

Second-harmonic generation by a partially coherent beam

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(Received 23 June 1986)

The effect of degradation of the coherence of the pump beam by a random phase screen on the conversion efficiency in the second-harmonic generation process is considered. It is shown that, under suitable conditions, the conversion efficiency can be increased significantly.

I. INTRODUCTION

It is well known that the nonlinear optical processes are significantly influenced by the coherence properties of the source.^{1,2} Consequently, there has been a great deal of interest in the study of the effect of fluctuations in the second-harmonic generation. These include the quantum fluctuations as well as the classical fluctuations of the pump field.³⁻⁶

Many studies have also been made to consider the spatial coherence properties of the beams generated by a random phase screen.⁷⁻¹¹ In this paper we consider the effect of degrading the spatial coherence of the pump beam on the conversion efficiency in the second-harmonic-generation process. In particular, we consider the situation where a coherent Gaussian laser beam scatters from a random phase screen, such as a rotating ground glass plate, and then interacts with the nonlinear medium to generate a second-harmonic beam. Our results indicate that by an appropriate choice of the parameters it should be possible to enhance the conversion efficiency by this method. We restrict ourselves to the situation where the conversion is not very large so that the parametric approximation is valid. For the description of the random phase screen we consider a model due to Berry¹² and which was later studied extensively by Jake-man and Pusey.^{13,14}

II. CONVERSION EFFICIENCY OF SECOND-HARMONIC BEAM

We consider a lossless nonlinear crystal of length l between the planes $z=0$ and $z=l$. A spatially partially coherent pump beam of frequency ω is incident at plane $z=0$ which gives rise to a second-harmonic beam of frequency 2ω after interaction with nonlinear crystal. In the nonlinear medium the pump and second-harmonic waves

satisfy the following set of coupled differential equations in the paraxial approximation:

$$\frac{\partial^2 \epsilon^\omega}{\partial x^2} + \frac{\partial^2 \epsilon^\omega}{\partial y^2} - 2ik_1 \frac{\partial \epsilon^\omega}{\partial z} = -K_1 (\epsilon^\omega)^* \epsilon^{2\omega}, \quad (1a)$$

$$\frac{\partial^2 \epsilon^{2\omega}}{\partial x^2} + \frac{\partial^2 \epsilon^{2\omega}}{\partial y^2} - 2ik_2 \frac{\partial \epsilon^{2\omega}}{\partial z} = -K_2 (\epsilon^\omega)^2, \quad (1b)$$

where $\epsilon^\omega(\mathbf{r})$ and $\epsilon^{2\omega}(\mathbf{r})$ are the slowly varying amplitudes of the pump and the second-harmonic waves, respectively; k_1 and k_2 are the corresponding wave numbers such that $k_2 = 2k_1$; and

$$K_1 = \frac{1}{2} K_2 = \frac{4\pi\omega^2}{c^2} d. \quad (2)$$

ω is the frequency of the pump wave and d is the non-linearity coefficient for second-harmonic generation.

In the parametric approximation, the pump depletion is neglected by making the right-hand side (RHS) of Eq. (1a) to be zero. This approximation is valid for small conversion efficiency. The solution of Eqs. (1a) and (1b) is therefore

$$\epsilon^\omega(\mathbf{r}) = \int \epsilon_0^\omega(\boldsymbol{\rho}_1) \Delta_1(\mathbf{r} - \boldsymbol{\rho}_1) d^2 \rho_1, \quad (3a)$$

$$\epsilon^{2\omega}(\mathbf{r}) = -K_2 \int [\epsilon_0^\omega(\mathbf{r}_1)]^2 \Delta_2(\mathbf{r} - \mathbf{r}_1) d^3 r_1, \quad (3b)$$

where $\boldsymbol{\rho} = (x, y)$ is the transverse two-dimensional vector, $\epsilon_0^\omega(\boldsymbol{\rho})$ is the pump field amplitude in the plane $z=0$, and the Green's functions Δ_1 and Δ_2 are

$$\Delta_1(\mathbf{r} - \boldsymbol{\rho}_1) = \frac{k_1}{2\pi iz} \exp \left[\frac{ik_1(\boldsymbol{\rho} - \boldsymbol{\rho}_1)^2}{2z} \right], \quad (4a)$$

$$\Delta_2(\mathbf{r} - \mathbf{r}_1) = -\frac{1}{4\pi(z - z_1)} \exp \left[\frac{ik_2(\boldsymbol{\rho} - \boldsymbol{\rho}_1)^2}{2(z - z_1)} \right]. \quad (4b)$$

