Obtainment of thermal noise from a pure quantum state

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It is shown that if one has access to only one mode of a two-mode squeezed-vacuum state, the photon statistics of this mode is indistinguishable from that of a thermal distribution. Parametric interactions that give rise to two-mode squeezing thus provide a mechanism for thermalization that is intrinsically quantum mechanical.

Two-mode squeezed-coherent states have received considerable theoretical attention[1–3] and have recently been generated experimentally via four-wave mixers[4–6] and parametric down converters.[7] In such parametric processes photons are generated in pairs with one photon at frequency $\omega_0+\omega$ and the other at $\omega_0-\omega$ where ω_0 is a carrier frequency which is half the pump frequency for a parametric down converter or equal to the pump frequency for four-wave mixers. We will arbitrarily call the mode at frequency $\omega_0+\omega$ the signal and the mode at $\omega_0-\omega$ the idler. The signal and idler modes can be spatially separate as in the case of the photon-correlation experiments of Friberg, Hong, and Mandel.[8,9] For squeezed-state experiments the optics is generally arranged so that the signal and idler beams are collinear. Since the signal and idler modes have different frequencies they can be spatially separated with a dispersive element even if generated collinearly. Hence one can have access to the signal mode or idler mode separately. Here it is shown that the light in the signal mode or the idler mode is indistinguishable from thermal light when the incoming light is in the vacuum state. This is remarkable since the squeezing Hamiltonian which transforms the vacuum state into a two-mode squeezed state does not give rise to classical chaotic behavior and the two-mode squeezed state itself is a pure quantum state. This pure state has two parts, the signal and idler which when viewed separately have photon statistics that are indistinguishable from thermal light.

Four-wave mixing or parametric down conversion is a process by which an intense pump beam modulates the susceptibility of a nonlinear medium on the time scale of an optical cycle. The time-dependent susceptibility does parametric work on the vacuum to create photon pairs. Equivalently one can regard the pump as modulating the index of refraction or the optical path length of the medium. In fact, parametric photon pair production could in principle be achieved by wiggling one of the mirrors of an empty optical cavity[10] at twice the cavity's resonant frequency. Hence the mechanism by which a parametric process generates thermal noise from the vacuum can be regarded as being similar to the mechanism by which a mirror undergoing constant acceleration generates thermal noise from the vacuum. This latter system has played a significant role in the discussion of black-hole evaporation.[11–13]

The calculations here are carried out in the Schrödinger picture. Let a_1 and a_2 denote the usual boson annihilation operators for the signal and idler mode, respectively, i.e.,

$$[a_i,a_j^\dagger]=\delta_{ij} \tag{1}$$

and

$$[a_i,a_j]=0 \tag{2}$$

for i, $j\in\{1,2\}$. For later use in determining the classical correspondence it is useful to note that the electric field operator or vector potential, when expressed in terms of the operators a_1 and a_2, scales as $\hbar^{1/2}$. For example, the signal mode electric field has the form

$$E_s=\hbar^{1/2}[E_0a_1+E_0^*a_1^\dagger]\ . \tag{3}$$

The Hamiltonian for parametric pair production has the form[1]

$$H=H_0+H_I\ , \tag{4}$$

where

$$H_0=\hbar(\omega_0+\omega)a_1^\dagger a_1+\hbar(\omega_0-\omega)a_2^\dagger a_2\ , \tag{5}$$

and

$$H_I=i\hbar k(t)[a_1a_2e^{-2i(\phi-\omega_0t)}-a_1^\dagger a_2^\dagger e^{2i(\phi-\omega_0t)}]\ , \tag{6}$$

where $k(t)$ is an arbitrary function of time.

An initial state $|i\rangle$ at time $t_i=0$ will evolve, under the action of H, into the final state $|f\rangle$ at time $t_f=t$ via the unitary transformation[1–3,14,15]

$$|f\rangle=S(\xi,\phi-\omega_0t)e^{-iH_0t/\hbar}|i\rangle\ , \tag{7}$$

where

$$\xi=\int_0^t k(t')dt'\ , \tag{8}$$

and

$$S(\xi,\phi)=\exp[\xi(a_1a_2e^{-2i\phi}-a_1^\dagger a_2^\dagger e^{2i\phi})] \tag{9}$$

is the two-mode squeezing operator.

