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Destabilizing effect of temperature modulation on time-dependent convection in ^3He-^4He mixtures

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The effect of periodic modulation of the vertical temperature gradient on the Hopf bifurcation in ^3He-^4He mixtures is studied experimentally. The threshold for the onset of convection is found to be reduced by modulation, with the amount of reduction linearly proportional to the amplitude of modulation. The results can be understood on the basis of a phenomenological model.

Liquid ^3He-^4He mixtures have been attracting increasing attention[1,2] through their versatility as Rayleigh-Bénard convection systems, initially for studying the onset problem[3] and also, more recently, for investigating nonlinear phenomena.[4,5] In this Rapid Communication we describe experiments designed to investigate the effect of temporal modulation of the applied vertical heat current on the critical Rayleigh number for the onset of time-dependent convection. This work is motivated by linear stability analyses[6,7] demonstrating that the threshold for the Hopf bifurcation from the conduction state to an oscillatory state will be lowered in proportion to the modulation amplitude if the modulation frequency is twice that of the natural frequency of the system.

The experimental cell is the same as that used by Lee, Lucas, and Tyler[3] and is a right cylinder of height 0.21 cm and diameter 2.48 cm with oxygen-free high-conductivity copper upper and lower boundary plates and thin stainless steel walls. The cryogenic equipment is also similar except that the base temperature of the experiment is that of a continuously running ^4He refrigerator[8] and the experiment was run under computer control. The temperature T_c of the cell upper plate was kept fixed to within 10^{-5} K using a temperature regulator with carbon resistance temperature sensors and was measured using a ^3He vapor pressure thermometer. T_c was always maintained below the codimension-two point.[4] The lower plate was thermally shielded from the inside of the vacuum can and attained a temperature T_F determined by a heat current H_F of a few microwatts from an electric heater mounted on the lower boundary. This heat current could be sinusoidally modulated, ramped linearly, stepped in time, or a combination of these using appropriate software, since it was derived from the output of a digital-to-analog converter (DAC) interfaced to the microcomputer. The 16-bit resolution of the DAC was sufficiently high that a change in the least significant bit produced no detectable change in the heat current. The temperature difference ΔT between the plates, at most 7 mK, was monitored by measuring to seven decades the ratio of two resistance sensors mounted on the upper and lower cell boundaries. This measurement was made once a second using an automatically balanced ratio transformer bridge[9] interfaced to the microcomputer.

The ^3He-^4He mixture was introduced into the experimental cell in stages through a filling capillary in the upper plate, the progress of the liquid level inside the cell being monitored by measuring the thermal resistance between the plates a few mK below the transition. In this way filling-capillary flooding was avoided and hence any subsequent fractionation effects, and the liquid-vapor interface lay within the temperature-regulated part of the apparatus. The molar concentration of ^3He in our mixture was 3.24% determined by measuring the mixture's λ-temperature T_λ for a vanishingly small heat current.

The critical-temperature difference $\Delta T_c(0)$, above which convection appeared with no modulation, was determined by ramping the heat current H_F at a steady rate with T_c fixed to generate a heating curve of ΔT against H_F, and observing the departure from linearity as in our earlier work.[3] An example is shown in Fig. 1. The dependence of $\Delta T_c(0)$ on $t = T_c - T_\lambda$ is shown in Fig. 2 and is consistent with our earlier results[3] for molar concentrations X of 0.016 and 0.079. The temperature at which $\Delta T_c(0)$ appears to diverge corresponds to $t = t_v = 11$ mK and convection does not occur in the range $0 < t < t_v$ when heating from below. Time dependence was not clearly developed until $t - t_v < 4$ mK, a result also consistent with our earlier work.[3] At $t - t_v = 3.4$ mK the amplitude and time scale of the fluctuations in ΔT were 60 μK and 20 min, respectively, changing to 80 μK and 17 min at $t - t_v = 1.9$ mK. Ramp rates were always sufficiently small that the $\Delta T_c(0)$ determination was independent of the rate. In practice, this meant rates of less than 1 $\mu W h^{-1}$ so that ΔT was ramped at less than 200 $\mu K h^{-1}$. The $\Delta T_c(0)$ determination was independent of ramp direction to within the scatter of our data, although the first departure from linearity appeared more marked when the power was increasing.

