculate the electron's energy and momentum after this loss. Scatter the electrons according to Eq. (2), the azimuthal angle of scatter ϕ is chosen randomly between 0 and 2π, whereas the scattering angle θ is chosen according to the probability distribution

$$p(\theta)=\frac{1}{\sigma_T}\int_0^{2\pi}\int_0^{\theta}\left[\frac{d\sigma}{d\Omega}\right]\sin\theta\, d\theta\, d\phi =\frac{(1-\alpha)(1-\cos\theta)}{(1+2\alpha-\cos\theta)}. \quad (5)$$

The above process is repeated starting at the new scattering center until the electron is either backscattered out of the sample, transmitted through it or its energy is completely absorbed. Note that radiation losses are not included in these calculations because of the low

TABLE I. Comparison of the experimental and Monte Carlo results.

Thickness (μm)	Number of β particles transmitted		Mean energy of the Transmitted β particles (keV)		Backscattered β particles calculated		Absorption calculated	
	Expt.	Calc.	Expt.	Calc.	Number	Mean energy (keV)	Number stopped	Mean energy absorbed (keV)
0	1.13×10^5		2.12×10^2					
8.5	9.2×10^4	9.6×10^4	2.2×10^2	2.19×10^2	3.9×10^3	62	5.3×10^3	8.7
24.8	8.05×10^4	8.0×10^4	2.33×10^2	2.35×10^2	8.03×10^3	79.5	1.9×10^4	29
49.3	6.03×10^4	6.1×10^4	2.4×10^2	2.47×10^2	11.4×10^3	93	3.4×10^4	55
75.9	4.9×10^4	4.9×10^4	2.4×10^2	2.55×10^2	13.4×10^3	102	4.7×10^4	79
283.7	9.9×10^3	9.4×10^3	2.26×10^2	2.4×10^2	15.9×10^2	113	8.7×10^4	175
831	3.7×10^3	2.0×10^3	2.0×10^2	2.0×10^2	16×10^3	113	9.5×10^4	193
457	5.0×0^2	1.3×10^2	1.1×10^2	1.1×10^2	16×10^3	113	9.7×10^4	196

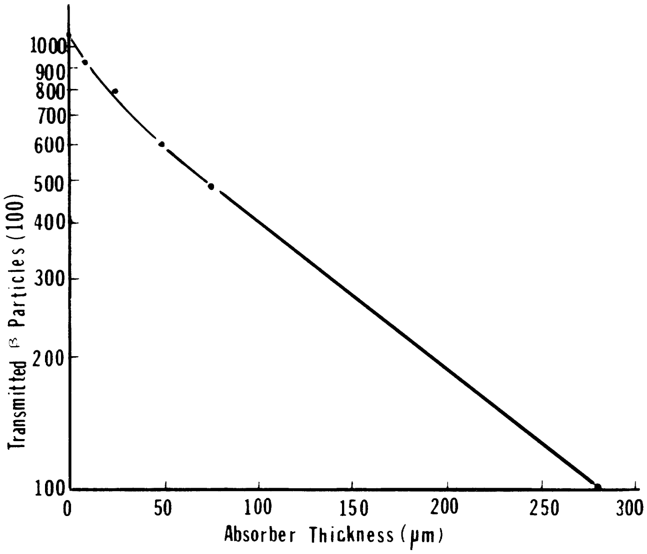

FIG. 2. Variation of number of transmitted β particles as a function of absorber thickness.

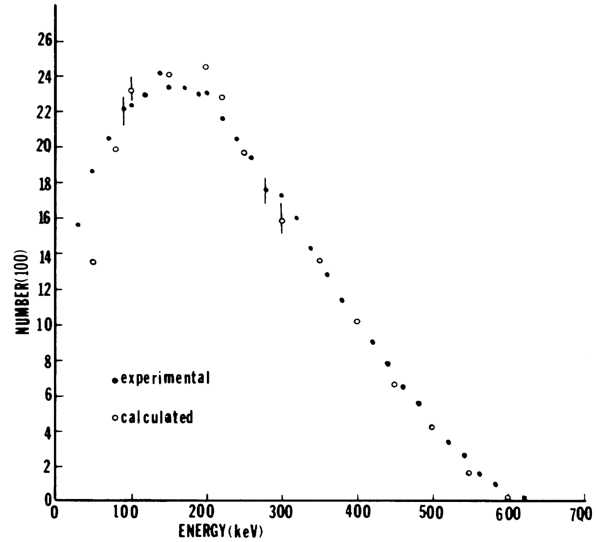

FIG. 3. Comparison of the experimental and calculated energy distribution of transmitted β particles for 24.8 μm thick absorbers.

energies of the β rays considered here. For a detailed description of the Monte Carlo procedure, the reader is referred to Ref. 7 after which this simulation was fashioned.

The input to the Monte Carlo process is the experimental electron energy spectrum obtained from the source with no absorber present. Normal incidence on the absorber is assumed. Results of the calculation are given in Table I and in Figs. 3, 4, and 5.

DISCUSSION

Experimental results show similarities between the β and γ interactions, especially in the approximate exponential decrease in the number of transmitted particles with thickness and in the shape of the spectra for different absorber thicknesses. We attempt to explain these similarities based on the calculated and experimental data presented in Table I.

