Photon and energy yields of bremsstrahlung produced by β particles from ^{90}Sr, ^{90}Y, and ^{204}Tl in thick targets of Cu, Cd, Ta, and Pb

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External bremsstrahlung spectra produced by the complete absorption of β particles from the decay of ^{90}Sr, ^{90}Y, and ^{204}Tl in thick targets of Cu, Cd, Ta, and Pb are measured using a NaI(Tl) detector. The photon and energy yields and energy-yield constant are calculated using the unfolded bremsstrahlung spectra. The experimental data are compared with the numerical calculations based on the formulation of Pratt and his collaborators and the Born approximation theory of Bethe and Heitler. The experimental data agree with the Elwert-corrected Bethe-Heitler theory within experimental uncertainties for Cu and Cd, and a systematic positive deviation is found for Ta and Pb. The experimental data show good agreement with the numerical screened calculations of Pratt and his collaborators. The energy-yield constant is found to exhibit an energy dependence.

I. INTRODUCTION

The data on absolute intensity, spectral and angular distribution of bremsstrahlung produced in thick targets are important for studies in nuclear radiation transport, shielding, dosimetry and other application purposes. When energetic electrons pass through matter, they are accelerated in Coulomb fields of nuclei and atomic electrons, emitting bremsstrahlung radiation. The intensity, energy spectrum and angular distribution of bremsstrahlung depend on incident energy and angle of incidence of the electrons as well as on the material and shape of the target.[1]

The classical theory of bremsstrahlung has been given by Kramers.[2] The quantum-mechanical theory for the bremsstrahlung of relativistic electrons has been developed by Bethe and Heitler[3] using Dirac's relativistic theory of the electron and the first approximation of Born. Elwert[3] gave a correction factor for the Bethe-Heitler theory (EBH) taking into account the effects of the Coulomb field towards the low-energy end of the spectrum. The nonrelativistic theory has been developed by Sommerfeld[4] using (unscreened) hydrogenlike wave functions. Tseng, Pratt, and collaborators[5-7] developed a quantum-mechanical theory for the bremsstrahlung of relativistic electrons using relativistic (screened) self-consistent-field wave functions. Exact numerical screened calculations (ES) have been done by Pratt and his collaborators, resulting in an extensive tabulation of the photon-energy spectrum from 1 keV to 2 MeV.[6] A comprehensive set of bremsstrahlung cross sections (differential in the energy of the emitted photons) has been tabulated[8] for electrons with energies from 1 keV to 10 GeV incident on neutral atoms with atomic numbers $Z=1-100$. In constructing this set, use has been made of the results of Pratt, Tseng, and collaborators[6] at energies below 2 MeV, and the analytical high-energy theory (with Coulomb corrections) of Davis, Bethe, Maximon, and Olsen[9] at energies above 50 MeV. A numerical interpolation scheme, applied to suitably scaled cross sections, was used to bridge the gap between the low-energy and high-energy theoretical results, and thus to obtain improved cross sections in the intermediate energy regions 2–50 MeV. These tabulations provide what is expected to be the best prediction of the photon-energy spectrum or singly differential cross section $d\sigma/dk$. An early useful review of the experimental and theoretical information on bremsstrahlung is given by Koch and Motz[10] and a more recent review by Pratt.[11] An exhaustive survey of the experiments is given by Nakel.[12] A comparison between theoretical single-collision cross sections and experiment is presented by Tseng and Pratt.[5] These comparisons show that simpler analytical approximations[3,4] agree with experimental data in some particular ranges of electron and photon energies and ES theory agrees satisfactorily with experimental data for all energies of the electron and photon. Recently good agreement has been found with ES theory (Hippler *et al.*,[13] Semann and Quarles[14]), although some discrepancies have been reported for the case of high-Z target materials for incident energy of 10 keV.[13]