All the modulation experiments were carried out at $t = 12.9$ mK, where there was clear time dependence. There is, however, a complication involved in working at this temperature in that the parameter region in which the time dependence is most marked happens to be the region in which the limit cycle formed due to the Hopf bifurcation is unstable. Consequently, the bifurcation is back-

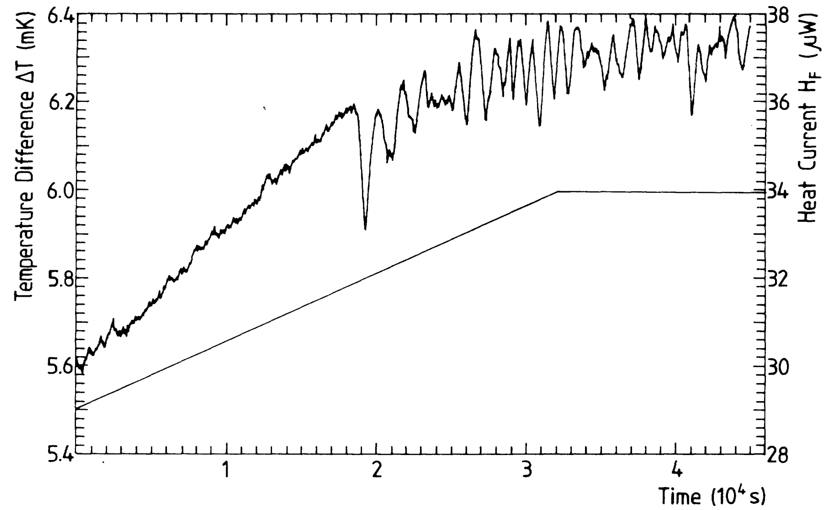

FIG. 1. Heating curve for a mixture with $X=0.0324$ and $t=12.9$ mK showing the time dependence of temperature difference ΔT between the cell boundaries (upper curve) produced by ramping the heat current H_F (lower curve). 1.3×10^4 s after the convection onset appears the ramp was halted to demonstrate the persistence of the time-dependent fluctuations in ΔT.

ward and the time dependence of the convection state will not be simply periodic with the frequency expected on the basis of the linear stability analysis. Although we intend this to be the subject of a future publication there is evidence for this from our earlier work[3] in that the experimentally determined critical Rayleigh numbers and the observed time scale of the ΔT fluctuations are, respectively, smaller and larger than expected from linear theory[10] in the parameter range $S<-1$, where S is the separation ratio. The nature of the unmodulated convection state at $t=12.9$ mK was investigated by taking a power spectrum of a 10-h time series of the ΔT fluctuations with ΔT set 9% above $\Delta T_c(0)$ and is shown in Fig. 3. The spectrum displays a major peak at 1024 s although several smaller components at larger periods are also present. The linear stability theory[10] yields a period which is an order of magnitude smaller. We note that such a discrepancy is a characteristic[11] of traveling-wave states, although their observation is not expected in our experiments since ΔT is an average over the horizontal boundaries.

In choosing a modulation frequency this departure from the linear regime forces a phenomenological description, where we assume that near the onset the Fourier component x_ω of $x(t)$ [where $x(t)=\int x_\omega(t)d\omega$] satisfies a damped oscillator equation

$$\ddot{x}_\omega+k\dot{x}_\omega+\omega^2x_\omega=0\ .$$

Here k is a small parameter which changes sign as the

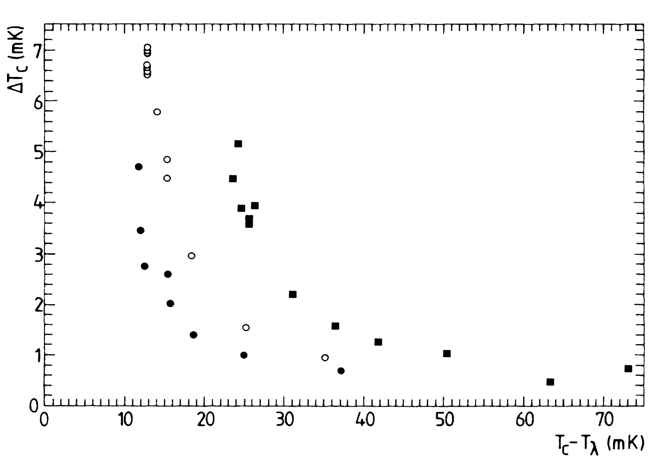

FIG. 2. Dependence of critical-temperature difference ΔT_c on $T-T_\lambda$ for molar concentration $X=0.0324$ (open circles). The data are compared with that obtained previously by Lee, Lucas, and Tyler (Ref. 3) for molar concentrations 0.016 (solid circles) and 0.079 (solid squares).