The total number of transmitted β particles experimentally obtained for the first five thicknesses agree within 5% with the calculated numbers. The big difference in the numbers for the last two thicknesses is caused by the increased contribution of noise pulses and pulses due to γ rays to the total number. The almost perfect agreement of the experimental and calculated energy spectra is also clear in Fig. 3. The mean energies of the transmitted β particles given in column 4 agree well with the mean energies of the calculated spectra given in column 5. The mean energy of the calculated spectra increases first and then decreases with thickness just as the mean energies of the experimental spectra. In effect, the transmitted β rays "harden" first with absorber thickness. As the β rays harden, the slope of the log N versus thickness curve is expected to decrease. A decrease in slope with thickness that parallels the increase in hardness can be clearly seen in Fig. 2.

Monte Carlo technique is also used to calculate the number and mean energy of the backscattered β rays.

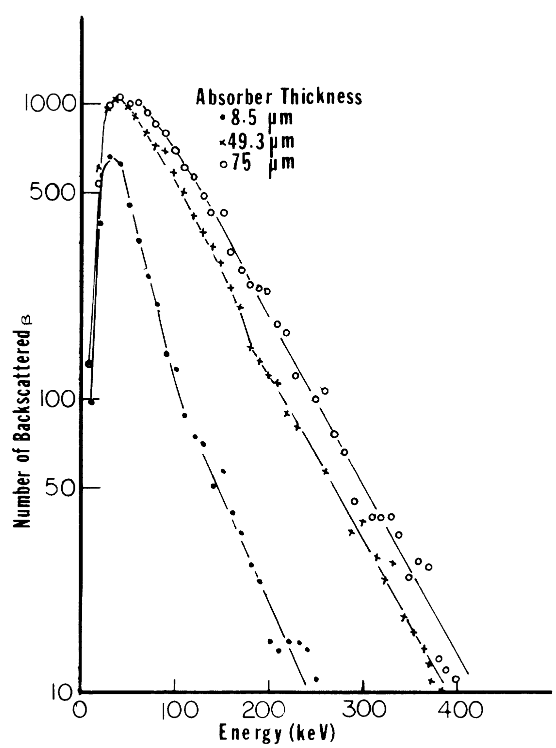

FIG. 4. Energy distribution of backscattered β particles for three different absorber thicknesses.

Results are summarized in columns 7 and 8 of Table I. The total number of backscattered β particles increases with thickness and reaches the saturation value for the absorber thickness of 283.7 μm. Mean energy also increases with thickness and reaches the maximum value of 113 keV. Energy distribution of the backscattered electrons for three thicknesses are shown in Fig. 4. As the absorber thickness increases, not only does the total number of backscattered β particles increase, but also more and more β rays with increasing energy are scattered. This partially accounts for the hardening of the transmitted β rays. Absorption of the low-energy β rays is the other reason. Columns 8 and 9 of Table I summarize results of the Monte Carlo calculation on energy absorption. Column 8 gives the number of β particles stopped in the absorber. The number increases with thickness. Column 9 gives the mean absorbed energy. The mean energy is the average of the energy absorbed from all particles, backscattered, stopped, and transmitted. Figure 5 shows the distribution of absorbed energy for three different thicknesses. Distribution of the absorbed energy broadens with thickness.

If the reduction of energy is the only result of interaction of β rays with matter, the consequences will be a shift in the energy spectrum of the transmitted β rays to the low-energy side. The other effects are backscattering and absorption (stopping). Both remove β rays from the low-energy side. A combination of the three effects allows the β spectrum of the transmitted β rays to appear similar to the spectrum with no absorber. The width and mean energy of the absorbed and backscattered β rays increase with thickness. This explains why the mean energy of the transmitted β rays first increases and then decreases with increasing absorber thickness.

In conclusion, if the energy distribution and intensity variation of β rays are considered, the experimental results are similar to those of γ rays. The β rays move to the low-energy side during interaction, but the number in the low-energy side is reduced by absorption and backscattering. The number moved to the low-energy region is almost balanced by the number removed from the region. The net effect is similar to the removal of an electron from the beam in one collision. Hence the exponential decrease in the intensity with absorber thickness.

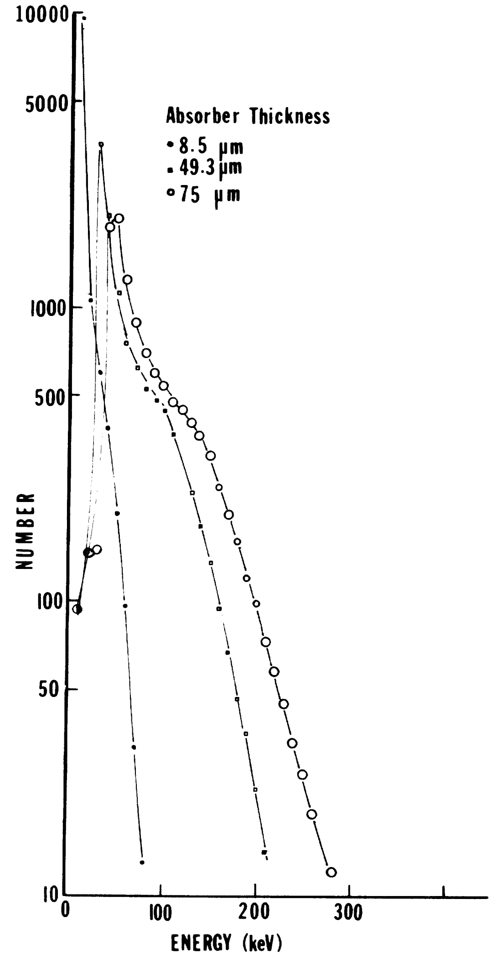

FIG. 5. Absorbed energy distribution for three different absorber thicknesses.

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