Spontaneous emission in a double dielectric slab for Čerenkov free-electron-laser operation

F. Ciocci, G. Dattoli, A. Doria, G. P. Gallerano, G. Schettini, and A. Torre*
Dipartimento Tecnologie Intersettoriali di Base, Comitato Nazionale per la Ricerca e per lo Sviluppo dell'Energia Nucleare e delle Energie Alternative, Divisione Fisica Applicata, Centro Ricerche Energia Frascati, C.P. 65-00044 Frascati, Rome, Italy

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The double-slab waveguide configuration for a Čerenkov free-electron laser is discussed. It is shown that it may allow higher emission brightness with respect to the single-slab configuration.

The possibility of realizing a free-electron-laser (FEL) device in the far-infrared (FIR) region exploiting a Čerenkov medium as the pump field, rather than an undulator magnet, has recently raised a certain deal of interest both theoretical[1] and experimental.[2]

The attention payed to this type of free-electron generator of coherent radiation is motivated by the suggestion that in the FIR region of the spectrum a Čerenkov FEL may provide a low-cost compact device with better performance than that of an undulator-based FEL.[1]

In a previous paper[3] a systematic analysis of the spectral features of the spontaneous emission in Čerenkov devices has been undertaken, both to understand the mechanism underlying the frequency selection and to dispel the role of the spontaneous emission "noise" on the laser start up.

In the above-quoted paper two types of configurations have been considered. The one relevant to a close cavity filled with dielectric and the second relevant to the so-called slab configuration (Fig. 1). In the first case the electron moves in the dielectric medium, while for the slab configuration the electron interacts with the evanescent field of the dielectric. The first configuration has been discarded as practical solution for a Čerenkov pump field for the following two reasons.

(a) Because the spectrum is noisy, i.e., a large number of modes are excited by the electrons;

(b) Because the qualities of the electron beam are degraded by the interaction with the dielectric and thus a strong inhomogenous broadening is induced on the emission line.

The slab configuration otherwise provides a pump field without the above drawbacks. Indeed, one mode in the spectral region of interest is selected and the quality of the beam is not affected since the electron couples to the external evanescent field of the dielectric.

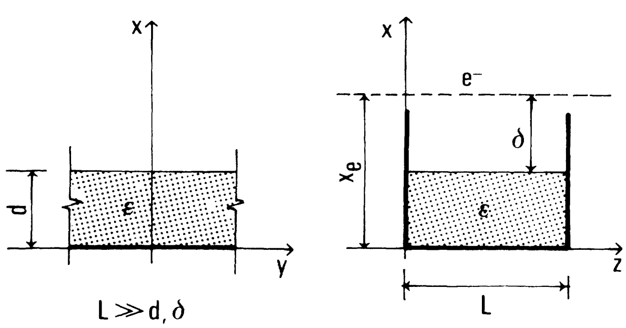

FIG. 1. Single-slab configuration.

We will not enter the mathematical details of the analysis but we sketch the essential steps of the formalism utilized in Ref. 3.

The basic mathematical tools have been those proposed by Linhart in Ref. 4 who calculated the radiation emitted by a pointlike charge in uniform motion on a straight line parallel to the plane face of a semiinfinite dielectric slab.

The field produced by a charge moving along the structure may be expanded according to the relation

$$\mathbf{A}_\lambda(t)=\sum_\lambda q_\lambda(t)\mathbf{A}_\lambda(\mathbf{r}) \ . \tag{1}$$

The coefficients $q_\lambda(t)$ contains the evolution of the λth mode while the spatial part of the vector potential can be conveniently specialized as

$$\mathbf{A}_\lambda(\mathbf{r})=\mathbf{a}_\lambda(\mathbf{r})\sin(kz) \tag{2}$$

and for a waveguide structure with plane conducting surfaces at $z=0$, L and k fulfill the discreteness condition

$$k=n\frac{\pi}{L} \ , \quad n=0,\pm 1,\pm 2,\ldots \ . \tag{3}$$

The time behavior of the λth mode will be obtained by solving the following forced harmonic oscillator equation:

$$\ddot{q}_\lambda(t)+\omega_\lambda^2 q_\lambda(t)=\frac{1}{c}\int \mathbf{j}\cdot\mathbf{A}_\lambda^* dv\equiv F_\lambda(t) \ , \tag{4}$$

whose solution can be written in the form

$$q_\lambda(t)=\int_{-\infty}^{\infty}\frac{\tilde{F}_\lambda(\omega)}{\omega_\lambda^2-\omega^2}e^{i\omega t}d\omega \ , \tag{5}$$

where $\tilde{F}_\lambda(\omega)$ is the Fourier transform of the forcing term in Eq. (4).