Experiments have been done to investigate the thick-target bremsstrahlung process using monoenergetic electrons and beta particles.[15] Dance *et al.*[16] have made extensive measurements at the incident energies $T_0=0.0-2.8$ MeV for the targets of Be, Al, Fe, Sn, and Au. Edelesack *et al.*[17] have reported the results for the electrons of $T_0=1.0$, 1.5, and 2.0 MeV incident on the targets of Al, Cu, Ag, and Au. O'Dell *et al.*[18] have made experiments at $T_0=5.3-20.9$ MeV using a Au-W target. Levy *et al.*[19] have obtained the result at $T_0=1-25$ MeV for the Pb target. Bremsstrahlung spectra from 25 MeV electrons on Ta as a function of radiator thickness and emission angle have been studied.[20]

Thin-target bremsstrahlung, although rarely met in practice, is covered by well-developed theory.[8] Thick-target bremsstrahlung is a more practical situation and is far less adequately covered by theory and experiment because of its complexity. Calculations as well as measurements of the absolute thick-target bremsstrahlung yield are rather scarce. Bustard and Silvermann[21] reported bremsstrahlung yield constants for ^{90}Sr-^{90}Y in secular equilibrium and ^{90}Y alone for targets intermediate between thin and thick. In this paper the bremsstrahlung yields produced by the complete absorption of ^{90}Sr-^{90}Y and ^{204}Tl β particles in thick targets of Cu, Cd, Ta, and Pb are presented and the variation of the bremsstrahlung yields with atomic number of the target is studied. The experimental bremsstrahlung yields and the energy-yield constant are compared with those obtained from EBH and ES theories.

II. EXPERIMENTAL METHOD

The experimental details and procedure are given in detail elsewhere[15] and are therefore only briefly summarized here. The detector was a 50.8 mm thick and 50.8 mm diameter NaI(Tl) crystal mounted on an RCA 6199 photomultiplier tube. The output of the detector was fed to the ND 100 multichannel analyzer. The β isotope was ^{204}Tl and the end-point energy of the β spectrum is 764 keV. The targets of elements Cu, Cd, Ta, and Pb of size 20 mm $\times$ 20 mm and thickness sufficient to stop all β particles of ^{204}Tl were prepared. A perspex sheet was placed in between the source and the detector with sufficient thickness to stop all β particles of ^{204}Tl. The targets of different thicknesses were interposed between the source and the detector and the resulting bremsstrahlung spectra were recorded. The counts recorded when the target was placed below the β stopper (perspex sheet) give the internal bremsstrahlung originating from the β source, background radiation and the external bremsstrahlung generated in the perspex β stopper. Similarly the counts recorded when the target was placed above the β stopper give the external bremsstrahlung generated in the target material, the internal bremsstrahlung, and the background radiation. The difference between these two counts gives the external bremsstrahlung generated in the target folded by the detector response. Compared with the bremsstrahlung generated in the target materials of interest in the present study, the bremsstrahlung generated in the perspex material used as a β stopper is negligible.

III. ANALYSIS OF BREMSSTRAHLUNG PULSE-HEIGHT SPECTRA

The measured pulse-height scintillation spectra differ from the true bremsstrahlung energy spectra. The unfolding of the pulse-height distribution and conversion to the true energy spectra was carried out by taking into account the detector resolution, detector efficiency, and the escape of Compton-scattered photons. The details of the unfolding method are given elsewhere.[15] The self-absorption of bremsstrahlung radiation in different target materials was studied. Using the unfolded bremsstrahlung spectra of different target thicknesses, the absorption factor for bremsstrahlung radiation was calculated as follows. By assuming an average depth of production of the bremsstrahlung in the target, the bremsstrahlung spectrum was attenuated through the remainder of the target using a mass-attenuation coefficient dependent on photon-energy spectra. On inspection of the absorption-corrected bremsstrahlung spectra, it is found that for the case of average depth of production of $0.25R$, the corresponding spectral points of different target thicknesses agree with each other, where R is the range of β particles in the target. The average energy of the β electrons was taken as one third of the end-point energy of the β spectrum. From this it is obvious that the average depth of production of bremsstrahlung radiation is approximately $0.25R$. This is in agreement with the value suggested by Brich *et al.*[22] This implies that a relatively large fraction of bremsstrahlung production takes place near the top of the target. This is understandable since the average energy of the β particles decreases rapidly with depth of penetration. The intrinsic efficiency of the crystal was determined for normal incidence of photons on the crystal. The unfolded spectrum was corrected for γ detection (photopeak) and geometric efficiency of the crystal. Isotropic emission of bremsstrahlung radiation from the target is assumed in the present measurement.[15]

The major errors in the present measurement are mainly of statistical origin in addition to errors in determining intrinsic and geometrical detection efficiencies and photofractions. The details of the errors computed are given elsewhere.[15] The uncertainty in the measurement of β activity of the source was about 10% and is included in the calculation of errors. The total uncertainty in the bremsstrahlung-yield measurement was about 22% for one standard deviation.

