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Macroscopic variables of the macroscopic system and the theory of measurement. II. Models

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Models are presented as examples in order to illustrate the previously proposed theory of measurement in quantum mechanics [R. Fukuda, Phys. Rev. A **35**, 8 (1987)]. It is based on the specific structure of the Hilbert space of the macroscopic system. It is shown that after the interaction of an object with a detector we have to prepare the exponential of the extensive operators to recover the interference which is lost in the measuring process.

I. INTRODUCTION

Recently much attention has been paid to the problem of measurement in quantum mechanics, which has a long history of controversy in theoretical physics.[1] We have presented in Ref. 2 a novel theory of measurement based on the elucidation of the structure of the Hilbert space of the macroscopic systems. The purpose of this paper is to give several examples. These models are selected by the criterion of simplicity so that they will not be the realistic models of measurement but we can clearly illustrate the ideas of I by these models.

Let us first summarize the arguments of I, which are essential for the discussions below. They are given in two steps.

A. The structure of the Hilbert space of the macroscopic system

It is convenient to discuss a quantum-mechanical macroscopic system by a second quantized field theory. (By macroscopic we mean that the number of degrees of freedom N or the volume of the system is infinite with N/V fixed.) The system is assumed for simplicity to be described by a local bosonic field $\phi(\mathbf{x})$ [and its conjugate field $\pi(\mathbf{x})$]. Now associated with any macroscopic system are extensive and intensive variables. In the macroscopic limit the extensive variables do not exist as operators in the Hilbert space since their matrix elements are infinite. The intensive variables are divided into two classes. Class I involves operators which are represented by the averaging over the macroscopic region V; for example $\int d^3x\,\phi(\mathbf{x})/V$, $\int d^3x \int d^3y\,\phi(\mathbf{x})C(\mathbf{x}-\mathbf{y})\phi(\mathbf{y})/V$, etc., where $C(\mathbf{x}-\mathbf{y})$ is a c-number function and V does not necessarily represent the volume of the whole system but may be a subregion as long as it is macroscopic.

Class II contains the remaining operators. They have local information; for example, $\phi(\mathbf{x})$, $\pi(\mathbf{x})$, $\int d^3y\,C(\mathbf{x}-\mathbf{y})\phi(\mathbf{y})$, etc. [Here we have to subtract $\int d^3x\,\phi(\mathbf{x})/V$ from $\phi(\mathbf{x})$, for instance.]

The intensive operators of class I are known to lose fluctuations in the macroscopic limit. Mathematically this is due to the law of large number or to the central-limit theorem and can be proved by the stationary phase. It means that the class-I operators become c numbers. For the proof of this statement and for the determination of the time development of these c-number variables, it is convenient to employ the effective action used extensively in particle physics; the general rule is known once the Hamiltonian H^M of our macroscopic system is given. Some examples of the effective action are given below.

Let the effective action of an arbitrary operator A of class I be $\tilde{\Gamma}[a]$, then $\tilde{\Gamma}[a]$ has the form $V\Gamma[a]$ (Γ is the action density). Now

$$\partial\Gamma[a]/\partial a(t)=0$$

is the equation of motion of $a(t)$. Here $a(t)$ is not the expectation value of A but A is an operator of unit matrix multiplied by the c number $a(t)$. We call $a(t)$ a trajectory.

Since the extensive variables and class-I intensive variables do not exist as operators, the Hilbert space is defined by the class-II intensive operators. The c-number values of class-I intensive variables play the role of parameters to fix the Hilbert space. There are numerous actual phenomena of this kind throughout particle physics, nuclear physics, and solid-state physics, etc.

We recall here that the conjugate variables of the class-I intensive variables are extensive so that the operators that shift the value of the class-I operator have extensive character. However, since we have no extensive operators in the macroscopic limit, the class-I operators take a single value in one Hilbert space; there does not exist an operator connecting two states having different values of the class-I operators.

B. Measuring process

The measuring devices are macroscopic systems so that they have the above-stated structure of the Hilbert space; the Hilbert space is specified by the c-number values of the class-I intensive operators of the detector and each Hilbert space is constructed by the class-II intensive operators of the detector *and* operators of the object. Before interaction of an object with the detector, the state vector of the whole system is given by $|0\rangle_t\,|\psi\rangle_t$ where $|0\rangle_t$ is the state vector at time t of an object and $|\psi\rangle_t$ is that of the detector. The c-number values should be specified in order to fix $|\psi\rangle_t$. Let Λ be a dynamical variable of the object we are going to measure and $|\lambda_i\rangle$ be the eigenvector of Λ satisfying $\Lambda|\lambda_i\rangle=\lambda_i|\lambda_i\rangle$. In general we can write $|0\rangle_t=\sum_i c_i(t)|\lambda_i\rangle$. We neglect in the following the influence of the detector on the motion of the object and assume that $c_i(t)$'s are given functions of t. (If this back reaction is large, our device does not act as a good detector. We discuss this problem below using models.)

We adopt the following two conditions which any measuring theory should satisfy.

(i) The state vector which is of the form $|0\rangle_t|\psi\rangle_t=\sum_i c_i(t)|\lambda_i\rangle|\psi\rangle_t$ before interaction becomes after interaction $\sum_i c_i(t)|\lambda_i\rangle|\psi,i\rangle_t \equiv \sum_i c_i(t)|i\rangle_t$. Here, $|\psi,i\rangle_t$ is the state of the detector specific to the state $|\lambda_i\rangle$ of the object.

(ii) Two states vectors $|i\rangle_t$ and $|i'\rangle_t$ are totally incoherent if $i\neq i'$.

The second condition is equivalent to the statement that any operator cannot connect $|i\rangle_t$ and $|i'\rangle_t$ if $i\neq i'$. This is a necessary condition for the probabilistic interpretation of quantum mechanics. After the interaction we have the result

$$\left[\sum_{i'} c_{i'}^*(t)_t\langle i'|\right]\Lambda\left[\sum_i c_i(t)|i\rangle_t\right]=\sum_i |c_i(t)|^2\lambda_i$$

so that $|c_i(t)|^2$ is the probability for the system to be in the state $|i\rangle_t$. It is simply the orthogonality relation of $|i\rangle_t$ that makes this interpretation possible. The condition (ii) becomes crucial when we postulate that the result of the observation of *any* operator P after the interaction, i.e., after (i) has occurred, should be that

$$\left[\sum_{i'} c_{i'}^*(t)_t\langle i'|\right]P\left[\sum_i c_i(t)|i\rangle_t\right] = \sum_i |c_i(t)|^2_t\langle i|P|i\rangle_t ;$$

interference term $\langle i'|P|i\rangle$ $(i\neq i')$ should vanish for any P.

