Solitons in strongly magnetized electron-positron plasmas and pulsar microstructure

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The propagation of an electromagnetic wave in weakly nonlinear but strongly magnetized electron-positron plasmas is investigated. A quasistatic slow plasma response to the wave is shown to result in solitons with the height directly proportional to the strength of the ambient magnetic field. A dispersion relation is derived in this case of magnetized plasma and the result is used to confirm the recent suggestion of self-modulational formation of pulsar microstructure.

I. INTRODUCTION

Nonlinear propagation of intense electromagnetic waves in magnetized plasma is of interest in connection with laser-induced fusion, ionosphere modification by radar, and the interaction of pulsar radiation with the plasma environment. The case of unmagnetized plasma has been investigated by several authors for the last few years (Shukla *et al.*;[1] see also references therein). Compressional, rarefactional, subsonic, supersonic, single-hump and double-hump solitons etc., are found to exist in electron-ion plasma. But for the large amplitude field, electron-ion plasma also behaves like electron-positron plasma to first order in the mass ratio;[2] the oscillatory velocity of the particle approaches the speed of light and the resulting mass variation causes strong nonlinearity. The latter competes with the self-interaction (ponderomotive) nonlinearity and gives rise to solitary wave structures, either density humps (solitons) or density holes (cavitons) in plasma.

According to current polar-cap pulsar models,[3,4] the pulsar magnetosphere is composed of secondary electrons and positrons resulting from pair production induced by high-energy curvature radiation photons emitted by primary positrons or electron beams coming from the pulsar surface. Recently, Chian and Kennel[5] explained the ultrashort intensity variations within individual pulses in pulsar radio emission. In their model nonlinearities arising from wave-intensity-induced particle-mass variation may excite a modulational instability of circularly polarized pulsar radiation. The result is a modulating solitonic envelope on a high-frequency carrier wave, and the microstructure analysis shows that the number N of micropulses within an individual pulse and the temporal pulse width τ are within the observed ranges ($N\sim 10^2-10^3$, $\tau\sim 1$ μsec), provided that the emission takes place in the low-density region. Such an investigation was carried out in our earlier paper[6] and similar results were obtained.

Since plasma in laser fusion, in the upper atmosphere, and in the pulsar environment is strongly magnetized, a strong magnetic field is to be taken into account. Such an attempt has been made in this paper with the approximation of quasistatic slow plasma response.

In Sec. II we derive the governing equation which describes the nonlinear evolution of the intense electromagnetic wave propagating in a magnetized electron-positron plasma. In Sec. III the governing equation is solved analytically for the case of a highly intense ambient magnetic field. Plasma response to the wave is found to result in a soliton with the height directly proportional to the strength of the ambient field. A dispersion relation for this solution is also derived following the method of Karpman and Krushkal.[7]

In Sec. IV our solution is used to confirm the recent data on pulsar microstructure. We have shown that microstructure of the individual pulse is related to the ambient magnetic field and the temperature of the pulsar environment. Some other aspects of the present theory, its short comings, and future developments are noted in the discussion.

II. BASIC EQUATIONS

Consider an electromagnetic (e.m.) wave propagating in an electron-positron plasma immersed in an ambient magnetic field. The direction of wave propagation is taken along the ambient field $\mathbf{B}_0$ (z axis) and it is assumed that all quantities do not depend on x and y but on z and time t. This system is described by the two fluids relativistic equations for the plasma, the wave equation for the vector potential $\mathbf{A}$, Poisson's equation for the scalar potential Φ, and the continuity equation

$$\left[\frac{\partial}{\partial t}+v_{jz}\frac{\partial}{\partial z}\right]\mathbf{u}_j=\frac{q_j}{mc^2}\left[-\frac{\partial \mathbf{A}}{\partial t}-c\nabla\Phi+\mathbf{v}_j\times(\nabla\times\mathbf{A})+\mathbf{v}_j\times B_0\hat{\mathbf{z}}\right]-\frac{T_j}{mc}\nabla\ln n_j\ , \quad (1)$$

$$\frac{\partial^2\mathbf{A}}{\partial t^2}-c^2\frac{\partial^2\mathbf{A}}{\partial z^2}=4\pi c\sum_j n_j q_j\mathbf{v}_j\ , \quad (2)$$

$$\frac{\partial^2\Phi}{\partial z^2}=-4\pi e(n_p-n_e)\ , \quad (3)$$

$$\frac{\partial n_j}{\partial t}+\frac{\partial}{\partial z}(n_j v_j z)=0\ , \quad (4)$$

where j refers to $p=e^{+}$ and $e=e^{-}$, $q_j=\pm e$ is the charge, $\mathbf{v}_j$ is the particle velocity, $\mathbf{u}_j=\mathbf{p}_j/mc$ is the dimensionless momentum (m is the electron rest mass):

$$\mathbf{u}_j=\gamma_j\frac{\mathbf{v}_j}{c},\quad \gamma_j=(1+|\mathbf{u}_j|^2)^{1/2}\ . \tag{5}$$

n_j is the particle density and T_j is the temperature. For convenience, we have assumed isothermal electrons and positrons, although other adiabaticies can easily be included.

