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Diffraction of atoms moving through a standing light wave

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The diffraction of sodium atoms from a standing wave of near-resonant light is studied as a function of the atomic velocity component along the **k** vector of the wave. A simple theory is presented which quantitatively predicts the observed diffraction patterns and the decrease of the rms momentum transfer to the atoms with increasing velocity. This decrease results from the dephasing between the two counterpropagating traveling waves due to the first-order Doppler shift. In addition, these measurements have resolved the previous discrepancy between theory and experiment concerning the overall momentum transferred by a standing wave.

In the last year or so, the many theories of light forces[1] have been subjected to the first real confrontations with experiment.[2-6] Of particular interest have been studies of a standing wave where induced-dipole (also called gradient or stimulated) forces have been quantitatively studied both without[2] and with[3] the occurrence of spontaneous emission. These have revealed the previously predicted phenomena of Kapitza-Dirac diffraction[7-9] and blue molasses,[10] respectively. Although most of the theory[11-15] investigating momentum transfer between atoms and a standing-wave radiation field has addressed only the case in which the atoms had no velocity component along the **k** vector of the standing light wave, the few theoretical treatments of the effects of such movement predict new phenomena including Bragg scattering,[16] "Doppleron" resonances,[17,18] and Landau-Zener transitions.[19]

We report here the first experimental investigation of the diffraction of atoms moving in a standing light wave, a study at low velocities, v_x, up to 2 m/sec along the **k** vector of the standing wave. This regime has been overlooked in previous theoretical treatments; fortunately we are able to present a theory based on Refs. 14 and 16, which gives an analytic expression for the final momentum distribution as a function of this velocity. This theory correctly predicts both the observed rapid decrease in the rms momentum transfer with increasing v_x and the detailed diffraction patterns as well.

A further motivation for our work was to resolve the discrepancy of approximately a factor of 2 between the predicted and previously observed rms values of the Kapitza-Dirac diffraction patterns,[2] which could have arisen due to nonorthogonal alignment of the standing waves with respect to the atomic beam, miscalibration of the power measurements, or aberrations in the Gaussian waist of the light beam. We find that improvements to our apparatus and techniques have resolved the previous discrepancy in the overall amount of deflection.

Our calculation for the momentum transferred to a moving, two-level atom (energy difference $\hbar\omega_0$, dipole moment μ) by a standing-wave electric field follows the semiclassical approach of Bernhardt and Shore.[16] The electric field that the atom experiences from a standing wave with temporal envelope $f(t)$ is given by

$$E(x,t)=2E_0 f(t)\cos(kx+kv_x t)\cos(\omega t)\,. \tag{1}$$

In this expression, v_x is the velocity of the atom along the **k** vector of the standing wave. In the interaction picture, the Schrödinger equation for the amplitudes of the ground (1) and excited (2) states reads (using the rotating-wave approximation)

$$i\frac{\partial}{\partial t}\begin{bmatrix} a_2 \\ a_1 \end{bmatrix}=\begin{bmatrix} \dfrac{-\hbar}{2M}\dfrac{\partial^2}{\partial x^2}-\Delta & f(t)\,\Omega_0\cos(kx+kv_x t) \\ f(t)\,\Omega_0\cos(kx+kv_x t) & \dfrac{-\hbar}{2M}\dfrac{\partial^2}{\partial x^2} \end{bmatrix}\begin{bmatrix} a_2 \\ a_1 \end{bmatrix}. \tag{2}$$