It is easy to see the validity of the above two conditions in our case. Let the Hamiltonian of the object be H^o and the interaction between the object and the detector be described by $H^{oM}(\Lambda,\phi,\pi)$. Then the total Hamiltonian H is given by $H=H^o+H^M+H^{oM}$. The total system evolves in time according to the law

$$\exp[-iH(t'-t)]|0\rangle_t|\psi\rangle_t = \sum_i c_i(t')\exp[-iH(t'-t)]|\lambda_i\rangle|\psi\rangle_t .$$

For each channel specified by $|\lambda_i\rangle$, $H^{oM}(\Lambda,\phi,\pi)$ can be replaced by $H^{oM}(\lambda_i,\phi,\pi)$. Now we observe the following important point.

Any variable whose value we read off on the detector belongs to the class-I intensive variable.

Examples are the position of the needle, grain density, or the current density, etc. The essential reason for the above statement is that there is no fluctuation associated with the class-I operators. (This may also be related to the usual assertion that there exist unavoidable uncertainties in the measuring process.) In our theory the measurement amounts to the determination of the Hilbert space to which our macroscopic system belongs.

Let A be a class-I operator of the detector chosen to measure Λ of the object. The c-number equation of motion of A is determined by the effective action density $\Gamma[\lambda_i,a]$ calculated by the Hamiltonian $H^M(\phi,\pi)+H^{oM}(\lambda_i,\phi,\pi)$. We solve $\partial\Gamma[\lambda_i,a]/\partial a(t)=0$ to get $a_i(t)$ which depends on i so that the condition (i) is satisfied. Since for different $a_i(t)$, the Hilbert space is different so that if $i\neq i'$, the two states $|i\rangle_t$ and $|i'\rangle_t$ are completely incoherent which is the condition (ii).

Here we observe the following two points.

(a) Since we have a deterministic equation of motion, there is no fluctuation in the time development of the detector variable $a_i(t)$; once λ_i is fixed we can predict definitely the state of the detector system. This is a required condition for the measuring apparatus.

(b) The equation which determines $a_i(t)$ is in general nonlocal in time. It involves time derivatives of an infinite order and we have to specify an infinite number of the initial conditions in order to determine $a_i(t)$. This seems to lead to a difficulty but actually it does not. In fact, in the actual experiment $a_i(t)$ need not be specified precisely but we have only to distinguish $a_i(t)$ from $a_j(t)$ for different i and j. For that purpose it will be sufficient to know a few initial values at $t=t_0$, say $a_i(t_0)$ or $da_i(t_0)/dt_0$. Remember that the terms with time derivatives of high order come from the quantum effects and contain factors of $\hbar$ and hence are small.

For the actual situation, N (or V) is not infinite and the interference remains. For finite N, extensive variables exist and class-I intensive variables receive fluctuations. These assertions are illustrated in the following by taking simple examples. We have in mind the Stern-Gerlach type of experiment where the observed variable Λ is the z component of the spin. The object particle is prepared in the form of the wave packet which is separated in two beams by the magnetic field. One (or both) of the beams interact with a detector. We do not discuss the process of the separation of beam and investigate the interaction of the one of the beams with the detector. The problem is thus reduced to measuring the position (of the wave packet) of the object. Our system is described by

$$(|\lambda_1\rangle+|\lambda_2\rangle)|\psi\rangle \tag{1}$$

before interaction where $|\lambda_1\rangle$ ($|\lambda_2\rangle$) represents the wave packet traveling through the first (second) region which we call the first (second) beam. These two regions are separated macroscopically.

II. MODEL 1

We first consider a model in which an object particle interacts with a macroscopic detector system consisting of N particles. The coordinate of the object is determined by measuring the center of mass of the detector—a class-I intensive variable. It is not necessary to employ the field theoretical method.

Let us introduce the coordinates $\mathbf{r}$ and $\mathbf{x}_i$ $(i=1-N)$ for the object particle and for the particles of the detector, respectively. The interaction potential is written as

$$V=g\sum_{i=1}^{N} W(\mathbf{r}-\mathbf{x}_i)+\sum_{i,j(\neq i)=1}^{N} v(\mathbf{x}_i-\mathbf{x}_j)\ . \tag{2}$$

The following assumptions are made here.

(i) The coupling constant g is small, so that the calculations are done in the lowest (nontrivial) order in g.

(ii) The detector has a compact (macroscopic) size in the sense that W does not change appreciably over the detector region.

Now we denote the center-of-mass coordinate of the detector system by $\mathbf{X}$ and the relative coordinates by $\mathbf{x}'_i$. Introducing $\mathbf{x}_i=\mathbf{X}+\mathbf{x}'_i$ $(\sum_i \mathbf{x}'_i=0)$ into W and then expanding in powers of $\mathbf{x}'_i$, the assumption (ii) allows us to write V as

$$V\simeq gNW(\mathbf{r}-\mathbf{X})+V^{\rm rel}\ , \tag{3}$$

where $V^{\rm rel}$ contains relative coordinates only. The precise statement of (ii) is that $\nabla^2 W/W \ll \langle \mathbf{x}'^2\rangle^{-1}$, where $\langle \mathbf{x}'^2\rangle$ is the expectation value of the square of the relative coordinate of any particle in the detector. Equation (2) leads us to the problem of separable coordinates. We could have started our model discussions from Eq. (3) neglecting $V^{\rm rel}$. In this case the detector system is a single particle with the coupling constant gN which is macroscopic. The total Hamiltonian is thus

$$H=\frac{1}{2\mu}\mathbf{p}^2+\frac{1}{2M}\mathbf{P}^2+gNW(\mathbf{r}-\mathbf{X})+H^{\rm rel}\ , \tag{4}$$

where $\mathbf{p}(\mu)$ is the momentum (mass) of the object and $\mathbf{P}$ $(M=Nm)$ is the total momentum (total mass) of the detector. We have assumed the equal mass m for particles in the detector. In Eq. (4) $H^{\rm rel}$ refers to relative coordinates. We take

$$W(\mathbf{r}-\mathbf{X})=\exp[-(\mathbf{r}-\mathbf{X})^2/a^2]\ , \tag{5}$$

where a represents the microscopic interaction range. This is chosen to make the analytic calculation possible. We do not write the wave function of the relative coordinates in the following.

Now the discussions are given in several steps. We first study the interaction of an object particle with a detector system and then come to the situation given in Eq. (1).

A. Wave function of the detector

We consider the case where the velocity of the center $\mathbf{r}_c(t)$ of the wave packet of the object is large so that the shape of the wave packet is not altered appreciably during the interaction. We solve first the Schrödinger equation for $\mathbf{X}$ under the potential $gNW[\mathbf{r}_c(t)-\mathbf{X}]$, where $\mathbf{r}_c(t)$ is assumed to be a given c-number function of t. The effect of the nonzero width of the wave packet of the object will be studied later in the discussion in Sec. II E below.

