Doppler splitting of electron cyclotron absorption resonance in plasmas

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The Gaussian broadening of absorption lines due to the Doppler effect is well known. However, it is shown here for the first time that Doppler splitting occurs for the ordinary-mode electron-cyclotron-resonance absorption in plasmas. Although this absorption is due to the finite size of the electron Larmor orbits it is mainly determined by $T_{\parallel}$ and is only weakly dependent on $T_{\perp}$ via cyclotron-overstability-type terms. This is in contrast to intuitive expectations which would suggest that finite-Larmor-radius effects should depend strongly on $T_{\perp}$.

When an electromagnetic (EM) wave of frequency ω and wave vector $\mathbf{k}$ is absorbed by a particle moving with a velocity $\mathbf{v}$, both the energy and the momentum must of course be conserved. The simultaneous conservation of both the energy and the momentum yields the familiar Doppler-shift condition $\omega=\mathbf{k}\cdot\mathbf{v}$. Consequently, the probability of absorption of the EM wave by the particle is proportional to $\delta(\omega-\mathbf{k}\cdot\mathbf{v})$. If the absorbing particles obey the Maxwell-Boltzman velocity distribution function $F(\mathbf{v})$, then the intensity of absorption is proportional to the velocity integral of $F(\mathbf{v})\delta(\omega-\mathbf{k}\cdot\mathbf{v}) = f(v=\omega/k) \propto \exp[-(\omega/kv_T)^2]$, where $f(v)$ is the one-dimensional distribution function in the direction of $\mathbf{k}$, and v_T is the particle thermal velocity. This is the well-known Gaussian broadening due to the Doppler effect.[1] However, Doppler splitting of absorption lines does not exist in the plasma physics literature.

In this paper we will show that Doppler splitting does indeed occur for the ordinary- (O-) mode electron-cyclotron-resonance (ECR) absorption in plasmas. Furthermore, in contrast to intuitive expectations, although this absorption is due to the finite size of the electron Larmor orbits it is mainly determined by $T_{\parallel}$ and is only weakly dependent on $T_{\perp}$ via cyclotron-overstability-type terms. Here $\parallel$ and $\perp$ refer to directions parallel and perpendicular, respectively, to the confining magnetic field $\mathbf{B}=B\hat{i}_z$. These results are applicable to any plasma such as astrophysical, solid state, stellerator, mirror machine, tokamak, etc., plasmas. Finally, we will examine some of the consequences of these unique O-mode absorption features on electron-cyclotron-resonance heating (ECRH), magnetohydrodynamic (MHD) behavior, and steady-state rf current drive in tokamaks, since these problems are of considerable current interest in fusion research.

After Fourier analysis in space and time, the Maxwell electromagnetic field equations yield, for plane waves of the form

$$\mathbf{E}=\mathbf{E}_0\exp[i(\mathbf{k}\cdot\mathbf{r}-\omega t)] ,$$

the result

$$\mathbf{k}\times(\mathbf{k}\times\mathbf{E})+(\omega^2/c^2)\overleftrightarrow{\mathbf{D}}\cdot\mathbf{E}=\mathbf{0} . \tag{1}$$

Here $\overleftrightarrow{\mathbf{D}}(\omega,\overleftrightarrow{\mathbf{k}})$ is the hot-plasma dielectric tensor.[2,3] The condition for a nontrivial solution of Eq. (1) is obtained by setting the determinant of the coefficient of E_x, E_y, and E_z equal to zero. Thus knowing all the nine dielectric tensor elements D_{ij}, $i,j=x,y$, and z, one can get a closed-form expression for the allowed wave number k as a function of ω. Then from the imaginary part of k (i.e., Im k), one obtains the optical depth τ for the mode of propagation under study.

We now consider the case of O-mode radiation ($\mathbf{E}\parallel\mathbf{B}$) of frequency $\omega\approx\omega_c=(eB/mc)$ propagating through a low-density (n) plasma $[\omega_p=(4\pi ne^2/m)^{1/2}<\omega_c]$ nearly perpendicular to $\mathbf{B}$ (i.e., $k_\perp > k_\parallel$). Then Eq. (1) becomes

$$[-(c^2k_\perp^2/\omega^2)+D_{zz}]E_z\approx 0 , \tag{2}$$

which gives the dispersion relation

$$(c^2k_\perp^2/\omega^2)\approx D_{zz} . \tag{3}$$

Using the Maxwell-Boltzmann zeroth-order distribution, and taking the large-argument expansion for the real part of the dispersion function,[2,3] the dielectric tensor element D_{zz} may be written as