The rate of energy exchange between the electron and fields can be easily evaluated and reads

$$\frac{d\mathscr{E}_\lambda}{dt}=\frac{1}{2}\,\mathrm{Re}[F_\lambda^*(t)\dot{q}_\lambda(t)] \ . \tag{6}$$

Finally the total energy radiated by the electron can be evaluated integrating (6) over the time and summing over all the modes. However, as already shown[3] a more convenient form is provided by

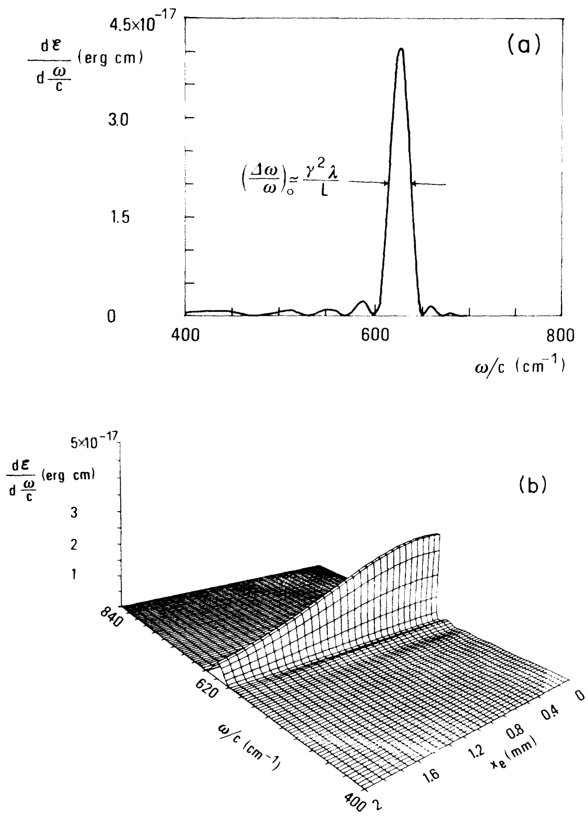

FIG. 2. Single-slab spontaneous-emission spectrum: (a) Single particle; (b) Gaussian-distributed e beam with rms $\sigma_x = 1$ mm.

$$\mathcal{E} = -\int d\omega \sum_{\{s\}} \frac{\pi^2}{c} \frac{|F_{\{s\}}(\omega)|^2}{d/dk(\omega_{[k,\{s\}]}/c) - \beta_0} \times \rho_k(\{s\},\omega)\,|_{\omega_{[k,\{s\}]}=\omega} , \tag{7}$$

where $\rho_k(\{s\},\omega)$ is the k-density function whose properties have been discussed in Ref. 3. The above result specialized to the fields of the slab configuration have provided the spectrum of the radiation emitted by an electron moving near and parallel to the surface of the dielectric. The results are shown in Fig. 2 where we have reported the single-particle energy spectrum for a dielectric thickness $d = 3.2$ μm and a distance of the electron from the

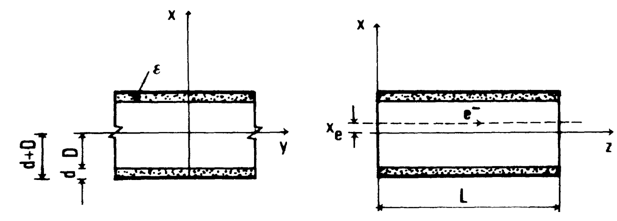

FIG. 3. Double-slab configuration.

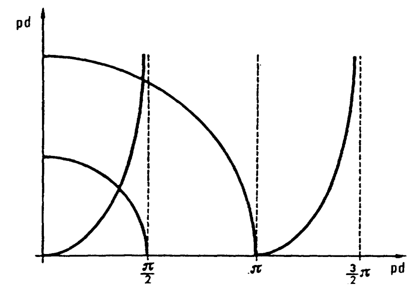

FIG. 4. Dispersion relations.

surface of 0.25 mm [Fig. 2(a)]. A "global" view is given by the three-dimensional (3D) plot of Fig. 2(b) where the spectrum radiated by a Gaussian-distributed e beam with $\sigma_x = 1$ mm is reported versus the frequency and the distance between the center of the distribution and the dielectric surface.

It should be noted that the homogeneous linewidth of the spontaneous emission is approximately given by

$$\left|\frac{\Delta\omega}{\omega}\right|_0 \simeq \frac{\gamma^2\lambda}{L} , \tag{8}$$

where λ is the central emission wavelength.