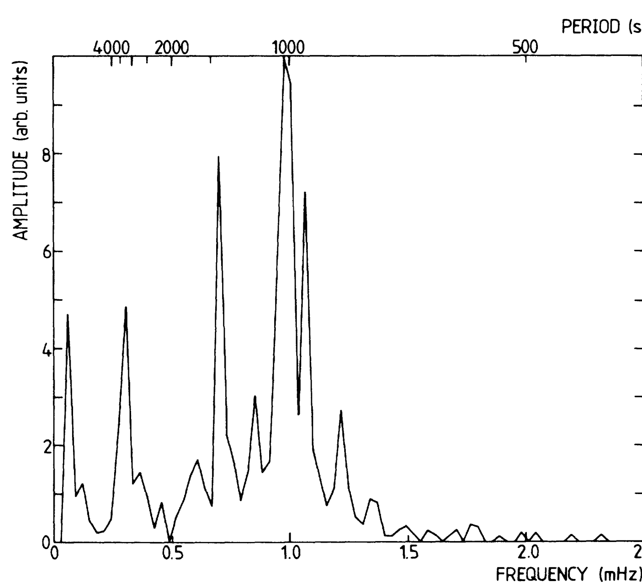

FIG. 3. Power spectrum of a 10-h ΔT time series at $t=12.9$ mK.

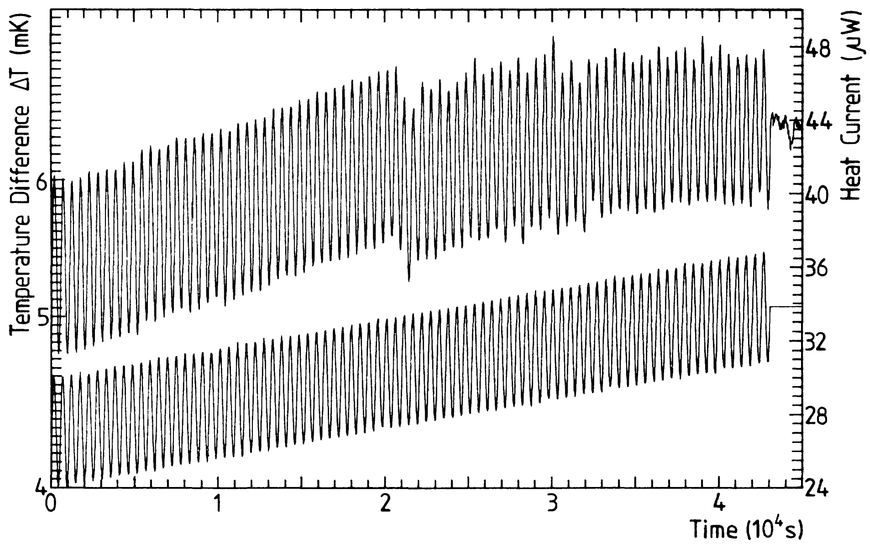

FIG. 4. Heating curve for a 3.24% mixture at $t = 12.9$ mK with H_F modulated at a period of 525 s and an amplitude of 3 μW. The ΔT data (upper curve) exhibit a convection onset at mean heating current 30.3 μW depressed below that obtained when modulation is absent (31.7 μW). The slight irregularities in the heat current (lower curve) are not present in the experimental system.

thermal Rayleigh number crosses the threshold. Clearly for $k > 0$, $x_\omega = 0$ is the stable solution. It loses stability to the state $x_\omega \sim e^{i\omega t}$ at $k = 0$. For the modulated system, the description is in terms of a damped Mathieu equation which reads

$$\ddot{x}_\omega + k\dot{x}_\omega + \omega^2(1 + \epsilon \cos \Omega t)x_\omega = 0 \ .$$

It is well known[12] that for $\Omega \simeq 2\omega$, the trivial state $x_\omega = 0$ loses stability at $k = \epsilon/2$, i.e., before k reaches its zero value. This implies that on modulation, the convective state will appear at a lower Rayleigh number and the fractional depression of the Rayleigh number is expected to be $\epsilon/2$ where ϵ is the modulation amplitude. With this in mind, we chose to modulate with a period of 525 s, i.e., close to twice the frequency of the major peak in the power spectrum, rather than any frequency in the range of those predicted by linear stability analysis.