IV. THEORY

The accurate evaluation of the bremsstrahlung produced in thick targets depends on knowledge of the entire electron-photon cascade set up in the target. However, Bethe and Heitler[3] gave the simple expression $n(W_e,k)$ for the spectral distribution of bremsstrahlung when an electron of total energy W_e is completely absorbed in a target with N atoms per unit volume. Here k is the energy of the bremsstrahlung photon emitted. In the present work, the distribution $n(W_e,k)$ was computed from the EBH and ES theories. For this purpose the single-collision differential cross section for the bremsstrahlung production integrated over all photon directions and electron energy loss per unit path length of the target were employed. The details of the method and the formula for $n(W_e,k)$ are given elsewhere.[15]

In the case of a β emitter with a continuous spectrum of electrons the bremsstrahlung spectrum is given as $S(k)$, the number of photons of energy k per unit energy interval per β disintegration. The bremsstrahlung spectral distribution $n(W_e,k)$ was integrated over respective β spectra of ^{204}Tl and ^{90}Sr-^{90}Y to arrive at $S(k)$.

An electron beam passing through matter can produce bremsstrahlung not only in the Coulomb field of a nucleus but also in the collision with an atomic electron

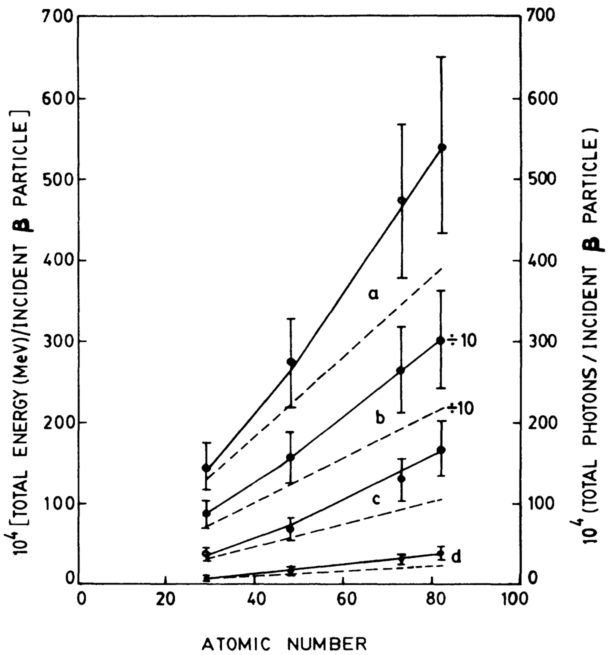

FIG. 1. Plot of experimental total bremsstrahlung yield as a function of atomic number of the target for ^{90}Sr-^{90}Y and ^{204}Tl β particles. The solid curve is based on the ES theory and the dotted curve on the EBH theory. Error bars represent the root mean square value of uncertainties at various points. (a) Energy yield of ^{90}Sr-^{90}Y β particles; (b) photon yield of ^{90}Sr-^{90}Y β particles; (c) photon yield of ^{204}Tl β particles; and (d) energy yield of ^{204}Tl β particles.

which then absorbs the recoil momentum and is ejected. The electron-electron bremsstrahlung generally gives only a small contribution to the total bremsstrahlung emission.[15] In the present calculations the contribution of this process is ignored.