We carry out a Fourier expansion of the amplitudes as $a_i(x,t)=\sum_n a_{i,n}(t)e^{inkx}$, where $a_{i,n}$ is the amplitude for the atom to be in state $i=1,2$ with momentum $p_x=n\hbar k$. We eliminate adiabatically the excited-state amplitude with the assumption that the detuning Δ is large compared to the peak traveling-wave Rabi rate: $\Omega_0=\mu E_0/\hbar$.[14] We also neglect the excited-state kinetic energy term since it is much smaller than Δ in the experiment. The amplitude for the ground-state atom to have momentum $p_n=n\hbar k$ is then found to satisfy

$$i\dot{a}_{1,n}=\frac{\Omega_0^2 f^2(t)}{4\Delta}[a_{1,n-2}\cos(2kv_x t)+2a_{1,n}+a_{1,n+2}\cos(2kv_x t)]+\frac{\hbar k^2 n^2}{2M}a_{1,n}\,, \tag{3}$$

with the initial condition $a_{1,n}(-\infty)=\delta_{n,0}$. A key assumption of this theory, the absence of spontaneous decays, was experimentally achieved by detuning far from resonance.[2] This assured that the dipole force dominated the momentum transfer.

Equation (3) expresses the fact that momentum transfer to the atom in its ground state is quantized in units of $2\hbar k$ from absorption and stimulated emission of photon pairs from the counterpropagating traveling waves. We can neglect the kinetic energy term in this equation since experimentally it is much smaller than $\Omega_0^2/2\Delta$ [a condition which has been checked by numerical solutions of Eq. (3)]. The solution to this equation is then

$$a_{1,n}(t)=\exp\left[\frac{-i\Omega_0^2}{2\Delta}\int_{-\infty}^{t}f^2(t')dt'\right]e^{in\pi/4}\cos\left[\frac{n\pi}{2}\right]J_{n/2}\left[\frac{\Omega_0^2}{2\Delta}\int_{-\infty}^{t}f^2(t')\cos(2kv_xt')dt'\right], \tag{4}$$

which yields the final probability ($t=+\infty$) for the atom initially ($t=-\infty$) in the ground state to gain momentum $p_n=n\hbar k$ as

$$P_n=|a_{1,n}(\infty)|^2=J_{n/2}^2(z), \tag{5}$$

where

$$z=\frac{\Omega_0^2}{2\Delta}\int_{-\infty}^{+\infty}f^2(t)\cos(2kv_xt)dt$$

for $v_x=0$; this result agrees exactly with that given in Refs. 2 and 14. The Gaussian field profile realized in the experiment, $f(t)=e^{-(t/\tau)^2}$, yields for the argument of the Bessel function,

$$z=\frac{\Omega_0^2}{2\Delta}\int_{-\infty}^{\infty}e^{-2(t/\tau)^2}\cos(2kv_xt)dt$$
$$=\left[\frac{\pi}{8}\right]^{1/2}\frac{\Omega_0^2\tau}{\Delta}\exp\left[\frac{-(kv_x\tau)^2}{2}\right]. \tag{6}$$

From the sum rules of Bessel functions,[2] we find that the rms momentum transfer is

$$p_{\mathrm{rms}}=\sqrt{2}\hbar kz=\left[\frac{\pi}{4}\right]^{1/2}\frac{\Omega_0^2\tau}{\Delta}\exp\left[\frac{-(kv_x\tau)^2}{2}\right]. \tag{7}$$

The dependence of Eqs. (3) and (4) on velocity, v_x, can be interpreted physically in either momentum space or position space. In momentum space (used in the above calculation), an eigenstate of transverse momentum, $a_{1,n}$, is unlocalized in space. For a given velocity, v_x, this momentum eigenstate sees two counterpropagating traveling waves, one blue shifted by $\omega_{\mathrm{Doppler}}=+kv_x$ and one red shifted by $\omega_{\mathrm{Doppler}}=-kv_x$. The amplitude for transfer from this momentum state, $a_{1,n}$, to a neighboring state, $a_{1,n+2}$, or $a_{1,n-2}$ (via the absorption and stimulated emission of photons from these waves) is modulated at the frequency that the atom perceives as the Doppler shift between these two traveling waves, i.e., $\Delta\omega_{\mathrm{Doppler}}=2kv_x$. This leads to the reduction of the rms momentum transfer as the velocity is increased. This reduction occurs even when the Doppler shift kv_x is much less than the natural decay rate Γ because the interaction is coherent over times much larger than Γ^{-1} (interaction time $\sim 10\Gamma^{-1}$ in our experiment).