At this point it is useful to introduce the operators

$$K_+ = a_1^\dagger a_2^\dagger \ , \tag{10}$$

$$K_- = a_1 a_2 \ , \tag{11}$$

and

$$K_z = \tfrac{1}{2}[a_1^\dagger a_1 + a_2 a_2^\dagger] \ . \tag{12}$$

These operators satisfy the commutation relations for the SU(1,1) lie algebra,[3,16,17]

$$[K_-, K_+] = 2K_z \ , \tag{13}$$

$$[K_z, K_\pm] = \pm K_\pm \ . \tag{14}$$

When the initial state $|i\rangle$ is the vacuum state $|0\rangle$ specified by

$$a_1 |0\rangle = 0 \tag{15}$$

and

$$a_2 |0\rangle = 0 \ , \tag{16}$$

Eq. (7) can be simplified to

$$|f\rangle = \exp[\gamma K_+ - \gamma^* K_-] |0\rangle \ , \tag{17}$$

where

$$\gamma = -\xi e^{2i(\phi - \omega_0 t)} \ . \tag{18}$$

It can be shown[3,17] that the unitary operator appearing in Eq. (17) can be factored as follows:

$$\exp[\gamma K_+ - \gamma^* K_-] = \exp\left[\left[\frac{\gamma}{|\gamma|} \tanh|\gamma| \right] K_+ \right] \times \exp\{-2[\ln(\cosh|\gamma|)] K_z\} \times \exp\left[\left[\frac{-\gamma^*}{|\gamma|} \tanh|\gamma| \right] K_- \right] \ . \tag{19}$$

From Eq. (11)

$$K_- |0\rangle = 0 \ , \tag{20}$$

consequently

$$\exp\left[\left[\frac{-\gamma^*}{|\gamma|} \tanh|\gamma| \right] K_- \right] |0\rangle = |0\rangle \ . \tag{21}$$

Further, from Eq. (12)

$$K_z |0\rangle = \tfrac{1}{2} |0\rangle \tag{22}$$

so

$$\exp\{-2[\ln(\cosh|\gamma|)] K_z\} |0\rangle = \operatorname{sech}|\gamma| \, |0\rangle \ . \tag{23}$$

Substituting Eq. (19) into (17) and using Eq. (21) and (23) one obtains

$$|f\rangle = \operatorname{sech}|\gamma| \exp\left[\left[\frac{\gamma}{|\gamma|} \tanh|\gamma| \right] K_+ \right] |0\rangle \ . \tag{24}$$

Expanding the exponential, this state can be written as

$$|f\rangle = \operatorname{sech}|\gamma| \sum_{n=0}^{\infty} \left[\frac{\gamma}{|\gamma|} \tanh|\gamma| \right]^n |n\rangle_1 |n\rangle_2 \ , \tag{25}$$

where $|n\rangle_1$ is the normalized number operator eigenstate for the signal mode

$$|n\rangle_1 = \frac{[a_1^\dagger]^n}{\sqrt{n!}} |0\rangle_1 \ . \tag{26}$$

Similarly, $|n\rangle_2$ is the normalized number operator eigenstate for the idler mode.

Let σ_1 denote an operator which operates on the signal mode state space, i.e., the space spanned by the state-vectors $|n\rangle_1$. Then it is straightforward to show from Eq. (25) that

$$\langle f | \sigma_1 | f \rangle = \operatorname{sech}^2|\gamma| \sum_{n=0}^{\infty} (\tanh|\gamma|)^{2n} \langle n | \sigma_1 | n \rangle \ . \tag{27}$$

We now compare this expectation value with the thermal average

$$\langle \sigma_1 \rangle_\beta = \mathrm{Tr}\{\rho \sigma_1\} \ , \tag{28}$$

where the density matrix ρ has the form

$$\rho = \sum_{n=0}^{\infty} P_n |n\rangle\langle n| \tag{29}$$

and

$$P_n = e^{-n\beta\hbar\omega_s}(1 - e^{-\beta\hbar\omega_s}) \ , \tag{30}$$

and

$$\beta = \frac{1}{k_B T} \ . \tag{31}$$

Here k_B is Boltzmann's constant, T is the temperature, and $\omega_s = \omega_0 + \omega$ is the signal mode frequency. One then has

$$\langle \sigma_1 \rangle_\beta = \sum_{n=0}^{\infty} P_n \langle n | \sigma_1 | n \rangle \ . \tag{32}$$

Comparing Eq. (27) with (29) one sees from Eq. (30) that the averages are equal provided

$$\tanh^2|\gamma| = e^{-\beta\hbar\omega_s} \ . \tag{33}$$

Since this equivalence holds for an arbitrary operator σ_1 we have shown that the photon statistics of the signal mode is indistinguishable from the photon statistics of a thermal source with temperature

$$T_s = \frac{\hbar\omega_s}{2k_B \ln(\coth|\gamma|)} \ . \tag{34}$$

Similar expressions result when one considers the photon statistics of the idler mode.