Let the wave function of the detector at time t_a be $\psi(t_a,\mathbf{X}_a)$. Then at later time we have

$$\psi(t_b,\mathbf{X}_b)=\int d\mathbf{X}_a K(t_b,\mathbf{X}_b;t_a,\mathbf{X}_a)\psi(t_a,\mathbf{X}_a)\ . \tag{6}$$

In Appendix A the kernel K is evaluated up to the first order in g. With $T=t_b-t_a$, the result is

$$K(t_b,\mathbf{X}_b;t_a,\mathbf{X}_a)=\left[\frac{mN}{2\pi i\hbar T}\right]^{3/2}\exp\frac{iN}{\hbar}S\ ,$$

$$S=m(\mathbf{X}_b-\mathbf{X}_a)^2/2T+g\int_{t_a}^{t_b}dt'(1+L)^{-1/2}\exp(\{(t_b-t')(t'-t_a)(\mathbf{X}_b-\mathbf{x}_a)^2/T-(t_b-t')[\mathbf{r}_c(t')-\mathbf{X}_a]^2$$
$$-(t'-t_a)[\mathbf{r}_c(t')-\mathbf{X}_b]^2\}/a^2T(1+L))\ , \tag{8}$$

$$L=2i\hbar(t_b-t')(t'-t_a)/a^2TmN\ . \tag{9}$$

We assume $\mathbf{r}_c(t)=\mathbf{r}_0+\mathbf{v}t$ with $\mathbf{v}$ constant and take $t_b=-t_a=T/2$. Then in the limit $|\mathbf{v}|\to\infty$, the region $t'\approx 0$ dominates the integral over t' in Eq. (8). By changing the integration variable from t' to $\tau=|\mathbf{v}|t'$, we get

$$K(t_b,\mathbf{X}_b;t_a,\mathbf{X}_a)$$
$$\sim(mN/2\pi i\hbar T)^{3/2}\exp\frac{iN}{\hbar}S'\quad \text{for}\ |\mathbf{v}|\to\infty\ , \tag{10}$$

$$S'=m(\mathbf{X}_{b,}-\mathbf{X}_a)^2/2T$$
$$+\frac{g\sqrt{\pi}aD}{|\mathbf{v}|}\exp\frac{D}{4a^2}[(\mathbf{X}_{b\perp}-\mathbf{X}_{a\perp})^2-2(\mathbf{r}_{0\perp}-\mathbf{X}_{b\perp})^2$$
$$-2(\mathbf{r}_{0\perp}-\mathbf{X}_{a\perp})^2]\ , \tag{11}$$

where $D=(1+i\hbar T/2mNa)^{-1}$ and $\mathbf{X}_{a\perp}(\mathbf{X}_{a\parallel})$, for example, is the component of $\mathbf{X}_a$ perpendicular (parallel) to $\mathbf{v}$. Note that in the limit $|\mathbf{v}|\to\infty$, the parallel components are unaltered by the interaction. S' is the effective action written as a function of t_a, $\mathbf{X}_a$, t_b, and $\mathbf{X}_b$. It is more convenient than the usual effective action Γ which is functional of $\mathbf{X}(t')$ for $t_a \le t' \le t_b$. It is easy to see that S' is obtained from Γ by solving $\partial\Gamma/\partial\mathbf{X}(t')=0$ for $t_a < t' < t_b$ and substituting the solution into Γ.

B. Macroscopic limit

Although our argument is independent of the initial wave function of the detector, it is convenient to take it as a wave packet of the form

$$\psi(t_a,\mathbf{X}_a)=(\sqrt{\pi}\eta_0)^{-3/2}\exp[-(\mathbf{X}_a-\mathbf{X}_0)^2/2\eta_0^2] \ . \quad (12)$$

In the limit $N\to\infty$, the integral over $\mathbf{X}_a$ in Eq. (6) is dominated by the stationary phase, which determines $\mathbf{X}_b$ as a function of T and $\mathbf{X}_a$ —a trajectory,

$$\begin{aligned}&\mathbf{R}\equiv\mathbf{X}_b-\mathbf{X}_a-\boldsymbol{\Delta}=0 \ ,\\ &\boldsymbol{\Delta}=\frac{gT\sqrt{\pi}}{m\,|\mathbf{v}|}\frac{\mathbf{r}_{0\perp}-\mathbf{X}_{a\perp}}{a}\exp[-(\mathbf{X}_{a\perp}-\mathbf{r}_{0\perp})^2/a^2] \ .\end{aligned} \quad (13)$$

Note that $\mathbf{X}_{b\parallel}=\mathbf{X}_{a\parallel}$. The quantum diffusion is absent in this limit and the wave packet does not show spreading. These results are valid for any t_b so that at any instant of time $\mathbf{X}_b$ is a c-number-valued quantity which specifies the Hilbert space defined by the relative coordinates of the detector system. We continue to use the wave function as a function of $\mathbf{X}_b$ although it has not a conventional meaning in the macroscopic limit.

The wave function at time t_b is calculated in Appendix B for small g and large N. The result is

$$\begin{aligned}\psi(t_b,\mathbf{X}_b)\sim\psi_s\exp\left[-\frac{gTaD}{2m\,|\mathbf{v}|}(-1+D\xi_\perp^2)\exp\left[-\frac{\xi_\perp^2}{a^2}\right]\right]\\ \times\int\delta^3(\mathbf{R})\psi(t_a,\mathbf{X}_a)d\mathbf{X}_a\\ +O(1/\sqrt{N})^3 \ ,\end{aligned} \quad (14)$$

where $\xi_\perp=\mathbf{X}_{b\perp}-\mathbf{r}_{0\perp}$ and ψ_s is the value of ψ at the stationary point. As is given in (B5) ψ_s has an infinite phase $\exp(icN)$, where c is proportional to g. The wave function is smeared out by the width η_0 of the initial wave packet, which is related to the efficiency of the detector.

Actually Eq. (13) fluctuates due to the finite size of the wave packet of the object. But this fluctuation is to be measured; a detector should detect this fluctuation also.

C. Interference term

Let us apply the above results to the state of Eq. (1). The detector is prepared in the first region. We first assume that there does not exist a detector in the second region. For the state $|\lambda_1\rangle|\psi\rangle$ the wave function $\psi_1(t_b,\mathbf{X}_b)$ after the interaction is given by Eq. (14) whereas for $|\lambda_2\rangle|\psi\rangle$ we set $g=0$ there since the separation of the two regions are assumed to be macroscopic and the interaction is taken to be short ranged. We write this as $\psi_2(t_b,\mathbf{X}_b)$. In the macroscopic limit, due to the δ function in Eq. (14), matrix elements of any well-defined operator between ψ_1 and ψ_2 vanish. The condition for this to be ensured is

$$\eta_0 \ll |\boldsymbol{\Delta}| \ , \quad (15)$$

which determines the efficiency of the detector. In the following we assume (15) and set $\mathbf{X}_a=\mathbf{X}_0$ neglecting the width η_0.

If another detector is placed in the second region, the argument goes through with a minor modification; the state ψ_1 or ψ_2 becomes a direct product of the two states referring to the two regions if the interaction between particles contained in two regions is neglected. Therefore the same reasoning as given above can be applied.