$$D_{zz}=D_{zz}^{(c)}+\Delta_{zz}=(1+\omega_p^2\chi_0')+\Delta_{zz} , \tag{4}$$

where

$$\Delta_{zz}=\lambda\omega_p^2\left\{-\chi_0'+\left[\frac{\omega-\omega_c}{2\omega}\right]\left[1-\frac{\omega_c}{\omega}\left[1-\frac{T_\parallel}{T_\perp}\right]\right]\chi_{-1}'+\left[\frac{\omega+\omega_c}{2\omega}\right]\left[1+\frac{\omega_c}{\omega}\left[1-\frac{T_\parallel}{T_\perp}\right]\right]\chi_1'\right\} , \tag{5}$$

$$\chi_l(\omega)=\mathbf{P}\left[\frac{1}{\omega+l\omega_c}\right]-\frac{i\pi^{1/2}}{|k_\parallel|(2\kappa T_\parallel/m)^{1/2}}\exp\left[-\frac{m}{2\kappa T_\parallel}\left[\frac{\omega+l\omega_c}{k_\parallel}\right]^2\right] . \tag{6}$$

The prime denotes differentiation with respect to the argument ω, $\lambda=(k_\perp^2\kappa T_\perp/\omega_c^2 m)$, and P denotes the principal value. In Eq. (4), $D_{zz}^{(c)}$ is the cold-plasma dielectric tensor element appropriate to the retarded boundary conditions, and Δ_{zz} is the resonant hot-plasma contribution. From Eq. (6), it is relatively easy to see that in the limit $|k_\parallel|\to 0$, $\mathrm{Im}\chi_l(\omega)\to -\pi\delta(\omega+l\omega_c)$, and

$$\mathrm{Im}[(\omega+l\omega_c)\chi_l'(\omega)]\to -\pi(\omega+l\omega_c)\delta'(\omega+l\omega_c) = \pi\delta(\omega+l\omega_c)$$

since the Dirac δ function satisfies the relation $x\delta'(x)=-\delta(x)$. Thus, the real and imaginary parts of D_{zz} are, of course, related to each other through the well-known Kramers-Kronig relations[4] as a consequence of the laws of causality (i.e., the effect should not precede the cause).

From Eqs. (3)–(6), writing $k_\perp=\mathrm{Re}k_\perp+\mathrm{Im}k_\perp$ and assuming that $\mathrm{Re}k_\perp>\mathrm{Im}k_\perp$, we obtain

$$\left[\frac{c\,\mathrm{Re}k_\perp}{\omega}\right]^2=\mathrm{Re}D_{zz}=1-\frac{\omega_p^2}{\omega^2}\left[1+\mathrm{P}\left[\frac{k_\perp^2\kappa T_\parallel/m}{\omega^2-\omega_c^2}\right]\right]\approx 1-\frac{\omega_p^2}{\omega^2}\ , \tag{7}$$

and

$$2\,\mathrm{Im}k_\perp=\left[\frac{\omega^2\,\mathrm{Im}D_{zz}}{c^2\,\mathrm{Re}k_\perp}\right]\approx\left[1-\frac{\omega_p^2}{\omega^2}\right]^{1/2}\frac{\kappa T_\perp\omega^2\omega_p^2}{2mc^3\omega_c^2}\times\left\{\left[1-\frac{\omega_c}{\omega}\left[1-\frac{T_\parallel}{T_\perp}\right]\right](\omega-\omega_c)\,\mathrm{Im}\chi_{-1}'+\left[1+\frac{\omega_c}{\omega}\left[1-\frac{T_\parallel}{T_\perp}\right]\right](\omega+\omega_c)\,\mathrm{Im}\chi_1'\right\}\ . \tag{8}$$

From Eq. (6), we get

$$\mathrm{Im}\chi_l'(\omega)=\frac{2\pi^{1/2}(\omega+l\omega_c)}{|k_\parallel|\,k_\parallel^2(2\kappa T_\parallel/m)^{3/2}}\times\exp\left[-\frac{m}{2\kappa T_\parallel}\left[\frac{\omega+l\omega_c}{k_\parallel}\right]^2\right]\ . \tag{9}$$