In the complementary spatial picture, the atoms have a definite position and experience a dipole force which depends on position. When an atom traverses a node or antinode of the standing wave (due to v_x) the force reverses sign, causing a decrease of the total momentum transfer. For an ensemble of atoms which enter the field at different locations with respect to the nodes or antinodes, there is a monotonic reduction of rms momentum transfer as the velocity is increased. Both of these pictures yield the same dependence of the rms momentum transfer on the velocity v_x. However, the coordinate-space picture does not predict the quantized nature of the momentum transfer, which is understandable since the atom's momentum uncertainty must exceed $2\hbar k$ if it is localized to less than $\lambda/2$, the node spacing in the standing wave.

Our high-resolution experimental apparatus has been described in detail in previous papers.[2,20] It consists of a supersonically cooled sodium source [$\Delta v/v=11\%$ full width at half maximum (FWHM)], optically pumped to a two-state system ($3\,^2S_{1/2}$, $F=2$, $m_F=2\leftrightarrow 3\,^2P_{3/2}$, $F'=3$, $m_F'=3$) and collimated with two 10-μm slits spaced 0.9 m apart. The final momentum distributions are measured 1.2 m downstream from the interaction region by a 25-μm scanning hot-wire detector. The overall momentum resolution for the data presented here is $0.89\hbar k$ (FWHM).

Recent refinements to this apparatus for this experiment include: (1) Replacement of three cylindrical optical elements ($\lambda/4$), with two high-quality spherical elements ($\lambda/20$), and one cylindrical element to introduce the needed asymmetry to produce a one-dimensional Gaussian beam. This improved the focal line in the interaction region from 2.0 to 1.2 times the diffraction-limited size. (2) Laser power was measured in two independent ways. With the use of a Scientech model No. 360203 disk absorbing calorimeter, the laser power was measured relative to the Ohmic heat produced by a current passed through a known resistance. This measurement was checked by observing first moments of *traveling*-wave momentum-transfer profiles. In a traveling wave, the only force present is the spontaneous force. The absorption of a photon leads to a deflection of $\hbar k$ along the $\mathbf{k}$ vector of the wave. The subsequent spontaneous emission, being symmetric about this direction, leads only to a spreading of the momentum distribution. The first moment of a traveling-wave momentum-transfer profile divided by $\hbar k$, $\langle n\rangle$ is then equal to the average number of spontaneous events that an atom experiences as it passes through the traveling wave

$$\langle n\rangle=\frac{\Gamma}{2}\int_{-\infty}^{\infty}\frac{s_0e^{-2(t/\tau)^2}dt}{s_0e^{-2(t/\tau)^2}+1}, \tag{8}$$

where $s_0=2\Omega_0^2/(4\Delta^2+\Gamma^2)$ is the saturation parameter. For these measurements, the detuning was controlled by locking the interaction laser to the beat note of a reference laser, which was stabilized to the sodium beam. This technique, which allowed control of the detuning to ~ 1 MHz (rms) over a typical scan time of ~ 3 min, made it possible to probe the 10-MHz (FWHM) line shape of the sodium D_2 line in steps of ~ 2 MHz. Figure 1 displays data of $\langle n\rangle$ versus detuning for a constant laser power and

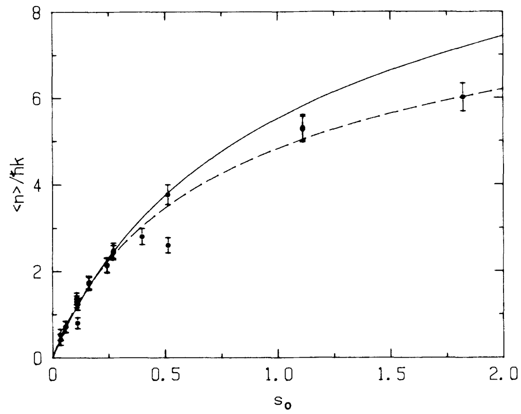

FIG. 1. Plot of $\langle n \rangle$ saturation parameter, s_0, for traveling-wave momentum-transfer profiles. Solid line is Eq. (8) calculated with independently measured values of power $(\Omega_0^2\tau)$ and waist parameter $\tau = 9.9\Gamma^{-1}$. Dashed line is Eq. (8) calculated with fitted values of power and waist parameter $\tau = 7.2\,\Gamma^{-1}$.