For finite N, interference remains, of course. Consider large but finite N and take the matrix element $(\psi_1,P\psi_2)$ of any operator P. If P consists of operators of the object and the relative coordinate of the detector then it is of the order $(1/\sqrt{N})^3$. In order to get the finite value of the order unity, P should involve the total momentum $\mathbf{P}=\hbar/i\partial/\partial\mathbf{X}_b$ of the detector in the form $\exp i\mathbf{P}\cdot\boldsymbol{\Delta}/\hbar$ to shift the center-of-mass coordinate. [It is further multiplied by $\exp(icN)$ to get rid of the infinite phase.]

It is the exponential of the macroscopic operator that recovers the interference.

Such an operator is hard to be prepared in the actual experiment. From the theoretical point of view, taking the expectation value of such an operator amounts to changing the Lagrangian (at some instant of time), i.e., to changing the theory. This is easily seen by writing $(\psi_1,e^{i\mathbf{P}\cdot\boldsymbol{\Delta}/\hbar}\psi_2)$ in the path-integral formula.

D. Back reaction on the object

Up to now, we have only considered the motion of the center of the wave packet of the object which is assumed to be a given function of t. We discuss here the motion of the wave packet and the effect of the interaction with the detector. Since we study the first-order effect in g, we are allowed to set $\mathbf{X}_b=\mathbf{X}_a=\mathbf{X}_0$ in the interaction term. The kernel $K(t_b,\mathbf{r}_b,t_a,r_a)$ for the object is given by Eqs. (7)–(9) with the replacement $mN\to\mu$, $\mathbf{X}_a\to\mathbf{r}_a$, $\mathbf{X}_b\to\mathbf{r}_b$, $\mathbf{r}_c(t)\to\mathbf{X}_0$. Now the following two limits are considered.

(i) $(\mathbf{r}_a-\mathbf{r}_b)^2\to\infty$, $(\mathbf{r}_a-\mathbf{X}_0)^2$ finite. This corresponds to the situation where the object passes near the detector at $t=t_a$. (This is not the case considered in Secs. II A–II C above.) In this case the integral over t' is dominated by the region $t'\approx t_a$. Then the kernel is proportional to

$$K\sim 1+i\frac{gN}{\hbar}\frac{(t_b-t_a)a^2}{\mathbf{r}_b^2}\alpha \ ,$$

where α is of the order unity. We can write

$$\frac{gN}{\hbar}\frac{Ta^2}{\mathbf{r}_b^2}=\frac{gN}{\mu\mathbf{v}^2}\frac{\mathbf{v}^2T^2}{\mathbf{r}_b^2}\frac{\mu a^2}{T\hbar} \ . \quad (16)$$

Let the object be an electron and v_0 be the velocity of the electron inside the atom and assume that a is of the order of the Bohr radius. Then the last factor is equal to

$$\frac{\mu v_0 a}{\hbar}\frac{a}{v_0}\frac{1}{T}\sim\left[\frac{a}{\mathbf{v}_0}\right]\frac{1}{T}$$

by uncertainty principle. Since a/v_0 is the typical atomic period, the last factor is of the order of 10^{-16} for $T=1$ sec. The second factor of Eq. (16) is of the order unity. Therefore the correction to the kernel due to the interaction is small if the kinetic energy $\mu\mathbf{v}^2$ of the object is comparable to the interaction energy gN, i.e., if it is of the macroscopic value.

(ii) $(\mathbf{r}_a-\mathbf{r}_b)^2\to\infty$, $(\mathbf{r}_a-\mathbf{X}_0)^2\to\infty$. This corresponds to the situation we have discussed in Secs. II A–II C, where $t_a=-T/2$, $t_b=T/2$. We take the limit $T\to\infty$. The integral over t' is dominated in this case by the region $t'\approx 0$ and the kernel is proportional to

$$K\sim 1+i\frac{gNTa}{|\mathbf{r}_b|}\beta\ ,$$

where β is a constant of the order unity. The magnitude of the correction term is given by Eq. (16) multiplied by $|\mathbf{r}_b|/a$ which is of the order 10^8 for $|\mathbf{r}_b|=1$ cm. It is still small if the object has the kinetic energy of the order N. Thus we are allowed to consider the object as a free wave packet, the shape of which does not change very much during the interaction if the velocity is large.

E. The total wave function

The wave function of the total system is written in terms of the path-integral formula where the path of the object is fixed and the path of the coordinate $\mathbf{X}$ is first summed up assuming the fixed trajectory of the object. Then it is given by

$$\psi(t_b,\mathbf{r}_b,\mathbf{X}_b)\propto\sum_{\text{Path } i}\int e^{(i/\hbar)A_0(P_i)}\psi_i(t_b,\mathbf{X}_b)\phi(t_a,\mathbf{r}_a)d\mathbf{r}_a\ , \tag{17}$$

where $\phi(t_a,\mathbf{r}_a)$ is the initial wave function (packet) of the object and $\sum_{\text{Path } i}$ is the summation over the possible paths of the object which connect $\mathbf{r}_a$ (at t_a) and $\mathbf{r}_b$ (at t_b). The function $\psi_i(t_b,\mathbf{X}_b)$ is given by Eq. (6) where K is evaluated with the fixed trajectory P_i of the object. $A_0(P_i)$ is the free action of the object particle corresponding to P_i. In the above discussions in Sec. II A–II D, we have assumed the free wave packet of the object with the center given by $\mathbf{r}_c(t)$. If the width of the packet is not large, the summation over the paths in Eq. (17) is dominated by the path $\mathbf{r}_c(t)$,

$$\psi(t_b,\mathbf{X}_b)\sim\int e^{(i/\hbar)A_0}\phi(t_a,\mathbf{r}_a)d\mathbf{r}_a\psi(t_b,\mathbf{X}_b)\ .$$

Here $\psi(t_b,\mathbf{X}_b)$ is given, in the limit $|\mathbf{v}|\to\infty$, by Eq. (14). For finite width of the object, we have to perform the summation over paths. Now different paths lead to different values of Δ given in Eq. (13). In order to detect the shape of the wave packet, the detector should satisfy

$$\eta_0\ll|\delta\Delta|$$

where $\delta\Delta$ is the variation of Δ due to the variation $\delta\mathbf{r}_0$ of $\mathbf{r}_0$, which is of the order of the width of the wave packet.