It is interesting and physically instructive to note that in the limit $|k_\parallel|\to 0$, Eq. (8) becomes

$$2\,\mathrm{Im}k_\perp\approx\left[1-\frac{\omega_p^2}{\omega^2}\right]^{1/2}\left[\frac{\pi\kappa T_\parallel\omega^2\omega_p^2}{2mc^3\omega_c^2}\right]\times[\delta(\omega-\omega_c)+\delta(\omega+\omega_c)]\ . \tag{10}$$

The absorption coefficient per unit path length $\alpha=2\,\mathrm{Im}k\approx 2\,\mathrm{Im}k_\perp$ for $k_\perp\gg k_\parallel$. Thus from Eqs. (8) and (9), it is seen that for finite $k_\parallel$ there are absorption resonances having maximum values at $|\omega|\approx|\omega_c|\pm k_\parallel(2\kappa T_\parallel/m)^{1/2}$ and that the absorption approaches zero as $\omega\to\pm\omega_c$ and also for $|\omega\pm\omega_c|\to\infty$. That is, for finite $k_\parallel$ the Doppler effect splits the twofold degenerate $k_\parallel=0$ resonance of Eq. (10) into two closely spaced resonances of Eq. (8) instead of the usually expected Gaussian broadening. This splitting is illustrated in Fig. 1 for three different angles of propagation. The frequency integral of these curves is independent of θ and yields a value of τ of 3.8 and is the same value obtained from Eq. (10) with $R=130$ cm. It is also seen from Eqs. (8) and (10) that although this absorption is due to the finite size of the electron Larmor orbits [i.e., note the appearance of λ in Eq. (5)], it is mainly determined by $T_\parallel$ [see Eq. (10)] and is only weakly dependent on $T_\perp$ via cyclotron-overstability-type terms [see Eq. (8)].

The optical depth τ is given by

$$\tau=\int 2\,\mathrm{Im}k_\perp\,|dR|\ , \tag{11}$$

and the power P_a absorbed on a single transit of the absorption region is $P_a\approx P_0[1-\exp(-\tau)]$, where P_0 is the incident microwave power. If the wall-reflection coefficient r is large, then the fractional absorbed power $F\approx[1-\exp(-\tau)]/[1-r\exp(-\tau)]$, and multiple transits will enhance the ECR absorption considerably. For tokamak plasmas the confining magnetic field $B\propto R^{-1}$, where R is the major radius of the torus, and $dR=-(R/\omega_c)d\omega_c$. Then from Eqs. (10) and (11), for near perpendicular propagation of the ordinary wave through the toroidal plasma, we get

$$\tau\approx(1-\omega_p^2/\omega^2)^{1/2}(\pi R\,\omega_p^2\kappa T_\parallel/2\omega mc^3)\ . \tag{12}$$

This result of Eq. (12) agrees with those of Chu[5] and An-

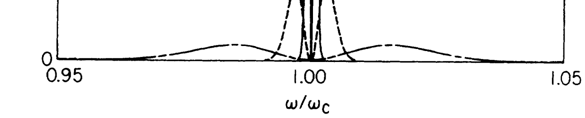

FIG. 1. Plots of 2 $\mathrm{Im}k_\perp$ of Eq. (8) as a function of ω/ω_c for three different angles of propagation: (——) is for $\theta\simeq 0.5°$, (— — —) for $\theta\simeq 2°$, and (—-—) for $\theta\simeq 10.0°$, where $(\pi/2-\theta)$ is the angle between **k** and **B**. Conditions are $n\simeq 3\times 10^{13}$ cm^{-3}, $T_\perp=3$ keV, $T_\parallel=2$ keV, and $(\omega_c/2\pi)=9.0\times 10^{10}$ Hz. The frequency integral of these curves is independent of θ and yields a value of τ of 3.8 and is the same as the value obtained from Eq. (10) with $R=130$ cm.

tonson and Manheimer,[6] for O-mode with $\omega \simeq \omega_c$ when the plasma has no pressure anisotropy (i.e., when $T_\perp = T_\parallel = T$), and is also valid for anisotropic plasma when $k_\parallel \approx 0$. Equation (12) was verified in a wave propagation experiment in the Princeton Large Torus (PLT).[7] It is seen from Eq. (8) that the cyclotron overstability terms $(1 \pm (\omega_c/\omega)\ [1-(T_\parallel/T_\perp)])$ tends to one as $T_\perp \approx T_\parallel$. Hence the result of Eq. (12) is valid also for an isotropic plasma when $k_\parallel$ is small but finite.