The circularly polarized wave is given by

$$\lambda^{1/2}\mathbf{A}_\perp=\alpha(z,t)(\hat{\mathbf{x}}+i\hat{\mathbf{y}})e^{-i\omega t}+\text{c.c.},\quad \lambda=\frac{e^2}{m^2c^4}\ . \tag{6}$$

The amplitude α in (6) is complex and a slowly varying function of z and t.

It is easily verified that the transverse motion is related to the field by

$$\mathbf{u}_{j\perp}=-\frac{q_j}{mc^2}\frac{\mathbf{A}_\perp}{(1+\nu\Gamma_j)}\ , \tag{7}$$

where

$$\nu=\frac{|e|B_0}{mc\omega},\quad \Gamma_j=\frac{1}{\gamma_j}\ .$$

From the wave equation we have

$$i\frac{2\delta}{\omega_p}\alpha_t+\frac{c^2}{\omega_p^2}\alpha_{zz}+\left[\delta^2-\frac{1}{n_0}\left[\frac{n_p}{\gamma_p+\nu}+\frac{n_e}{\gamma_e+\nu}\right]\right]\alpha=0\ , \tag{8}$$

where $\omega_p=(4\pi n_0e^2/m)^{1/2}$, n_0 is the background initial density, $\delta=\omega/\omega_p$, and the term $(1/\omega)\alpha_{tt}\sim 0$ is neglected. Let

$$\alpha(z,t)=\sigma(z,t)e^{i\phi(z,t)}$$

where σ,ϕ are real. From Eq. (8) we have two equations,

$$\frac{\partial}{\partial t}\sigma^2+\frac{c^2}{\omega}\frac{\partial}{\partial z}(\phi_z\sigma^2)=0\ , \tag{9a}$$

$$\frac{2\delta}{\omega_p}\sigma\phi_t-\frac{c^2}{\omega_p^2}\sigma_{zz}+\left[\frac{c^2\phi_z^2}{\omega_p^2}-\delta^2\right]\sigma+\frac{1}{n_0}\left[\frac{n_p}{\gamma_p+\nu}+\frac{n_e}{\gamma_e+\nu}\right]\alpha=0\ . \tag{9b}$$

In the linear phase approximation

$$\phi(z,t)=\psi(z)+\phi(t)\ , \tag{10}$$

ϕ_z,ϕ_t are constants. Equation (9a) gives the condition

$$|\alpha|^2=\sigma^2=f(\xi),\quad \xi=z-v_0t,\quad v_0=\frac{c^2\phi_z}{\omega}\ . \tag{11}$$

Our earlier investigations[6,8] show that the relativistic mass variation of the particle generates modulational instability of the wave. The wave is localized in the form of solitons. The speed of the soliton is superluminious ($v=v_0/c>1$). Longitudinal flow of the fluid occurs due to the radiation and the thermal pressure. Since the longitudinal force is a gradient force, the fluid flow may be considered much smaller than the speed of the soliton (i.e., $v_{jz}\ll v_0$). In such a case the theory of quasistatic slow plasma response[9,10] may be used to investigate the present problem. Furthermore, particle energy in this case is mostly determined by the transverse motion and it gives $\gamma_e\approx\gamma_p\approx\gamma$. Under such conditions particle energy in the high-frequency field and the ponderomotive force may be easily calculated. Particle energy in the field is

$$\gamma=[(1+\nu)^2+|\alpha|^2]^{1/2}-\nu\ . \tag{12}$$

The ponderomotive force is the averaged force which acts on the particle due to radiation:

$$F=-\nabla\Psi\ , \tag{13a}$$

where

$$\Psi=mc^2\{[(1+\nu)^2+|\alpha|^2]^{1/2}-(1+\nu)\} \tag{13b}$$

is the ponderomotive potential.