III. MODEL 2

As a second example, we take a macroscopic detector system which is described by a local Hermitian field $\phi(\mathbf{x})$ and the Hamiltonian is assumed to be

$$H^M=\int d^3x[\tfrac{1}{2}\dot\phi(\mathbf{x})^2+\tfrac{1}{2}\phi(\mathbf{x})\omega(\nabla^2)\phi(\mathbf{x})]\ , \tag{18}$$

where $\dot\phi\equiv d\phi/dt$ and $\omega(\nabla^2)$ represents the dispersion. The object is again given in terms of the wave packet but we concentrate our discussion on its center position $\mathbf{r}_c(t)$ which is assumed to be a given function of t. The interaction of the object and the detector is assumed to be of Yukawa type,

$$H^{oM}=e^2\int d^3x\frac{e^{-\mu|\mathbf{r}_c(t)-\mathbf{x}|}}{|\mathbf{r}_c(t)-\mathbf{x}|}\phi(\mathbf{x})\ , \tag{19}$$

where $1/\mu$ represents the microscopic interaction range. To measure the position of the object we choose the ϕ density defined by

$$\phi\equiv\frac{1}{V_0}\int_{(\mathbf{x}_0,V_0)}d^3x\ \phi(\mathbf{x})\ , \tag{20}$$

which belongs to class I. Here V_0 is an arbitrary macroscopic region around $\mathbf{x}_0$. If we choose V_0 as a rectangular region of the size L_{0x},L_{0y},L_{0z} so that $V_0=L_{0x}L_{0y}L_{0z}$, then

$$\phi=\frac{1}{\sqrt{V}}\sum_{\mathbf{k}}e^{i\mathbf{k}\cdot\mathbf{x}}\frac{\phi(\mathbf{k})}{V_0}M(\mathbf{k})\ , \tag{21}$$

where we have defined $\phi(\mathbf{x})=(1/\sqrt{V})\sum_k e^{i\mathbf{k}\cdot\mathbf{x}}\phi(\mathbf{k})$ and

$$M(\mathbf{k})=\prod_{i=x,y,z}\left[\frac{2\sin\frac{L_{0i}k_i}{2}}{k_i}\right]\ , \tag{22}$$

and V is the total volume.

A. The deterministic equation of ϕ

Now the kernel $K(t_b,\phi_b(\mathbf{x});t_a,\phi_a(\mathbf{x}))$ of this problem is a functional of $\phi_a(\mathbf{x})$ and $\phi_b(\mathbf{x})$. It is easily obtained by writing the Hamiltonian in terms of $\phi(\mathbf{k})$; by Eqs. (18) and (19) our problem is equivalent to the system of uncoupled forced harmonic oscillators. As is given in Appendix C, the result is expressed as

$$K=\prod_{\mathbf{k}}\left[\frac{\omega_k}{2\pi i\hbar\sin(\omega_k T)}\right]^{1/2}\times\exp\frac{i}{\hbar}S(t_b,\phi_b(\mathbf{k});t_a,\phi_a(\mathbf{k}))\ , \tag{23}$$

where $\omega_k\equiv\omega(\mathbf{k}^2)$. We write $\phi(\mathbf{k})$ in terms of ϕ given in Eq. (20) or (21) and the remaining independent variables $\phi'(\mathbf{k})$. Thus S becomes a functional of ϕ_a, ϕ_b, $\phi'_a(\mathbf{k})$, and $\phi'_b(\mathbf{k})$. In our case S can be separated as $S=S_0S_1$ where S_0 is a function of ϕ_a and ϕ_b which is nothing but the effective action of ϕ, and S_1 is of $\phi'_a(\mathbf{k})$ and $\phi'_b(\mathbf{k})$ (separation of variables). In the following we do not discuss S_1. A convenient way to obtain S_0 from S is to integrate over $\phi(\mathbf{k})$ with the constraint given in Eq. (20); $\exp(i/\hbar S_0)$ is proportional to

$$\exp\frac{i}{\hbar}S_0(t_b,\bar{\phi}_b;t_a,\bar{\phi}_a)$$
$$\sim\int\prod_{i=a,b}[d\phi_i(\mathbf{k})]\delta(\phi_i-\bar{\phi}_i)\exp\frac{i}{\hbar}S$$
$$\sim\int\prod_{i=a,b}[d\phi_i(\mathbf{k})]dJ_i\exp\frac{i}{\hbar}V_0J_i(\phi_i-\bar{\phi}_i)\exp\frac{i}{\hbar}S\ . \tag{24}$$

After integration over $\phi_a(\mathbf{k})$ and $\phi_b(\mathbf{k})$, we get the result shown in Eq. (C5). Let us write $\mathbf{r}_c(t)=\mathbf{r}_0+\mathbf{v}t$, where $\mathbf{v}$ is taken in the z direction, and consider the limit $|\mathbf{v}|\to\infty$. We assume that ω_k is independent of $\mathbf{k}$ in order to make the calculation simple, and write ω_k as ω. In $W(J_a,J_b)$ the term $\mathbf{D}$ becomes proportional to L_{0z} in the limit $|\mathbf{v}|\to\infty$ since in Eq. (C7) the summation $\sum_{\mathbf{k}}$ is dominated by the region $k_z\approx 0$ where 2 $\sin(L_{0z}k_z/2)/k_z\sim L_{0z}$. On the other hand, C in Eq. (C6) is proportional to V_0 which is easily seen by switching off the interaction; $e^2=0$. Therefore if we take the limit $L_{0z}\to\infty$, the integral in Eq. (C5) is dominated by the stationary phase,

$$\mathbf{J}\frac{C}{V_0}-\frac{1}{V_0}\mathbf{D}-\boldsymbol{\phi}=\mathbf{0}\ . \tag{25}$$

This is inserted in Eq. (C5) obtaining the effective action S_0,

$$S_0(t_b,\phi_b;t_a,\phi_a)=-\tfrac{1}{2}(\mathbf{D}+V_0\boldsymbol{\phi})C^{-1}(\tilde{\mathbf{D}}+V_0\tilde{\boldsymbol{\phi}})\ . \tag{26}$$

The wave function is obtained by

$$\psi(t_b,\phi_b)\sim\int\exp\frac{i}{\hbar}S_0(t_b,\phi_b;t_a,\phi_a)\psi(t_a,\phi_a)d\phi_a\ .$$

Since S_0 is of the order L_{0z} the integral over ϕ_a is again dominated by the stationary phase, which determines ϕ_b, in terms of t_a,t_b,ϕ_a;

$$\phi_b+(C_{22}^{-1}/C_{21}^{-1})\phi_a+\frac{1}{V_0}[D_1+(C_{22}^{-1}/C_{21}^{-1})D_2]=0\ . \tag{27}$$

Equation (27) is a c-number relation. Since $C_{22}^{-1}/C_{21}^{-1}=-\cos(\omega T)$, we get by using Eq. (C7),

$$D_1-D_2\cos(\omega T)=\frac{4\pi e^2}{V}\sum_{\mathbf{k}}M(\mathbf{k})\int_{t_a}^{t_b}dt\cos[\mathbf{k}\cdot(\mathbf{r}_c(t)-\mathbf{x}_0)]\sin[\omega(t_b-t)]$$
$$\to 2\frac{4\pi e^2}{\omega}L_{0z}\int\frac{dk_z}{2\pi}\int\frac{d\mathbf{k}_\perp}{(2\pi)^2}\frac{M(\mathbf{k}_\perp)}{\mathbf{k}_\perp^2+\mu^2}\int_{t_a}^{t_b}dt\sin[\omega(t_b-t)]$$
$$\times\mathrm{Re}\{\exp[ik_z(\mathbf{x}_{0z}-\mathbf{r}_{0z}-|\mathbf{v}|t)]$$
$$\times\exp[i\mathbf{k}_\perp\cdot(\mathbf{x}_{0\perp}-\mathbf{r}_{0\perp})]\}\quad\text{for }|\mathbf{v}|\to\infty\ , \tag{28}$$

where $\mathbf{k}_\perp$ is the component of $\mathbf{k}$ perpendicular to $\mathbf{v}$. Thus we finally arrive at

$$\phi_b-\phi_a\cos(\omega T)=-\frac{8\pi e^2}{\omega}\frac{1}{L_{0x}L_{0y}}\int_{t_a}^{t_b}dt\sin[\omega(t_b-t)]\delta(\mathbf{x}_{0z}-\mathbf{r}_{0z}-|\mathbf{v}|t)\int\frac{d\mathbf{k}_\perp}{(2\pi)^2}\frac{\cos[\mathbf{k}_\perp\cdot(\mathbf{k}_{0\perp}-\mathbf{r}_{0\perp})]}{\mathbf{k}_\perp^2+\mu^2}M(\mathbf{k}_\perp)\ . \tag{29}$$