It then follows, in a straightforward manner, that

$$\begin{aligned} I^{2\omega}(\mathbf{r}) &= \langle [\epsilon^{2\omega}(\mathbf{r})]^* \epsilon^{2\omega}(\mathbf{r}) \rangle \\ &= K_2^2 \int d^3 r_1 \int d^3 r_2 \int d^2 \rho_1 \int d^2 \rho_2 \int d^2 \rho_3 \int d^2 \rho_4 \Delta_2^*(\mathbf{r} - \mathbf{r}_1) \Delta_2(\mathbf{r} - \mathbf{r}_2) \Delta_1^*(\mathbf{r}_1 - \boldsymbol{\rho}_1) \\ &\quad \times \Delta_1^*(\mathbf{r}_1 - \boldsymbol{\rho}_2) \Delta_1(\mathbf{r}_2 - \boldsymbol{\rho}_3) \Delta_1(\mathbf{r}_2 - \boldsymbol{\rho}_4) \Gamma^\omega(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, \boldsymbol{\rho}_3, \boldsymbol{\rho}_4), \end{aligned} \quad (5)$$

where

$$\Gamma^\omega(\rho_1, \rho_2, \rho_3, \rho_4) = \langle [\varepsilon_0^\omega(\rho_1)]^* [\varepsilon_0^\omega(\rho_2)]^* \varepsilon_0^\omega(\rho_3) \varepsilon_0^\omega(\rho_4) \rangle \quad (6)$$

is the fourth-order correlation function of the pump field. In Eq. (5) r_1 and r_2 integrations can be done. The resulting expression for the mean intensity at the point $\mathbf{r}=(\rho, l)$ is¹⁵

$$I^{2\omega}(\rho, l) = \frac{K_2^2}{4l^2} \frac{k_2^2}{(4\pi)^4} \int d^2\rho_1 \int d^2\rho_2 \int d^2\rho_3 \int d^2\rho_4 E_1 \left[\frac{-ik_2}{8l} (\rho_1 - \rho_2)^2 \right] E_2 \left[\frac{ik_2}{8l} (\rho_3 - \rho_4)^2 \right] \\ \times \exp \left\{ \frac{ik_2}{2l} \left[[\rho - \frac{1}{2}(\rho_1 + \rho_2)]^2 - [\rho - \frac{1}{2}(\rho_3 + \rho_4)]^2 \right] \right\} \Gamma^\omega(\rho_1, \rho_2, \rho_3, \rho_4), \quad (7a)$$

where

$$E_1 \left[\frac{-ik_2}{8l} (\rho_1 - \rho_2)^2 \right] = \int_0^l \frac{dz}{z} \exp \left[\frac{ik_2}{8z} (\rho_1 - \rho_2)^2 \right]. \quad (7b)$$

Next we determine the correlation function $\Gamma^\omega(\rho_1, \rho_2, \rho_3, \rho_4)$. If a Gaussian laser beam scatters from a random screen at $z=0$, the field is given by

$$\varepsilon_0^\omega(\rho) = \varepsilon_{00} e^{-\rho^2/w_0^2} e^{i\phi(\rho)}, \quad (8)$$

where w_0 is the rms radius of the Gaussian beam and $\phi(\rho)$ is the random phase. We assume that $\phi(\rho)$ is described by a Gaussian process with zero mean. The correlation function is then

$$\Gamma^\omega(\rho_1, \rho_2, \rho_3, \rho_4) = |\varepsilon_{00}|^4 \exp \left[-\frac{1}{w_0^2} (\rho_1^2 + \rho_2^2 + \rho_3^2 + \rho_4^2) \right] g(\rho_1, \rho_2, \rho_3, \rho_4), \quad (9a)$$

where

$$g(\rho_1, \rho_2, \rho_3, \rho_4) = \exp \left\{ -\frac{1}{2} \langle [\phi(\rho_1) + \phi(\rho_2) - \phi(\rho_3) - \phi(\rho_4)]^2 \rangle \right\} \\ = \exp \left\{ \langle \phi^2 \rangle [2 + C(\rho_1 - \rho_2) + C(\rho_3 - \rho_4) - C(\rho_1 - \rho_3) - C(\rho_1 - \rho_4) - C(\rho_2 - \rho_3) - C(\rho_2 - \rho_4)] \right\}, \quad (9b)$$

with $\langle \phi^2 \rangle$ being the mean-square phase fluctuations and

$$C(\rho_1 - \rho_2) = \frac{\langle \phi(\rho_1) \phi(\rho_2) \rangle}{\langle \phi^2 \rangle} = C(\rho_2 - \rho_1). \quad (10)$$