It is to be noted that the ponderomotive force calculated in this case is exact and by letting $\nu=0$ the unmagnetized case is recovered.[10] In the slow plasma response, the radiation pressure is assumed to be balanced by the ambipolar potential and thermal pressure. Hence the density perturbation of the fluid may be calculated from the longitudinal motion dropping the inertia[10]

$$n_j=n_0e^{-1/T(q_j\Phi+\Psi)}\ . \tag{14}$$

The ambipolar field Φ is generated by the high-frequency radiation pressure which is charge independent. Hence the charge separation may be neglected and the plasma is quasineutral ($n_p=n_e=n$).

$$n=n_0e^{-\beta\{[(1+\nu)^2+|\alpha|^2]^{1/2}-(1+\nu)\}}\ , \tag{15}$$

with

$$\beta=\frac{mc^2}{T}\ .$$

The complex amplitude $\alpha(z,t)$ is described by the equation

$$i\frac{2\delta}{\omega_p}\alpha_t+\frac{c^2}{\omega_p^2}\alpha_{zz}+\left[\delta^2-\frac{2}{[(1+\nu)^2+|\alpha|^2]^{1/2}}\exp(-\beta\{[(1+\nu)^2+|\alpha|^2]^{1/2}-(1+\nu)\})\right]\alpha=0 \tag{16}$$

Under the approximation of quasistatic slow plasma response, Eq. (16) is the governing equation which describes the nonlinear evolution of the intense electromagnetic wave propagating in a magnetized electron-positron plasma. Relativistic and ponderomotive nonlinearities are the reasons to generate modulational instability to the wave.

III. SOLITONS IN STRONGLY MAGNETIZED PLASMA

We are considering the case of laser fusion or pulsar radiation where the ambient magnetic field is very intense (in the vicinity of 10^{12} G). In such cases $\nu \gg 1$, $\nu^2 \gg |\alpha|^2$, and Eq. (16) takes the form

$$i\alpha_t + \frac{c^2}{2\omega}\alpha_{zz} + \frac{\omega_p^2}{2\omega}\left[\delta^2 - \frac{2}{\nu}\right]\alpha + \frac{\omega_p^2}{2\omega\nu^2}\left[\frac{1}{\nu}+\beta\right]|\alpha|^2\alpha = 0 . \quad (17)$$

If the linear term is removed with the substitution

$$\alpha = \bar{\alpha} e^{iRT}, \quad R = \frac{\omega_p^2}{2\omega}\left[\delta^2 - \frac{2}{\nu}\right] \quad (18)$$

the equation for $\bar{\alpha}$ is the nonlinear Schrödinger equation

$$i\bar{\alpha}_t + P\bar{\alpha}_{zz} + Q|\bar{\alpha}|^2\bar{\alpha} = 0 , \quad (19)$$

with

$$P = \frac{c^2}{2\omega}, \quad Q = \frac{\omega_p^2}{2\omega\nu^2}\left[\frac{1}{\nu}+\beta\right] . \quad (20)$$

A well-known solution of Eq. (19) is the soliton[11]

$$\bar{\alpha}(z,t) = \alpha_0 \text{sech}\left[\left|\frac{Q}{2P}\right|^{1/2}\alpha_0(z - v_0 t)\right] \times e^{i(k_0 z - \omega_0 t)} , \quad (21)$$

with

$$k_0 = \frac{\omega v}{c}, \quad \omega_0 = \frac{1}{2}\left[\omega v^2 - \frac{\omega_p^2}{2\omega\nu^2}\left[\frac{1}{\nu}+\beta\right]\alpha_0^2\right] . \quad (22)$$

The solution for the wave is then

$$\lambda^{1/2}\mathbf{A}(z,t) = \alpha_0 \text{sech}\left[\left|\frac{Q}{2P}\right|^{1/2}\alpha_0(z - v_0 t)\right] \times [\hat{\mathbf{x}}\cos\theta(z,t) + \hat{\mathbf{y}}\sin\theta(z,t)] , \quad (23)$$

where

$$\alpha_0 = \frac{\nu}{\omega_p}\left[\frac{2\omega(\omega v^2 - 2\omega_0)}{\left[\frac{1}{\nu}+\beta\right]}\right]^{1/2}$$

is the dimensionless amplitude and the phase $\theta(z,t)$ is given by

$$\theta(z,t) = \frac{\omega v}{c}z - \left[\frac{1}{2}(1+v^2)\omega + \frac{\omega_p^2}{\omega\nu} - \frac{\omega_p^2}{4\omega\nu^2}\left[\frac{1}{\nu}+\beta\right]\alpha_0^2\right]t . \quad (24)$$

The solution is a solitonic envelope to the wave. The amplitude and the phase of the wave are modulated and differ significantly from the case of unmagnetized plasma.[6] The magnetic field strongly effects the solitonic envelope of the wave with the modification in the phase. The field energy is proportional to $\alpha_0^2\text{sech}^2 x$, where $x = (Q/2P)^{1/2}\alpha_0\xi$. For larger ν, we see that the height of the localized field is directly proportional to the strength of the field (ν).