If we take $L_{0x}=L_{0y}=1/\mu$ and approximate $M(\mathbf{k}_\perp)/L_{0x}L_{0y}\cong 1$, then the equation can be written as

$$0=\hat{\phi}\equiv\phi_b-\phi_a\cos(\omega T)+\Delta\ , \tag{30}$$

$$\Delta=\begin{cases}\Delta_0, & t_a<(\mathbf{x}_{0z}-\mathbf{r}_{0z})/|\mathbf{v}|<t_b\\ 0, & \text{otherwise}\ ,\end{cases} \tag{31}$$

$$\Delta_0=\frac{8\pi e^2}{\omega|\mathbf{v}|}\frac{1}{2\pi}K_0(\mu|\mathbf{x}_{0\perp}-\mathbf{r}_{0\perp}|)\times\sin\left[\omega\left[t_b-\frac{\mathbf{x}_{0z}-\mathbf{r}_{0z}}{|\mathbf{v}|}\right]\right]\ . \tag{32}$$

Here K_0 is the modified Bessel function.

Before interaction $\phi=0$ so that we set $\phi_a=0$, then the region of $\mathbf{x}_0$ giving the nonvanishing value of ϕ_b makes a tube of radius $1/\mu$ along the trajectory of the object. For large $|\mathbf{v}|$ the length of this tube becomes infinitely long so that L_{0z} can be taken to be arbitrarily large, of the order $|\mathbf{v}|T$. In this case we obtain a finite fluctuationless value for ϕ_b. Note that ϕ_b oscillates both in t_b and $\mathbf{x}_{0z}$, a consequence of our integrable system.

B. The wave function, the interference term, and the back reaction

Since in model 2 the separation of the variables ϕ and $\phi'(\mathbf{k})$ is possible, the discussions can be given in parallel with model 1 so that we present only the results. At any instant of time t_b, the value ϕ_b defines the Hilbert space which is constructed by the operators $\phi'(\mathbf{k})$ at t_b.

Suppose $|\lambda_i\rangle$ $(i=1,2)$ of Eq. (1) represents the wave packet at the center $\mathbf{r}_{ci}(t)=\mathbf{r}_{0i}+\mathbf{v}_i t$ with the macroscopic separation between $\mathbf{r}_{01}$ and $\mathbf{r}_{02}$. Both beams interact with the detector system. The wave function $|\lambda_i\rangle|\phi\rangle$ becomes, at time t_b, $\psi_i(t_b,\phi_b)$ which is proportional to $\delta(\hat{\phi}_i)\exp(iL_{0z}c_i)$ where $\hat{\phi}_i$ is given by Eg. (30) with the replacement $\mathbf{r}_0\to\mathbf{r}_{0i}$. This is multiplied by the initial wave function of ϕ_a and integrated over ϕ_a. Let the width of the initial wave function be $\Delta\phi$; then the criterion for our system to work as a detector is, using Eq. (31) and inserting the suffix i,

$$\Delta\phi_i \ll \Delta_{0i} \quad (i=1,2) .$$

If this is satisfied, then in the limit $L_{0z}\to\infty$ there does not exist an operator which has finite matrix elements between two states $\psi_1(t_b,\phi_b)$ and $\psi_2(t_b,\phi_b)$. For large but finite L_{0z} the recovery of the interference is possible by using the exponential of the extensive operator. Let us introduce the local momentum field $\pi(\mathbf{x}) =(\hbar/i)\partial/\partial\phi(\mathbf{x})$ and the extensive operator π_i integrated over the two tubelike regions V_{0i} determined by the two beams, $\pi_i=\int_{V_{0i}} d^3x\,\pi(\mathbf{x})$. The operator $\prod_{i=1,2} \times\exp(i\pi_i\Delta_{0i}/\hbar)\exp(ic_iL_{0z})$ is the desired one but it is not easy to conceive such an operator both experimentally and theoretically. It does not exist for infinite L_{0z}.

It is expected that as $|\mathbf{v}|\to\infty$ the influence of the interaction on the object becomes small. This is indeed the case since D of Eq. (28) is proportional to $1/|\mathbf{v}|$ in the limit of large $|\mathbf{v}|$. This allows us to consider the object as having a free particle wave packet.

IV. DISCUSSIONS

In actuality we do not live in the ideal macroscopic world. The parameters which characterize the size of the system, N (in model 1) or L_{0z} (in model 2), for example, are not infinite in the strict sense.

Take a large but finite system, then we have single Hilbert space for the entire theory. All the quantum-mechanical variables show the *unitary* time development. The class-I intensive operators are not c numbers and the exponential of the macroscopic operators is well defined and the interference terms definitely remain. There is not loss of information as long as the system size is finite.

Consider the entropy S defined by $S=-\mathrm{Tr}\,\ln\rho$ where ρ is the density matrix of the system. For a finite system the entropy is constant in time since taking the trace is an invariant operation under the unitary transformation.[4]

The situation drastically changes in the macroscopic limit as has been discussed above. We have many Hilbert spaces and the usual quantum rule applies in *each* Hilbert space. There can be no interferences or communications between different Hilbert spaces. This is what is required for the probabilistic interpretation of the quantum mechanics. For example, there is no meaning of the sum of the two vectors belonging to different Hilbert spaces.

The essential point is the fact that taking the macroscopic limit will be a good approximation to the actual situation where the number of degrees of freedom is finite but extremely large. We believe that each experiment can be described quite appropriately (both qualitatively and quantitatively) by the theory in which the macroscopic limit has been taken. We want to stress here that this resolves the *conceptual* difficulties in the theory of measurement.[5]

Let us study the structure of the density matrix as a function of the size of the system. Suppose that the system, object plus apparatus, is in a pure state $|\psi\rangle=\sum_i c_i|\psi_i\rangle$ at the time $t=t_0$; then the eigenvalue of ρ is unity or zero. After the interaction of the object with the apparatus, the eigenvalue of ρ is still unity or zero as long as the apparatus is a finite system. In the macroscopic limit the diagonal elements $|c_i|^2$ are of the order unity but off-diagonal elements are of the order of (some positive powers of) $1/\sqrt{N}$ and hence vanish. The density matrix now describes the mixed state with the eigenvalues $|c_i|^2$. The above apparent contradiction is resolved by our observation that the entire Hilbert space is broken up into separate ones and to each diagonal element corresponds one Hilbert space.