waist parameter [measured independently with an EG&G RL-123G Reticon to be $\tau = (9.9 \pm 0.5)\Gamma^{-1}$]. Also displayed is Eq. (8) calculated with experimentally measured values of both power and beam waist, as well as with best-fit parameters which yielded a fitted value of $\tau = (7.2 \pm 0.2)\Gamma^{-1}$. Although there is a discrepancy between the measured and fitted beam waist of $\sim 30\%$, the fitted laser power $(\Omega_0^2\tau)$ was 1.05 ± 0.05 times the power measured by the Scientech model No. 360203 power meter, thus increasing confidence in the absolute power measurement.

Experimental data for various velocities, v_x, along the **k** vector of the standing wave are presented in Fig. 2. The velocity, v_x, was controlled by tilting the standing wave with respect to the atomic beam (see Fig. 3). This involved adjusting the retro mirror and also the angle of the incoming light beam to maintain the standing wave perpendicular to the mirror. Also displayed in Fig. 2 is the theoretical prediction convolved with the machine and velocity resolution functions. We observe no experimentally significant discrepancy between the measurements and the prediction.

Various data points of $p_{\rm rms}$ versus velocity are shown in Fig. 4 for a constant laser power and detuning. The data points were fit to a Gaussian function,

$$p_{\rm rms}(v_x) = p_{\rm rms}(0)\exp\left(-\frac{v_x^2}{2v_0^2}\right), \tag{9}$$

which yielded the values $p_{\rm rms}(0) = (4.81 \pm 0.24)\hbar k$ and $v_0 = 1.30 \pm 0.04$ m/s. The independently measured values of laser power, $\Omega_0^2\tau = (152 \pm 8)\Gamma$, and detuning, $\Delta = (28.0 \pm 1.0)\Gamma$, yield the following results:

$$p_{\rm rms}(0) = \left(\frac{\pi}{4}\right)^{1/2} \frac{\Omega_0^2\tau}{\Delta} hk = (4.81 \pm 0.29)\hbar k\,,$$
$$v_0 = \frac{1}{k\tau} = 1.17 \pm 0.04 \text{ m/s}. \tag{10}$$