A dispersion relation for the wave in this case of strongly magnetized electron-positron plasma can be obtained with the method of Karpman-Krushkal:[7]

$$\omega = \omega(\mathbf{k}, \alpha_0^2), \quad P = \frac{1}{2}\frac{\partial^2\omega}{\partial k^2}, \quad Q = -\frac{\delta\omega}{\delta\alpha_0^2} , \quad (25)$$

where P,Q are given by Eq. (20). The result is

$$\omega^2 = k^2c^2 + \omega_p^2 - \frac{\omega_p^2}{2\nu^2}\left[\frac{1}{\nu}+\beta\right]\alpha_0^2 , \quad (26)$$

which shows that the increasing magnetic field significantly modifies the dispersion relation.

IV. PULSAR MICROSTRUCTURE

The microstructure of the individual pulse is related with the modulational instability of the wave. This can be seen to occur for $PQ > 0$ and in the magnetized case this condition is always satisfied.

We estimate the number N of micropulses within a single pulse and the pulse temporal width τ. From Eq. (23) we see that the envelope pulse width is

$$\delta = \left|\frac{2P}{Q}\right|^{1/2}\frac{1}{\alpha_0} . \quad (27a)$$

Then

$$N = \frac{\delta}{\lambda} \equiv \left|\frac{2P}{Q}\right|^{1/2}\frac{1}{\alpha_0}\frac{k}{2\pi}, \quad \tau = \frac{\delta}{v_0} \equiv \left|\frac{2P}{Q}\right|^{1/2}\frac{1}{\alpha_0 v_0} , \quad (27b)$$

where λ is the wavelength. Taking P and Q from Eq. (20) and the wave number k from Eq. (26), we have

$$N = \frac{\nu}{2\pi\alpha_0\omega_p}\left\{\left[\frac{2}{\frac{1}{\nu}+\beta}\right]\left[\omega^2 - \omega_p^2 + \frac{\omega_p^2}{2\nu^2}\left[\frac{1}{\nu}+\beta\right]\alpha_0^2\right]\right\}^{1/2} , \quad (28)$$

$$\tau = \frac{\nu}{\omega_p}\left[\frac{2}{\frac{1}{\nu}+\beta}\right]^{1/2}\frac{1}{\alpha_0 v} . \tag{29}$$

We see that the magnetic field and the temperature of the pulsar environment influence greatly the microstructure and the pulse width of the pulsar radiation. For an easy understanding, let us consider a limiting case where $\omega/\omega_p \gg 1$.

Then we have

$$N \sim \frac{\nu\omega}{\sqrt{2}\alpha_0\omega_p}\left[\frac{T}{mc^2}\right]^{1/2}, \quad \tau \sim \frac{\nu}{v\omega_p\alpha_0}\left[\frac{2T}{mc^2}\right]^{1/2} . \tag{30}$$

It shows that the number of micropulses and the pulse width are directly proportional to the magnetic field and the temperature. On the other hand, micropulses of higher intensities have a narrower pulse width as suggested by Ferguson.[12]

V. DISCUSSION

In this paper we have investigated the solitonic envelope to an intense electromagnetic wave propagating in a strongly magnetized electron-positron plasma. Such a situation prevails in laser fusion and in the pulsar environment. We have also studied pulsar microstructure here. A quasistatic slow plasma response is considered with the assumption that thermal pressure is exactly balanced by the ambipolar and the ponderomotive forces. Under the quasineutral approximation, plasma density is determined by the ponderomotive potential. The field-dependent nonlinear current is calculated and the governing equation describing the nonlinear evolution of the propagating wave in the magnetized electron-positron plasma is obtained.

For intense magnetic fields the equation is simplified and the evolution is governed by the nonlinear Schrödinger equation (NLS), which gives a solitonic envelope to the carrier wave with the amplitude-dependent nonlinear frequency shift. With the increase of ambient field strength, the soliton's height also increases. The dispersion relation for the wave propagating in the magnetized plasma is calculated to establish the pulsar microstructure. The resulting pulse structure, number of micropulses within one single pulse, and temporal pulse width are calculated with on account of the strong magnetic field and temperature in the pulsar environment.

However, a quantitative test of the modulational instability theory requires the extension to include other effects such as large wave amplitude, plasma temperature, plasma inhomogeneity, and the removal from the quasineutral approximation. Progress on the above points will be reported elsewhere.

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