PHYSICAL REVIEW A VOLUME 36, NUMBER 3 AUGUST 1, 1987

Simultaneous observation of Lyman-α and Balmer-β transitions in hydrogenic iron, Fe^{25+}: A novel technique for $1s$ Lamb-shift measurement

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(Received 16 October 1986)

The $n=2$ to $n=4$ Balmer-β transitions in hydrogenic Fe^{25+} have been observed in the first order of Bragg diffraction in a beam-foil light source. Simultaneous observation of the $1s$-$2p$ Lyman-α transitions from the same source in the fourth diffraction order leads to a new method for the measurement of the $1s$ ground-state Lamb shift which is less sensitive to uncertainties arising from Doppler shifts than previous work. With the use of the Balmer-β transitions for calibration, the Lyman-α wavelengths are, for the $1s\,^2S_{1/2}$–$2p\,^2P_{3/2}$ transition, $1.778\,15\pm0.000\,19$ Å, and for the $1s\,^2S_{1/2}$–$2p\,^2P_{1/2}$ transition, $1.783\,64\pm0.000\,19$ Å. Values of the Lamb-shift contributions to the $1s$-$2p$ transitions, and the $2p_{1/2,3/2}$ fine structure, are extracted.

INTRODUCTION

The energy-level separations between the $n=1$ and $n=2$ states of hydrogenic ions may be calculated using the Dirac equation, with additional small corrections for nuclear size and motion, for the anomalous magnetic moment of the electron, and for quantum electrodynamic (QED) radiative corrections.[1-4] A comparison of the measured values with theory serves as a test of the theory and is a test of the radiative QED contributions, in particular the $1s$ Lamb shift, insofar as the other contributions are well known. Though it was not appreciated at the time, the first such measurement of a $1s$ Lamb shift was made in the hydrogenic lithium ion in 1933.[5,6] QED tests of this type are now of particular interest for highly charged hydrogenic ions, since a comparison between theory and experiment for the Lamb shift in high-Z hydrogenic ions is the most precise and sensitive way to test QED at high-field strength.[7]

Recently, a number of measurements of the $1s\,^2S_{1/2}$–$2p\,^2P_{1/2,3/2}$ transition wavelengths have been reported for hydrogenic ions with Z between 17 and 36.[8-14] The accuracy to which the Lyman-α wavelengths were measured was limited either by the uncertainty in the Doppler-shift correction arising from the uncertainty in the ion beam velocity and the viewing angle[8-10,12] or from the uncertainty in allowing for the presence of satellites in the spectra.[13] We describe here a novel technique which is insensitive to Doppler shifts. From the low intensities of all heliumlike and lithiumlike transitions in our spectra, our technique also does not appear to be subject to serious contamination of the spectrum with satellite transitions from lower charge states. The technique is demonstrated in hydrogenic iron Fe^{25+}, but it may be applied to higher-Z hydrogenic ions.

The principle of the technique is to use the 4:1 ratio of the wavelengths of Balmer-β and Lyman-α transitions to make a comparison between the Balmer-β transition in the first diffraction order, and the Lyman-α transition in the fourth order.[15] This technique is similar in concept to that used by Wieman and Hänsch[16] in a measurement of the $1s\,^2S_{1/2}$ Lamb shift in atomic hydrogen, from which our technique actually takes its inspiration. A detailed consideration of the Lyman-α–Balmer-β spectrum[15] shows that, fortuitously, the separation between the components of interest is small enough that the wavelength of the Lyman-α transition relative to that of the Balmer-β transition may be accurately determined, yet sufficiently large that only moderate resolution is required to separate the transitions. In the present experiment, the hydrogenic transitions studied all arise from the same charge state in the same fast ion beam, so that the relative Doppler shifts $\delta\lambda/\lambda_0$ will be the same for all components. This is the major advantage of the technique compared with other work using calibration from stationary sources. In addition, the particular experiment described here employed a double spectrometer[17-19] which independently provides Doppler cancellation by simultaneous observation at two angles, ϕ and $180°+\phi$ to the ion beam direction. In this arrangement, the sum of the two observed Doppler-shifted wavelengths is $2\gamma\lambda_0$, independent of ϕ [the ion beam velocity is $v=\beta c$, c is the speed of light, $\gamma=(1-\beta^2)^{-1/2}$, and the rest-frame wavelength of the transition is λ_0].

EXPERIMENT

The experimental arrangement is shown in Fig. 1. A beam of bare iron nuclei (Fe^{26+}) at an energy of 0.489 GeV (8.74 MeV/A, $\beta=0.135$), from the Lawrence Berkeley Laboratory Super Heavy Ion Linear Accelerator (SuperHILAC) was passed through a thin (25 μg cm^{-2}) carbon foil. Some of these nuclei capture an electron in the foil, and we find that under these excitation conditions, lines in the Balmer series of hydrogenic Fe^{25+} are clearly seen for principal quantum numbers n up to about 10. In particular, there is a good probability for the production of the $n=2$ to $n=4$ hydrogenic Balmer-β transitions.