Even though the slab configuration presents undoubtful advantages with respect to the dielectric filled structures it has the main drawback that the field strongly decreases with the distance of dielectric and only a portion of the beam is utilized for the emission and, eventually, for the gain.

To prevent this problem the obvious solution is a symmetric slab configuration (see Fig. 3) where the electrons move between two single slabs facing each other. The double-slab configuration has been already studied in Ref. 5; here we will reconsider the problem from the point of view of the spectral properties of the emitted radiation and to compare the results to those of the single slab.

The fields of the double slab will be the sum of two single-slab contributions: a symmetric and an antisymmetric combination will result. The expression of the TM fields together with the dispersion relation are reported below.

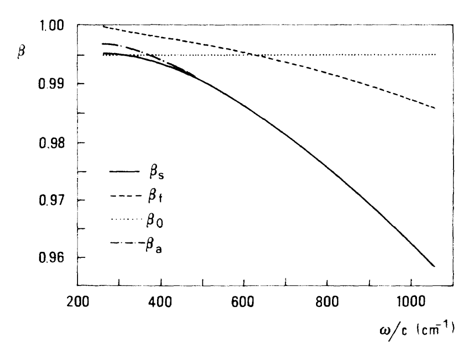

FIG. 5. Phase and group velocities.

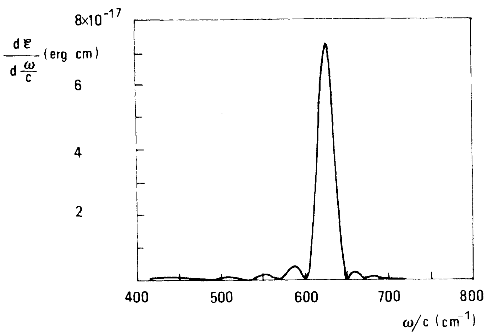

FIG. 6. Single-particle spontaneous-emission spectrum.

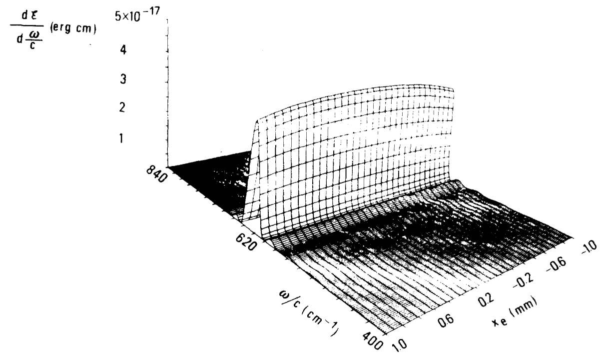

FIG. 8. Spectrum for a Gaussian-distributed e beam with rms $\sigma_x = 1$ mm.

The symmetric part is

$$A_x = -\frac{k}{q}\frac{Ac}{\omega}\sinh(qx)\sin(kz) ,$$
$$A_y = 0 , \tag{9a}$$
$$A_z = \frac{Ac}{i\omega}\cosh(qx)\sin(kz) .$$

The antisymmetric part is

$$A_x = \frac{k}{q}\frac{iA'c}{\omega}\cosh(qx)\sin(kz) ,$$
$$A_y = 0 , \tag{9b}$$
$$A_z = -\frac{A'c}{\omega}\sinh(qx)\sin(kz) .$$

The normalization coefficients are

$$A^2 = \frac{16\pi c^2 q}{Ly_0(c/\omega)^2\{[1-(k/q)^2]2qD + [1+(k/q)^2]\sinh(2qD)\}} ,$$
$$A'^2 = \frac{16\pi c^2 q}{Ly_0(c/\omega)^2\{-[1-(k/q)^2]2qD + [1+(k/q)^2]\sinh(2qD)\}} . \tag{10}$$

The dispersion relations are

$$\tanh(qD)\tan(pd) = q\varepsilon/p \quad \text{(symmetric)} ,$$
$$\tan(pd) = q\varepsilon/p\,\tanh(qD) \quad \text{(antisymmetric)} , \tag{11}$$
$$(pd)^2 + (qd)^2 = (\varepsilon - 1)d^2(\omega/c)^2 .$$

The plot relevant to the transverse relation is shown in Fig. 4 where the transverse wave numbers p and q for TM_n mode are determined by the intersection between the circumference and the nth branch of the circular tangent.

The synchronism condition between symmetric and antisymmetric mode is shown in Fig. 5 where the group and phase (β_f) velocities in units of c have been plotted versus the frequency together with electron velocity. In the region of interest ($\omega/c \sim 625$ cm^{-1}) the symmetric (β_s) and antisymmetric (β_a) group velocities are coincident, thus providing a single mode.