It can now be shown that the system described by the Hamiltonian, Eq. (4), is well-behaved in the limit $\hbar \to 0$. To this end, let the initial state $|i\rangle$ have the form

$$|i\rangle=|i\rangle_1|i\rangle_2\ , \tag{35}$$

where the signal is in the coherent state of the form

$$|i\rangle_1=e^{-|\alpha_1|^2/2\hbar}\sum_{n=0}^{\infty}\frac{\alpha_1^n}{\hbar^{n/2}\sqrt{n!}}|n\rangle_1\ , \tag{36}$$

where $\hbar^{1/2}\alpha_1$ is the coherent-state amplitude. The idler is taken to be in the coherent state obtained by replacing the subscripts 1 with 2.

In Eq. (36) $\hbar$ has been factored out of the coherent-state amplitude in such a way that the expectation value of E_s remains fixed as $\hbar\rightarrow 0$. That is, the classical limit is taken in a way that holds the classical part of the field amplitudes fixed. The final state $|f\rangle$ into which the state Eq. (35) evolves is determined by Eq. (7). The expectation value of E_s with respect to $|f\rangle$ is easily evaluated by using the standard result[1-3]

$$S^\dagger(\xi,\phi)a_1S(\xi,\phi)=\cosh(\xi)a_1-e^{2i\phi}\sinh(\xi)a_2^\dagger\ . \tag{37}$$

One then has

$$\langle f|E_s|f\rangle=E_0[\cosh(\xi)\alpha_1 -e^{2i\phi}\sinh(\xi)\alpha_2^*]e^{-i(\omega_0+\omega)t}+\text{c.c.} \tag{38}$$

which is independent of $\hbar$. On the other hand, the variance

$$(\Delta E_s)^2\equiv\langle f|E_s^2|f\rangle-\langle f|E_s|f\rangle^2 \tag{39}$$

scales as $\hbar$ and vanishes in the classical limit $\hbar\rightarrow 0$. Hence Eq. (38) characterizes the classical response of the squeezer. The response is linear in the amplitudes of the signal and idler input. Since the classical behavior of the squeezer is nonchaotic, the thermal noise characterized by the temperature given by Eq. (34) arises from an intrinsically quantum mechanism, i.e., the amplification of vacuum fluctuations.

In conclusion it has been shown that an initial vacuum state will evolve under the influence of the Hamiltonian, Eq. (4), into the two-mode squeezed state Eq. (25). This is a pure quantum state. However, if one looks at the quantum statistics of the signal mode or the idler mode alone the light is chaotic, exhibiting the statistics of thermal light. The Hamiltonian, Eq. (4), describes a system consisting of a nonlinear medium bathed with an intense classical pump for the case when pump depletion can be neglected. Hence a thermalization mechanism by which a very small fraction of the coherent pump energy is converted into chaotic light has been described. Since the system's behavior is nonchaotic in the classical limit $\hbar\rightarrow 0$ for all values $|\gamma|$ of the coupling parameter, a thermalization mechanism has been described which is intrinsically quantum mechanical.

[1]C. M. Caves and B. L. Schumaker, Phys. Rev. A **31**, 3068 (1985).
[2]B. L. Schumaker and C. M. Caves, Phys. Rev. A **31**, 3093 (1985).
[3]B. L. Schumaker, Phys. Rep. **135**, 317 (1986).
[4]R. E. Slusher, L. W. Hollberg, B. Yurke, J. C. Mertz, and J. F. Valley, Phys. Rev. Lett. **55**, 2409 (1985).
[5]R. M. Shelby, M. D. Levenson, S. H. Perlmutter, R. G. DeVoe, and D. F. Walls, Phys. Rev. Lett. **57**, 691 (1986).
[6]M. W. Maeda, P. Kumar, and J. H. Shapiro, Opt. Lett. **12**, 161 (1987).
[7]L. Wu, H. J. Kimble, J. L. Hall, and H. Wu, Phys. Rev. Lett. **57**, 2520 (1986).
[8]S. Friberg, C. K. Hong, and L. Mandel, Phys. Rev. Lett. **54**, 2011 (1985).
[9]C. K. Hong and L. Mandel, Phys. Rev. Lett. **56**, 58 (1986).
[10]H. A. Haus and Y. Yamamoto, Phys. Rev. A **34**, 270 (1986).
[11]S. A. Fulling and P. C. Davies, Proc. R. Soc. London, Ser. A **348**, 393 (1976).
[12]P. C. W. Davies and S. A. Fulling, Proc. R. Soc. London, Ser. A **356**, 237 (1977).
[13]W. R. Walker, Phys. Rev. D **31**, 767 (1985).
[14]B. R. Mollow and R. J. Glauber, Phys. Rev. **160**, 1097 (1967).
[15]B. R. Mollow, Phys. Rev. **162**, 1256 (1967).
[16]B. Yurke, S. L. McCall, and J. R. Klauder, Phys. Rev. A **33**, 4033 (1986).
[17]D. R. Truax, Phys. Rev. D **31**, 1988 (1985).