The bulk of our results on onset depression were obtained by modulating the heating current H_F with some amplitude w while ramping the mean value linearly in time. $\Delta T_c(w)$, the critical-temperature difference in the presence of modulation amplitude w, could then be determined as before by observing the departure from linearity of the envelope of the induced ΔT oscillations. An example of such a heating ramp is shown in Fig. 4.

In practice, $H_{Fc}(w)$, the value of H_F in the presence of modulation of amplitude w at the onset of convection, is a more useful onset marker because the measurement error on H_F is less than ΔT. Since $H_F \propto \Delta T$ in the conduction regime it follows that $H_{Fc}(w) \propto \Delta T_c(w)$. The data collected on $H_{Fc}(w)$ fall into two groups, fitting a pattern in which $H_{Fc}(0)$, initially 33.5 μW, changed to 31.7 μW halfway through the six-week experimental period. The cause for this has not been established but may be connected with system nonlinearity associated with working close to t_v. In Fig. 5 the dependence of the fractional onset depression $D(\epsilon) = 1 - H_{Fc}(w)/H_{Fc}(0)$ on the fractional modulation amplitude $\epsilon = w/H_{Fc}(w)$ is displayed demonstrating that data from the two groups are consistent within the scatter of the data. A best straight-line fit through the origin yields a gradient $D(\epsilon)/\epsilon = -0.47 \pm 0.08$.

Although the bulk of our data was taken with a forcing period of 525 s, a few measurements were made all with the same modulation amplitude of 2.0 μW at periods of 1050, 303, and 235 s. Although many more measure-

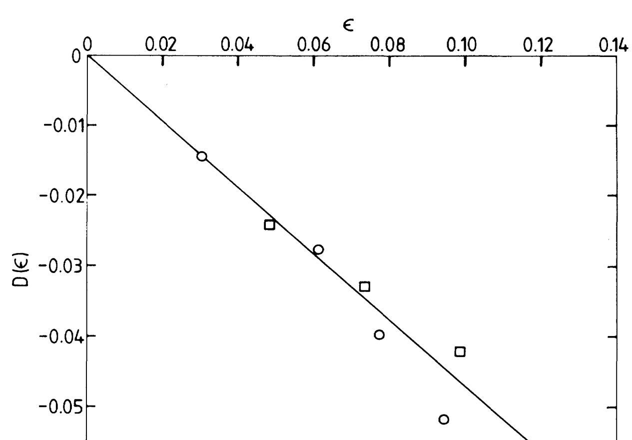

FIG. 5. Dependence of fractional onset depression $D(\epsilon) = 1 - H_{Fc}(w)/H_{Fc}(0)$ on fractional modulation amplitude $\epsilon = w/H_{Fc}(w)$. Open circles and open squares correspond, respectively, to data for which $H_{Fc}(0)$ are 33.5 and 31.7 μW. Assuming proportionality, a best fit for all data points gives gradient $D(\epsilon)/\epsilon = -0.47 \pm 0.08$. The scatter of the data for the larger values of ϵ may be caused by side-wall forcing (Ref. 13).

ments would be needed to establish a quantitative trend the data indicate a depression at 1050 s of about half the size measured at 525 s, while at 303 and 235 s the data are consistent with a much smaller depression. These results are in accord with the structure of the power spectrum shown in Fig. 3 in that the depression is greatest when the forcing period is half the period of the most prominent peak in the spectrum, while the depression is smaller at periods where the contribution to the spectrum is less.

For the concentration and temperature at which our modulation experiments were performed we estimate the numerical value of the expression for the gradient $D(\epsilon)/\epsilon$, given in Ref. 7 to be -0.487 ± 0.025. Since our time-dependent state appears to be chaotic, this estimate and our experimental results are not directly comparable. Nevertheless, our experimental result is roughly $\epsilon/2$, as expected from the phenomenological description.

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