For $k_\parallel \approx 0$, the wave absorption is localized near the resonance zone at $R \approx R_c$ where $\omega \approx \omega_c$. However, when $k_\parallel \neq 0$, there exist two closely spaced resonant layers centered at R_1 and R_2, respectively, such that $\omega = \omega_c(R_1) + k_\parallel(2\kappa T_\parallel/m)^{1/2}$ and $\omega = \omega_c(R_2) - k_\parallel(2\kappa T_\parallel/m)^{1/2}$, and there is no absorption at $R_c = (R_1+R_2)/2$. Thus, $|R_1 - R_2| \approx R_c[2k_\parallel(2\kappa T_\parallel/m)^{1/2}]$ and is linearly proportional to $k_\parallel$. This double-resonant layer might prove beneficial in suppressing plasma MHD instabilities with certain wavelengths; e.g., for $|R_1 - R_2| \approx p\lambda$ or $(p+\frac{1}{2})\lambda$, where p is an integer. This is similar to a feedback stabilization via a Fabry-Perot system. Furthermore, in the resonant layer R_1 the wave energy and momentum are transferred to comoving electrons (i.e., to electrons with z velocities v_z which are parallel to $\mathbf{B} = B\hat{\mathbf{i}}_z$); while in the resonant layer R_2 the wave energy and momentum are transferred to countermoving electrons (i.e., to electrons with v_z which are antiparallel to $\mathbf{B}$). This means that in ECR current drive experiments, the induced steady-state current will flow in opposite directions on either side of R_c (i.e., the current at R_1 will be antiparallel to the current at R_2), and there is no rf-induced current at R_c. Thus it appears with finite-$k_\parallel$ O-mode ECRH one can control not only the electron-pressure profile but also the current profile which may improve tokamak MHD behavior and confinement.

When $T_\perp \neq T_\parallel$ it is extremely difficult to obtain an analytic expression for τ from Eqs. (8) and (11). However, since the dominant absorption occurs only near the resonant layers R_1 and R_2, one can show that for the R_1 layer

$$\tau(R_1) \approx (\tau/2)\{1 + (k_\parallel/\omega_c)(2\kappa T_\parallel/m)^{1/2}[(T_\perp/T_\parallel)-1]\} ,$$

and for the R_2 layer

$$\tau(R_2) \approx (\tau/2)\{1 - (k_\parallel/\omega_c)(2\kappa T_\parallel/m)^{1/2}[(T_\perp/T_\parallel)-1]\} .$$

Thus, if the first absorption layer is optically thick [i.e., $\tau(R_1$ or $R_2) \gg 2$], the wave energy never reaches the second layer. In interpreting future ECRH experiments one must bear this point in mind.

The theory upon which O-mode ECRH experiments in toroidal plasmas are currently based is the linear theory for hot plasmas. For $T_\perp = T_\parallel$, it is shown elsewhere[5] that the result of Eq. (12) is unaltered even if one takes account of the broadening due to the relativistic mass variation when $k_\parallel \approx 0$. Since the relativistic effect broadens the resonance only towards lower values of ω_c, the Doppler splitting will always occur even for small $k_\parallel$. But, when

$$(k_\parallel/k) \leq [\kappa(2T_\perp + T_\parallel)]/[3(mc^2)^{1/2}(2\kappa T_\parallel)^{1/2}] ,$$

$\tau(R_1)$ will differ significantly from $\tau(R_2)$ keeping $\tau(R_1) + \tau(R_2) = \tau$. We have not taken this effect into account since the gross features presented here are always there even for very small $k_\parallel$. For very small $k_\parallel$ one must take account of the relativistic broadening in evaluating $\tau(R_1)$ and $\tau(R_2)$. In this paper we have examined the results of a finite $k_\parallel$ linear theory as a first step toward prescribing the conditions needed for efficient application of ECRH in tokamak plasmas. However, it should be noted that the quasilinear and nonlinear theories must be developed in conjunction with experiments to determine fully the effectiveness of ECR for the heating regimes of interest and for steady-state rf current drive in tokamaks.

In conclusion, it is therefore clear that the Doppler effect cannot only lead in most cases to a Gaussian broadening of the absorption lines as found in the literature, but also in some cases to a splitting into two distinct absorption lines. This is a new addition to the plasma-physics literature.

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