The information is lost in the following sense. Although each off-diagonal matrix element of ρ becomes vanishingly small the dimension of the matrix ρ becomes bigger and bigger as we take the macroscopic limit. The information is scattered all over the huge Hilbert space and each matrix element carries a tiny portion of the entire information. The whole information is contained in the whole ρ, of course. In the actual experiment, however, off-diagonal matrix elements do not contribute. This is actually a requirement for the good detector.

The time interval ΔT for the system to change from a pure state to a mixed state, i.e., the time required for the contaction of the wave is estimated by taking model 1 as an example. We use Eq. (8) and require that the phase $\exp(iNS)/\hbar$ is of the order unity. If we assume that the average velocity of the detector $\bar{v}\equiv|\mathbf{x}_b-\mathbf{x}_a|/T$ is finite then ΔT turns out to be of the order $1/N$; $\Delta T\sim 1/gN$, or $1/m\bar{v}^2N$. For infinite N, ΔT is zero and it is practically not possible to prepare a wave function which is defined in the whole Hilbert space.

In order to apply our theory to the realistic measuring processes detailed numerical studies are required together with the extension of the models to the case where the separation of variables is not possible.

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APPENDIX A

The kernel K is obtained by the path-integral formula.[3] Let us divide the time interval $T=t_b-t_a$ in discrete time steps of separation ϵ with $T=N_0\epsilon$, $t_n=t_a+n\epsilon(n=0\text{–}N_0)$, $t_0=t_a$, $t_{N0}=t_b$. Then

$$K(t_0,\mathbf{X}_b;t_a,\mathbf{X}_a) =\left[\frac{mN}{2\pi i\hbar\epsilon}\right]^{3(N_0-1)/2}\int\prod_{n=1}^{N_0-1} d\mathbf{X}_n\exp\frac{i}{\hbar}L \ . \quad \text{(A1)}$$

where

$$L=\sum_{n=0}^{N_0}\left[\frac{mN(\mathbf{X}_n-\mathbf{X}_{n-1})^2}{2\epsilon}-gN\epsilon\exp\{-[\mathbf{X}_n-\mathbf{r}_c(t_n)]^2/a^2\}\right],$$

with $\mathbf{X}_{-1}=0$. To zeroth order in g we get the usual diffusion kernel

$$K_0=\left[\frac{mN}{2\pi i\hbar T}\right]^{3/2}\exp\frac{mN(\mathbf{X}_b-\mathbf{X}_a)^2}{2\pi i\hbar T}\ . \tag{A2}$$

The first-order contribution is

$$K_1=-\frac{i}{\hbar}gN\epsilon\sum_n\left[\frac{mN}{2\pi i\hbar(t_b-t_n)}\frac{mN}{2\pi i\hbar(t_n-t_a)}\frac{mN}{2\pi i\hbar\epsilon}\right]^{3/2}\times\int_{-\infty}^{\infty}d\,\mathbf{X}_n\exp\left[\frac{imN(\mathbf{X}_b-\mathbf{X}_n)^2}{2\hbar\epsilon(t_b-t_n)}+\frac{imN(\mathbf{X}_n-\mathbf{X}_a)^2}{2\hbar\epsilon(t_n-t_a)}-\frac{[\mathbf{X}_n-\mathbf{r}_c(t_n)]^2}{a^2}\right]. \tag{A3}$$

The integral over $\mathbf{x}$ is Gaussian with the result

$$K_1=-\frac{i}{\hbar}gN\left[\frac{mN}{2\pi i\hbar t}\right]^{3/2}\epsilon\sum_n(1+L_n)^{-1/2}\exp Q_n\ , \tag{A4}$$

where

$$L_n=2i\hbar(t_b-t_n)(t_n-t_a)/a^2mNT\ , \tag{A5}$$

$$Q_n=\{(t_b-t_n)(t_n-t_a)(\mathbf{X}_b-\mathbf{X}_a)^2/T-(t_b-t_n)[\mathbf{r}_c(t_n)-\mathbf{X}_a]^2-(t_n-t_b)(\mathbf{r}_c(t_n)-\mathbf{X}_b)^2\}/(1+L_n)a^2T\ . \tag{A6}$$

Taking the limit $\epsilon\to0$, we get Eqs. (6)–(9) to first order in g.

APPENDIX B

In order to get the wave function in the limit $N\to\infty$, it is convenient to expand S' of Eq. (11) around the stationary point satisfying $\partial S'/\partial\mathbf{X}_a=0$, which is given by

$$0=\tilde{\mathbf{R}}\equiv\mathbf{X}_b-\mathbf{X}_a-\frac{gT\sqrt{\pi}}{m\,|\mathbf{v}|}D\frac{\mathbf{r}_{0\perp}-X_{b\perp}}{a}\exp\left[-\frac{D}{a^2}(\mathbf{X}_{b\perp}-\mathbf{r}_{0\perp})^2\right] \tag{B1}$$

to first order in g. S' becomes, up to quadratic terms in $\tilde{R}$,

$$S=S_0'+N\sum_{i,j=x,y,z}\tilde{R}_iM_{ij}\tilde{R}_j\ ,$$

where S_0' is the value of S' at $\tilde{R}_i=0$. After straightforward calculations we arrive at

$$S_0'=\frac{gN\sqrt{\pi}a}{|\mathbf{v}|}\left[1-\frac{i\hbar T}{2a^2mN}\left[1-\frac{(\mathbf{X}_{b\perp}-\mathbf{r}_{0\perp})^2}{a^2}\right]\right]\times\exp\left[-\frac{D}{a^2}(\mathbf{X}_{b\perp}-\mathbf{r}_{0\perp})^2\right], \tag{B2}$$

$$M_{ij}=\frac{m}{2T}\delta_{ij}+\frac{gD\sqrt{\pi}}{4\,|\mathbf{v}|\,a}[-\delta_{ij}^{\perp}+2D(\mathbf{X}_{b\perp i}-\mathbf{r}_{0\perp i})\cdot(\mathbf{X}_{b\perp j}-\mathbf{r}_{0\perp j})/a^2]\times\exp[-(\mathbf{X}_{b\perp}-\mathbf{r}_{0\perp})^2/a^2]\ , \tag{B3}$$

where $\delta_{ij}^{\perp}$ is the two-dimensional Kronecker's δ function, which is defined to be zero if i or j refers to z. By the formula

$$\lim_{N\to\infty}\exp\frac{i}{\hbar}N\sum_{i,j}R_iM_{ij}R_j=\left[\frac{i^3\pi^3}{N^3\det M}\right]^{1/2}\delta^3(\mathbf{R})\ , \tag{B4}$$

we get Eq. (14) where $\psi_s=\exp(iS_0'/\hbar)$, which has an infinite phase $\exp(icN)$ with

$$c=\frac{g\sqrt{\pi}a}{|\mathbf{v}|}\exp\left[-\frac{(\mathbf{X}_{b\perp}-\mathbf{r}_{0\perp})^2}{a^2}\right], \tag{B5}$$