The value measured for $p_{\rm rms}(0)$ agrees within error with

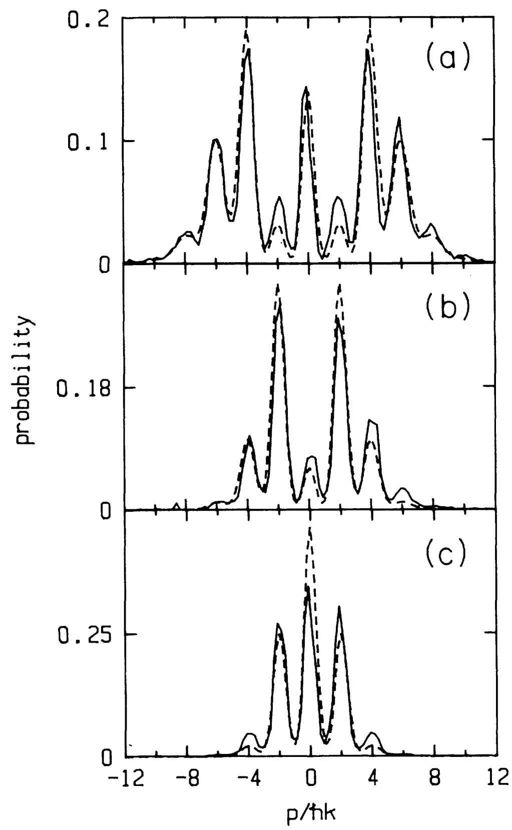

FIG. 2. Diffraction patterns for different velocities, v_x, of atomic beam along the **k** vector of the standing light wave. (a) $v_x = -0.06$ m/s, (b) $v_x = -1.22$ m/s, (c) $v_x = 1.68$ m/s. $\Omega_0 = 5.49\Gamma$ and $\Delta = 28.0\Gamma$ for all scans. Solid lines are experimental data and dashed lines are theoretical predictions convolved with experimental resolution.

the predicted value; however, the observed value for v_0 is larger than that predicted by ~ 1.5 combined errors. Although spontaneous decays were suppressed for this experiment, the small amount of residual decays could play a role in this small discrepancy.

Figure 5 displays the rms values of momentum transfer

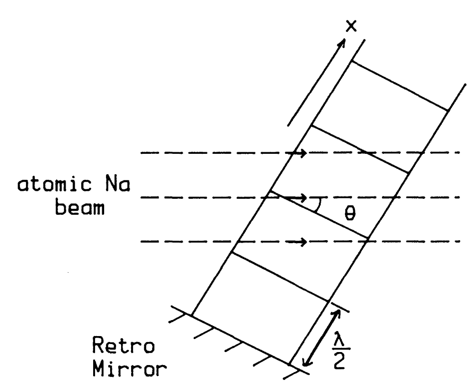

FIG. 3. Diagram of interaction region. Standing light wave has a Gaussian intensity profile $(e^{-2} \rightarrow e^{-2} = 2v\tau)$ in direction perpendicular to **k** vector.

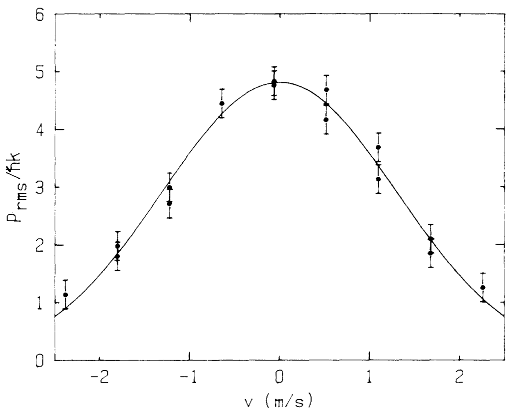

FIG. 4. Plot of $p_{rms}(v_x)$ vs velocity v_x, for constant laser intensity Ω_0 and detuning Δ.

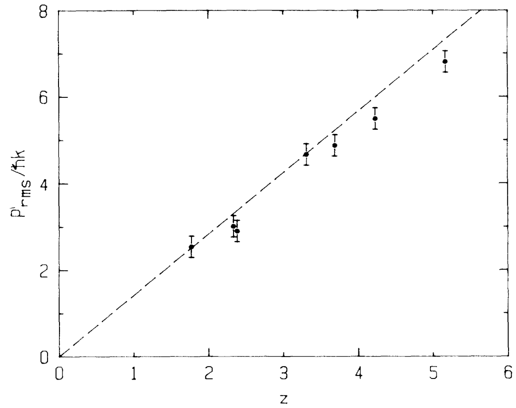

FIG. 5. rms momentum vs z for scans where the standing light wave was orthogonal to the atomic beam. The dashed line is Eq. (7) with $v_x = 0$.

versus the argument of the Bessel function z for a variety of diffraction patterns in the case where the standing wave was orthogonal to the atomic beam. With the refinements noted previously, the predicted and observed values of rms momentum transfer agree consistently within 10%, thus resolving the previous discrepancy[2] in favor of the theory.