Utilizing the fields (9a) and (9b) and the relations (4)–(7), we can display the spectral details of the radiation emitted in the double slab configuration (the mathematical details are omitted and will be discussed

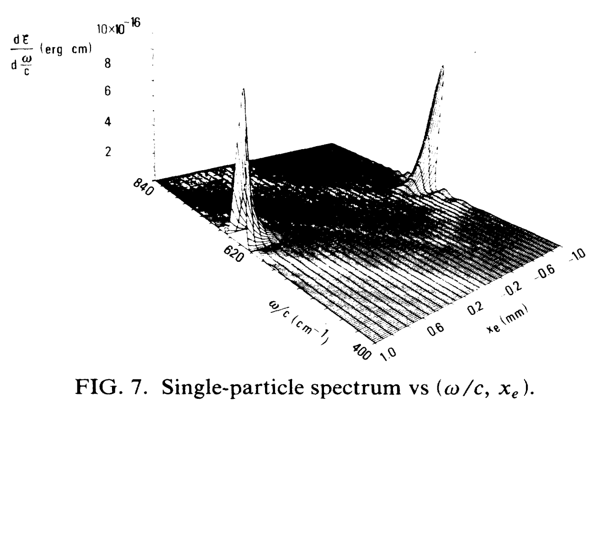

FIG. 7. Single-particle spectrum vs (ω/c, x_e).

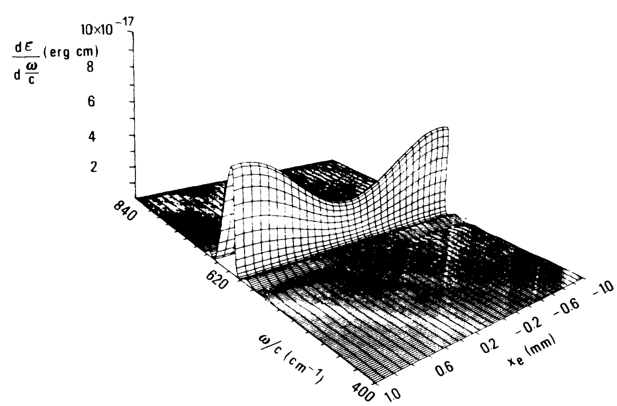

FIG. 9. Spectrum for a Gaussian-distributed e beam with rms $\sigma_x = 0.5$ mm.

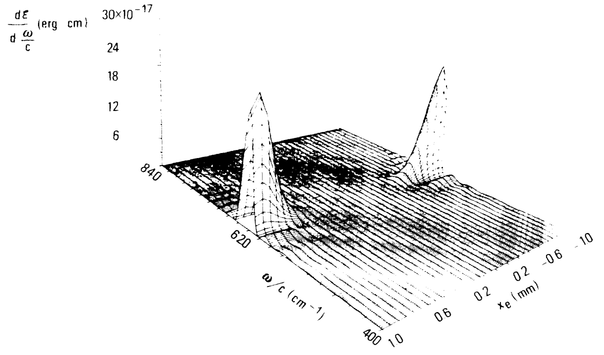

FIG. 10. Spectrum for a Gaussian-distributed e beam with rms $\sigma_x = 0.1$ mm.

elsewhere).

In Fig. 6 we report the single-particle spectrum of an electron moving near the surface of a dielectric at a distance of 200 μm, as expected, the spectrum is practically identical to that of the single slab configuration since the electron experiences the field of the nearest dielectric surface only. In Fig. 7 we report the 3D plot of the spectrum versus ω/c and distance from the center of the guide. The spectrum has a double-peak intensity, which follows the pattern of the field. In Fig. 8 the situation changes dramatically with respect to the single slab configuration. Indeed, assuming a Gaussian-distributed beam with an rms transverse dimension of the same order of the distance between the slabs, the x profile of the spectrum is almost constant.

Unlike the single-slab case there is not an exponential decay because of the possibility of getting contributions from both sides of the dielectric surfaces. However, taking a beam with smaller transverse dimension the spectral intensity decreases at the center of the waveguide (see Fig. 9) and becomes identical to the single-particle case for a pointlike beam (see Fig. 10).

The advantages of the double-slab configuration are therefore evident. They allow the possibility of utilizing a larger portion of the electron beam with the consequent higher brightness of the spectrum.

The impact of the double-slab configuration on a Čerenkov FEL device will be discussed elsewhere.

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*Also at Dipartimento di Fisica, Università di Salerno, Salerno, Italy.

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