APPENDIX C

We first write $\phi(\mathbf{k})=\mathrm{Re}\phi(\mathbf{k})+i\,\mathrm{Im}\phi(\mathbf{k})$, then because of Hermiticity $\phi(-\mathbf{k})=\mathrm{Re}\phi(\mathbf{k})-i\,\mathrm{Im}\phi(\mathbf{k})$. H^{0M} is thus given as

$$H^{0M}=\frac{4\pi e^2}{\sqrt{V}}\sum_{\mathbf{k}}{}'\frac{1}{\mathbf{k}^2+\mu^2}\{\mathrm{Re}\phi(\mathbf{k})\cos[\mathbf{r}_c(t)\cdot\mathbf{k}]-\mathrm{Im}\phi(\mathbf{k})\sin[\mathbf{r}_c(t)\cdot\mathbf{k}]\}\ , \tag{C1}$$

where $\sum'_{\mathbf{k}}$ implies the summation over the half $\mathbf{k}$ space. We obtain ϕ as

$$\phi=\frac{1}{V_0}\frac{2}{\sqrt{V}}\sum_{\mathbf{k}}{}'M(\mathbf{k})\{\mathrm{Re}\phi(\mathbf{k})\cos[\mathbf{r}_c(t)\cdot\mathbf{k}]-\mathrm{Im}\phi(\mathbf{k})\sin[\mathbf{r}_c(t)\cdot\mathbf{k}]\}\ . \tag{C2}$$

Applying the well-known results[3] of the forced harmonic oscillator to the real and imaginary part of ϕ separately, we obtain with the notation $\mathrm{Re}\phi(\mathbf{k})=\phi^r(\mathbf{k})$, $\mathrm{Im}\phi(\mathbf{k})=\phi^i(\mathbf{k})$,

$$S[t_b,\phi_b(\mathbf{k});t_a,\phi_a(\mathbf{k})]=\sum_{\mathbf{k}}{}' \frac{\omega_k}{2\sin(\omega_k T)}(F^r+F^i)\ , \tag{C3}$$

$$\begin{aligned}F^{r,i}=&[\phi_b^{r,i}(\mathbf{k})^2+\phi_a^{r,i}(\mathbf{k})^2]\cos(\omega T)-2\phi_b^{r,i}(\mathbf{k})\phi_a^{r,i}(\mathbf{k})\\&+2\frac{\phi_b^{r,i}(\mathbf{k})}{\omega_k}\int_{t_a}^{t_b} f^{r,i}(\mathbf{k},t)\sin[\omega_k(t-t_a)]dt+2\frac{\phi_a^{r,i}(\mathbf{k})}{\omega_k}\int_{t_a}^{t_b} f^{r,i}(\mathbf{k},t)\sin[\omega_k(t_b-t)]dt\\&+(\text{terms independent of } \phi_{a,b}^{r,i})\ ,\end{aligned} \tag{C4}$$

where

$$(f^r,f^i)=-\frac{4\pi e^2}{\sqrt{V}}\frac{1}{\mathbf{k}^2+\mu^2}(\cos[\mathbf{r}_c(t)\cdot\mathbf{k}],-\sin[\mathbf{r}_c(t)\cdot\mathbf{k}])\ .$$

The integration over $\phi_a(\mathbf{k})$ and $\phi_b(\mathbf{k})$ in Eq. (18) can be done as

$$\begin{aligned}&\exp\frac{i}{\hbar}S_0(t_b,\phi_b;t_a,\phi_a)\sim\int dJ_a dJ_b \exp\frac{i}{\hbar}[W(J_a,J_b)-V_0\boldsymbol{\phi}\cdot\tilde{\mathbf{J}}]\ ,\\&W(J_a,J_b)=\tfrac{1}{2}\mathbf{J}C\tilde{\mathbf{J}}-\mathbf{D}\cdot\tilde{\mathbf{J}}\end{aligned} \tag{C5}$$

apart from a factor independent of J_a,J_b. Here we have defined vectors $\mathbf{J}=(\mathrm{J_b},\mathrm{J_a})$, $\boldsymbol{\phi}=(\phi_b,\phi_a)$, $\mathbf{D}=(D_1,D_2)$, and the tilde denotes the transposed vector. C and $\mathbf{D}$ are evaluated as

$$C=\sum_{\mathbf{k}}\frac{\omega_k}{2\sin(\omega_k T)}\frac{2}{V\omega_k^2}M(\mathbf{k})^2\begin{bmatrix}\cos(\omega_k T) & 1\\ 1 & \cos(\omega_k T)\end{bmatrix}\ , \tag{C6}$$

$$\begin{aligned}D_1(\text{or } D_2)=\frac{4\pi e^2}{V}\sum_{\mathbf{k}}\frac{M(\mathbf{k})}{\omega_k(\mathbf{k}^2+\mu^2)}\int_{t_a}^{t_b} dt\cos[\mathbf{k}\cdot(\mathbf{x}_0-\mathbf{r}_c(t))]\Bigg[&-(\text{or }+)\frac{\sin[\omega_k(t-t_a)]-\sin[\omega_k(t_b-t)]}{1+\cos(\omega_k T)}\\&+\frac{\sin[\omega_k(t-t_a)]+\sin[\omega_k(t_b-t)]}{1-\cos(\omega_k T)}\Bigg]\ .\end{aligned} \tag{C7}$$

[1]See, for example, *Foundation of Quantum mechanics,* Proceedings of the International School of Physics "Enrico Fermi," Course IL, edited by B. d'Espagnat (Academic, New York, 1971; Proceedings of the International Symposium on "Foundation of Quantum Mechanics — In the Light of New Technology" (ISQM), edited by S. Kamefuchi *et al.* (Physical Society of Japan, Tokyo, 1984); *ibid.*, edited by M. Namiki *et al.* (Physical Society of Japan, Tokyo, 1986).

[2]R. Fukuda, Phys. Rev. A **35**, 8 (1987).

[3]R. P. Feynman, Rev. Mod. Phys. **20**, 367 (1948); R. P. Feynman and A. R. Hibbs, *Quantum Mechanics and Path Integrals* (McGraw-Hill, New York, 1965).

[4]For the discussions on the informations in quantum theory we refer the reader to the paper D. Bohm and B. J. Hiley, Found. Phys. **14**, 255 (1983).

[5]There are many papers on the study of the macroscopic limit: H. Wakita, Prog. Theor. Phys. **27**, 1156 (1962); F. Haake and W. Weidlich, Z. Phys. **213**, 451 (1968); M. Cini, Nuovo Cimento B **73**, 27 (1983).