An interesting model for the normalized second-order phase correlation function $C(\rho)$ is given by a Gaussian, i.e.,

$$C(\rho) = \exp(-\rho^2/\xi^2), \quad (11)$$

where ξ is the phase correlation length. This choice of $C(\rho)$ is, however, rather inconvenient in the calculation of $I^{2\omega}(\rho, l)$ according to Eq. (7a). However, when $\langle \phi^2 \rangle \gg 1$, we can use the approximation^{13,14,16}

$$e^{\langle \phi^2 \rangle C(\rho_1 - \rho_2)} \simeq 1 + (e^{\langle \phi^2 \rangle} - 1) e^{-\langle \phi^2 \rangle (\rho_1 - \rho_2)^2 / \xi^2}. \quad (12)$$

The intensity of the second-harmonic beam in the output plane $z=l$ is then obtained by substituting for Γ^ω from Eqs. (9)–(12) in Eq. (7).

A quantity of interest is the conversion efficiency η which is the ratio of the power of the second-harmonic beam in the plane $z=l$ to the power of the fundamental beam in the plane $z=0$, i.e.,

$$\eta_{pc} = \frac{\int I^{2\omega}(\rho, l) d^2\rho}{\int I^\omega(\rho, 0) d^2\rho}. \quad (13)$$

In Eq. (13), the subscript pc represents partial coherence. It can be shown (see Appendix) that, when $\alpha \gg 1$,

$$\eta_{pc} = \frac{K_2^2 |\varepsilon_{00}|^2}{128} \left[w_0^4 \left| \ln \left[1 + \frac{4il}{k_2 w_0^2} \right] \right|^2 \right. \\ \left. + \frac{1}{4} \int_0^l \frac{dz_1}{z_1} \int_0^l \frac{dz_2}{z_2} \left[\frac{1}{\beta_1 \beta_2^*} \right] \right. \\ \left. \times (e^{-(\beta_1 + \beta_2^*) \xi^2} - e^{-\beta_1 \xi^2} - e^{-\beta_2^* \xi^2}) \right], \quad (14)$$

where

$$\beta_i = \frac{1}{2W_0^2} - \frac{ik_2}{8z_i} \quad (i=1, 2). \quad (15)$$

For the coherent light, the conversion efficiency is given by

$$\eta_{coh} = \frac{K_2^2 |\varepsilon_{00}|^2 w_0^4}{128} \left| \ln \left[1 + \frac{4il}{k_2 w_0^2} \right] \right|^2. \quad (16)$$

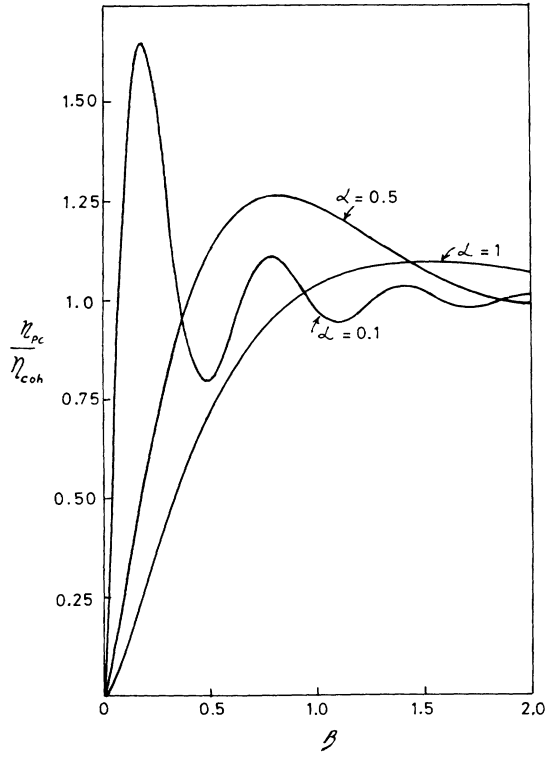


FIG. 1. η_{pc}/η_{coh} vs β for $\alpha=0.1, 0.5,$ and 1.0 .