The bremsstrahlung photon yield N, is defined as the number of photons generated per incident β particle. Similarly the energy yield I is the total bremsstrahlung energy radiated per incident β particle. These can be easily deduced from the bremsstrahlung spectrum $S(k)$, using the relations

$$I=\int_{k_{\min}}^{k_{\max}} kS(k)dk \ ,$$

$$N=\int_{k_{\min}}^{k_{\max}} S(k)dk \ .$$

The $S(k)$ values for ^{204}Tl is obtained from the present experiment and for ^{90}Sr-^{90}Y the experimental values are obtained from elsewhere.[15]

It has been shown, using monoenergetic electrons in carefully controlled experiments, that the bremsstrahlung energy I, in MeV per incident electron, can be expressed by

$$I=MZE^2 \ ,$$

where M is a constant and E is the initial kinetic energy of the electron. This relationship was first given by Roentgen and has been demonstrated to hold true for relativistic electrons up to 2.5 MeV. One can utilize this relation to predict the total bremsstrahlung by the absorption of any continuous β-ray spectrum which is given by Evans[23] as

$$I=MZ(E_{\mathrm{rms}})^2 \ \mathrm{MeV}/\beta \text{ disintegration} \ ,$$

where E_{rms} is the root mean square value of the kinetic energy of the continuous β-ray spectrum in MeV.

V. RESULTS AND DISCUSSION

The experimental bremsstrahlung-yield values are compared with those obtained from EBH and ES theories. The photon and energy-yield values are plotted as a function of atomic number of the target for ^{90}Sr-^{90}Y and ^{204}Tl β particles and is shown in Fig. 1. It is evident from Fig. 1 that the agreement between EBH theory and experimental points is good within experimental uncertainties in the case of Cu and Cd. For high-Z elements

TABLE I. Theoretical and experimental energy-yield constant data M (MeV$^{-1}\times10^{-3}$), for Cu, Cd, Ta, and Pb.

Electron energy	Element	Present experiment	EBH theory	ES theory	Value recommended by Evans
100–700 keV	Cu	0.252 + 0.055	0.214	0.242	0.700
(^{204}Tl)	Cd	0.286 + 0.063	0.237	0.302	0.700
	Ta	0.355 + 0.078	0.253	0.379	0.700
	Pb	0.405 + 0.080	0.253	0.402	0.700
50–2000 keV	Cu		0.392	0.427	0.700
(^{90}Y)	Cd		0.405	0.479	0.700
	Ta		0.419	0.556	0.700
	Pb		0.414	0.574	0.700
50–2000 keV	Cu	0.479 + 0.104	0.425	0.463	0.700
(^{90}Sr-^{90}Y)	Cd	0.544 + 0.119	0.441	0.524	0.700
	Ta	0.620 + 0.124	0.457	0.610	0.700
	Pb	0.631 + 0.126	0.452	0.630	0.700

Ta and Pb the EBH theoretical values are systematically lower compared with the experimental points. On the other hand, the experimental values show good agreement within experimental uncertainties with ES theory for all the elements studied.

From the energy-yield values, the energy-yield constant is calculated using the Evans's equation. The values of the energy-yield constant are given in Table I for Cu, Cd, Ta, and Pb. In practical usage this constant is directly proportional to spectral yields, and is therefore important to any shielding design determination. The energy-yield constant is found to exhibit an energy dependence. The values of energy-yield constant discussed and tabulated by Evans[23] from various theories and experiments lie in the domain of $0.4–1.1\times10^{-3}$ MeV^{-1}. The present theoretical values based on ES theory varies from 0.242 to 0.643×10^{-3} MeV^{-1} and 0.214 to 0.452×10^{-3} MeV^{-1} in the case of EBH theory. The present values of experimental energy-yield constant vary from 0.252 to 0.631×10^{-3} MeV^{-1} in the investigated energy region of 50 to 2274 keV, whereas Evans has recommended 0.7×10^{-3} MeV^{-1} up to 5 MeV electron energy. The breakdown of the Bethe-Heitler theory is due essentially to the use of oversimplified Born approximation wave functions in estimating the cross section for bremsstrahlung process. The theory of Pratt and his collaborators, where electron wave functions have been obtained in partial-wave series by numerically integrating the Dirac equation, describes the experimental data very well in the energy range studied, i.e., from 50 to 2000 keV in the atomic number range $Z=29-82$.

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