The demonstration that small velocities of atomic motion over the standing wave have a large effect on the amount of momentum transfer has several implications. Since such motion can result from small departures from perfect orthogonality ($\theta \sim 10^{-3}$ rad here, even smaller for larger interaction times), it is imperative that this orthogonality be carefully assured in future experiments involving standing waves. In particular, the recently observed asymmetry induced in resonance curves[4] by these forces should be very sensitive to small misalignments of the standing wave nodes. In general, it would be wise to include the possibility of such small misalignment in future theories of radiative forces wherever ground-state coherence can persist. It would seem, for example, that a small tilt would rapidly reduce coherent momentum transfer from the dipole force but not from the induced diffusion predicted for standing-wave momentum transfer in the diffusive regime.[15]

In conclusion, we have investigated the behavior of the standing-wave dipole force on an atomic beam in the case where the atoms initially have a velocity along the **k** vector of the standing wave. We have measured the effect of a single physical process—the dephasing of successive absorption and stimulated-emission events in the standing wave due to the velocity—and have shown that the process is predicted quantitatively by the theory presented here. We have also refined our apparatus and techniques and have resolved the previous discrepancy between predicted and observed rms momentum transfer from a standing light wave to an atomic beam.

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[1] *The Mechanical Effects of Light* [J. Opt. Soc. Am. B **2**, (1985), special edition].
[2] P. L. Gould, G. A. Ruff, and D. E. Pritchard, Phys. Rev. Lett. **56**, 827 (1986).
[3] A. Aspect, J. Dalibard, A. Heidmann, C. Salomon, and C. Cohen-Tannoudji, Phys. Rev. Lett. **57**, 1688 (1986).
[4] M. Prentiss and S. Ezekiel, Phys. Rev. Lett. **56**, 46 (1985).
[5] W. Yuzhu, Z. Rufang, Z. Zhiyiao, C. Weiquan, N. Guoquan, Z. Shanyu, W. Changsheng, and Z. Weijun, Sci. Sin. **27**, 881 (1984).
[6] P. Bucksbaum, M. Bashkansky, and T. McIlrath, Phys. Rev. Lett. **58**, 349 (1987).
[7] P. L. Kapitza and P. A. M. Dirac, Proc. Cambridge Philos. Soc. **29**, 297 (1933).
[8] S. Altshuler, L. M. Frantz, and R. Braunstein, Phys. Rev. Lett. **17**, 231 (1966).
[9] R. J. Cook and A. F. Bernhardt, Phys. Rev. A **18**, 2533 (1978).
[10] J. Dalibard and C. Cohen-Tannoudji, J. Opt. Soc. Am. B **2**, 1707 (1985).
[11] E. Arimondo, A. Bambini, and S. Stenholm, Phys. Rev. A **24**, 898 (1981).
[12] A. P. Kazantsev, G. I. Surdutovich, and V. P. Yakovlev, Pis'ma Zh. Eksp. Teor. Fiz. **31**, 542 (1980) [JETP Lett. **31**, 509 (1980)].
[13] C. Tanguy, S. Reynaud, and C. Cohen-Tannoudji, J. Phys. B **17**, 4623 (1984).
[14] A. Dulcie, J. Eberly, H. Huang, J. Javanainen, and L. Roso-Franco, Phys. Rev. Lett. **56**, 2109 (1986).
[15] C. Tanguy, S. Reynaud, M. Matsuoka, and C. Cohen-Tannoudji, Opt. Commun. **44**, 249 (1983).
[16] A. F. Bernhardt and B. W. Shore, Phys. Rev. A **23**, 1290 (1981).
[17] V. G. Minogin and O. T. Serimaa, Opt. Commun. **30**, 373 (1979).
[18] D. E. Pritchard and P. L. Gould, J. Opt. Soc. Am. B **2**, 1799 (1985).
[19] A. P. Kazantsev, V. S. Smirnov, G. I. Surdutovich, D. O. Chudesnikov, and V. P. Yakovlev, J. Opt. Soc. Am. B **2**, 1731 (1985).
[20] P. L. Gould, G. A. Ruff, P. J. Martin, and D. E. Pritchard, Phys. Rev. A **36**, 1478 (1987).