It follows from Eqs. (14) and (16), after some rearrangement, that the ratio of the conversion efficiencies due to beams generated by a partially coherent source and by a coherent source of identical intensity is

$$\frac{\eta_{pc}}{\eta_{coh}} = 1 + \left\{ \frac{1}{4} [\ln(1+\alpha^2)]^2 + (\tan^{-1}\alpha)^2 \right\}^{-1} \times [F_1^2(\alpha, \beta) + F_2^2(\alpha, \beta) - \ln(1+\alpha^2)F_1(\alpha, \beta) - 2(\tan^{-1}\alpha)F_2(\alpha, \beta)], \quad (17)$$

where

$$\alpha = \frac{4l}{k_2 w_0^2}, \quad (18a)$$

$$\beta = \frac{\xi^2}{2w_0^2}, \quad (18b)$$

$$F_1(\alpha, \beta) = e^{-\beta} \int_0^1 \frac{dx}{(x^2 + 1/\alpha^2)} \times \left[x \cos \left[\frac{\beta}{\alpha x} \right] - \frac{1}{\alpha} \sin \left[\frac{\beta}{\alpha x} \right] \right], \quad (18c)$$

$$F_2(\alpha, \beta) = e^{-\beta} \int_0^1 \frac{dx}{(x^2 + 1/\alpha^2)} \times \left[x \sin \left[\frac{\beta}{\alpha x} \right] - \frac{1}{\alpha} \cos \left[\frac{\beta}{\alpha x} \right] \right]. \quad (18d)$$

In Fig. 1 we have plotted η_{pc}/η_{coh} versus β for different values of α . It is clear that, under suitable conditions, it should be possible to increase the conversion efficiency by 50% or more by degrading the coherence of the laser beam.

III. DISCUSSION

The enhanced conversion efficiency for a partially coherent pump field has been predicted before (Ref. 17). The difference between the present paper and Ref. 17 is the model for the pump field. Whereas in the present paper we assume that the random phase of the scattered laser beam is described by a Gaussian random process, in Ref. 17 we describe the random amplitude of the scattered beam by a Gaussian random process. With a Gaussian random amplitude, the fourth-order correlation function $\Gamma^\omega(\rho_1, \rho_2, \rho_3, \rho_4)$ of the pump beam is then given in terms of the second-order correlation $\Gamma^\omega(\rho_1, \rho_2)$ as follows:

$$\Gamma^\omega(\rho_1, \rho_2, \rho_3, \rho_4) = \Gamma^\omega(\rho_1, \rho_3)\Gamma^\omega(\rho_2, \rho_4) + \Gamma^\omega(\rho_1, \rho_4)\Gamma^\omega(\rho_2, \rho_3), \quad (19)$$

where $\Gamma^\omega(\rho_1, \rho_2) = \langle [\varepsilon_0^\omega(\rho_1)]^* \varepsilon_0^\omega(\rho_2) \rangle$. This is a result of the moment theorem of the Gaussian random processes. The enhancement of the second-harmonic generation conversion due to amplitude fluctuations arises simply due to the decomposition (19) of $\Gamma^\omega(\rho_1, \rho_2, \rho_3, \rho_4)$. In the present case, it is the phase noise which leads to an enhanced conversion efficiency.

The predicted conversion efficiency shown in Fig. 1 has several interesting features. The most obvious is the dependence of the ratio on both α , the normalized crystal length, and β , the normalized spatial coherence length. This strong coupling disappears in the two limits where the beam is fully coherent ($\beta \rightarrow \infty$) or fully incoherent ($\beta \rightarrow 0$). The ratio η_{pc}/η_{coh} goes to 1 and 0, respectively, in these limits, as it should. The peak conversion efficiency depends strongly on both parameters.

ACKNOWLEDGMENTS

This research was supported in part by the Pakistan Science Foundation and the International Laser System, a division of Litton System, Inc.

APPENDIX: DERIVATION OF EQ. (14)

We define a new set of variables,

$$\begin{aligned} \mathbf{r}_1 &= \rho_1 - \rho_2, \quad \mathbf{r}_2 = \rho_3 - \rho_4, \\ \mathbf{R}_1 &= \frac{1}{2}(\rho_1 + \rho_2), \quad \mathbf{R}_2 = \frac{1}{2}(\rho_3 + \rho_4), \end{aligned} \quad (A1)$$

in Eq. (7). Since

$$\int d^2\rho \exp \left[\frac{ik_2}{2l} [(\rho - \mathbf{R}_1)^2 - (\rho - \mathbf{R}_2)^2] \right] = \left[\frac{2\pi l}{k_2} \right]^2 \delta^{(2)}(\mathbf{R}_1 - \mathbf{R}_2), \quad (\text{A2})$$

it follows from Eq. (7) that

$$\begin{aligned} \int d^2\rho I^{2\omega}(\rho, l) &= \frac{K_2^2}{1024\pi} |\varepsilon_{00}|^4 w_0^2 \int_0^l \frac{dz_1}{z_1} \int_0^l \frac{dz_2}{z_2} d^2r_1 d^2r_2 \\ &\quad \times \exp[-(\beta_1 r_1^2 + \beta_2^* r_2^2)] \exp \left\{ -\langle \phi^2 \rangle \left[2 + C(\mathbf{r}_1) + C(\mathbf{r}_2) - 2C \left[\frac{\mathbf{r}_1 - \mathbf{r}_2}{2} \right] \right. \right. \\ &\quad \left. \left. - 2C \left[\frac{\mathbf{r}_1 + \mathbf{r}_2}{2} \right] \right] \right\}, \quad (\text{A3}) \end{aligned}$$

where β_i ($i=1,2$) is given by Eq. (15). In deriving Eq. (A3), we have used the explicit forms of E_1 and Γ^ω from Eqs. (7a) and (9), respectively.

The terms associated with the phase fluctuations in Eq. (3) can be simplified considerably in the limit $\langle \phi^2 \rangle \gg 1$. On substituting for $\exp[\langle \phi^2 \rangle C(r)]$ from Eq. (12), this term becomes, in the limit $\langle \phi^2 \rangle \gg 1$,

$$\begin{aligned} D &= \exp \left\{ -\langle \phi^2 \rangle \left[2 + C(\mathbf{r}_1) + C(\mathbf{r}_2) - 2C \left[\frac{\mathbf{r}_1 - \mathbf{r}_2}{2} \right] - 2C \left[\frac{\mathbf{r}_1 + \mathbf{r}_2}{2} \right] \right] \right\} \\ &= \frac{e^{-2\langle \phi^2 \rangle} [1 + (e^{\langle \phi^2 \rangle} - 1) e^{-(\langle \phi^2 \rangle / 4\xi^2)(r_1 - r_2)^2}]^2 [1 + (e^{\langle \phi^2 \rangle} - 1) e^{-(\langle \phi^2 \rangle / 4\xi^2)(r_1 + r_2)^2}]^2}{[1 + (e^{\langle \phi^2 \rangle} - 1) e^{-(\langle \phi^2 \rangle / \xi^2)r_1^2}] [1 + (e^{\langle \phi^2 \rangle} - 1) e^{-(\langle \phi^2 \rangle / \xi^2)r_2^2}]} \\ &\approx \frac{e^{2\langle \phi^2 \rangle} e^{-(\langle \phi^2 \rangle / \xi^2)(r_1^2 + r_2^2)}}{(1 + e^{\langle \phi^2 \rangle} e^{-(\langle \phi^2 \rangle / \xi^2)r_1^2})(1 + e^{\langle \phi^2 \rangle} e^{-(\langle \phi^2 \rangle / \xi^2)r_2^2})}, \quad (\text{A4}) \end{aligned}$$

so that, after carrying out the angular integrations in Eq. (A3), we obtain

$$\int d^2\rho I^{2\omega}(\rho, l) = \frac{\pi K_2^2}{1024} |\varepsilon_{00}|^4 w_0^2 \int_0^l \frac{dz_1}{z_1} \int_0^l \frac{dz_2}{z_2} T(\beta_1) T(\beta_2^*), \quad (\text{A5})$$

where

$$T(\beta_i) = e^{\langle \phi^2 \rangle} \int_0^\infty dx \frac{e^{-[\beta_i + (\langle \phi^2 \rangle / \xi^2)]x}}{1 + e^{\langle \phi^2 \rangle (1 - x / \xi^2)}} \quad (i=1,2). \quad (\text{A6})$$

The major contribution to the integration in Eq. (A6) comes from the region $x=0$ to $x=\xi^2$ when $\langle \phi^2 \rangle \gg 1$. In this situation we can change the limits of integration from 0 to ξ^2 and ignore 1 in the denominator so that

$$T(\beta_i) \simeq \int_0^{\xi^2} dx e^{-\beta_i x} = \frac{1}{\beta_i} (1 - e^{-\beta_i \xi^2}). \quad (\text{A7})$$

We therefore obtain

$$\int d^2\rho I^{2\omega}(\rho, l) = \frac{\pi K_2^2}{1024} |\varepsilon_{00}|^4 w_0^2 \int_0^l \frac{dz_1}{z_1} \int_0^l \frac{dz_2}{z_2} \frac{1}{\beta_1 \beta_2} (1 - e^{-\beta_1 \xi^2})(1 - e^{-\beta_2^* \xi^2}). \quad (\text{A8})$$

Since

$$\int d^2\rho I^\omega(\rho, 0) = \frac{\pi |\varepsilon_{00}|^2 w_0^2}{2}, \quad (\text{A9})$$

the resulting expression for the conversion efficiency for the second-harmonic field [cf. Eq. (13)] is given by Eq. (14).

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