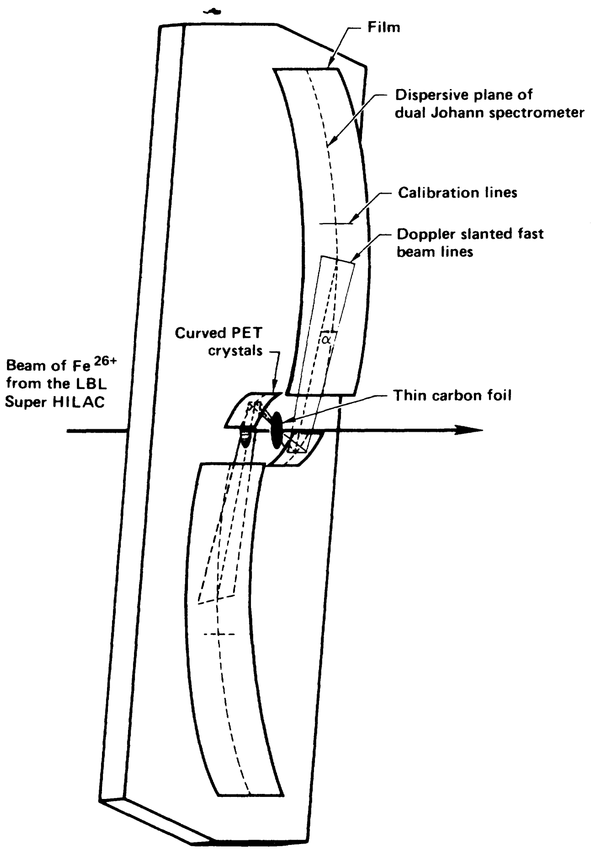

FIG 1. The experimental setup.

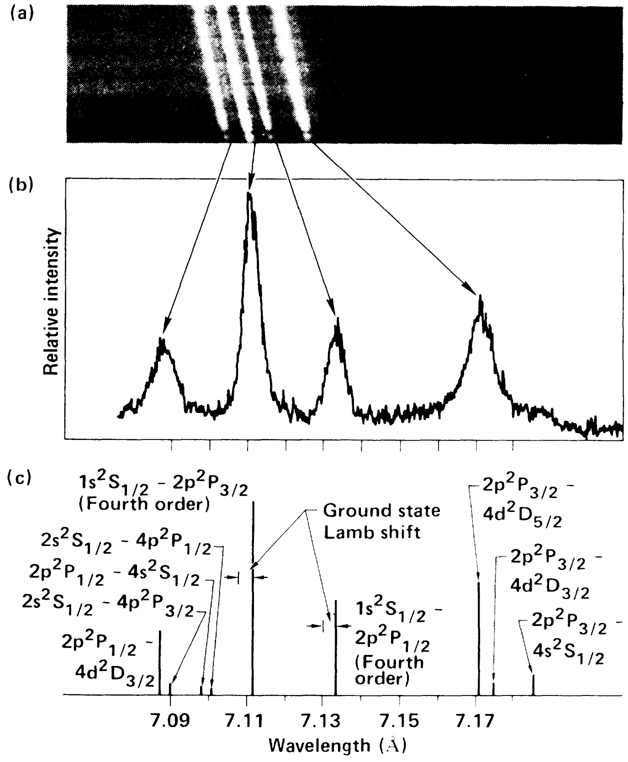

FIG. 2. (a) A print of a typical film in the Lyman-α or Balmer-β region. (b) A microdensitometer scan of the Lyman-α or Balmer-β region. (c) The predicted spectrum, using the theoretical wavelengths from Table I and relative intensities of the Balmer-β transitions from H. A. Bethe and E. E. Salpeter, *Quantum Mechanics of One- and Two-Electron Atoms* (Springer-Verlag, New York, 1957), Sec. 63, Table 16.

From earlier experiments with hydrogenic Kr^{35+} at Grand Accelerateur National d'Ions Lourds (GANIL), Caën, France, and hydrogenic Ge^{31+} at Gesellschaft für Schwerionenforschung Darmstadt m.b.H. (GSI), Darmstadt, Germany, it is now clear that the combination of a fully stripped ion beam and a thin exciter foil gives the best excitation conditions so far realized for the simultaneous observation of Lyman-α and Balmer-β radiation.[20]

The spectrometer used was the dual-arm Johann curved crystal spectrometer developed by Lawrence Livermore National Laboratory.[17-19] The dispersing crystal used in the experiment was PET(002), with a $2d$ spacing of 8.757 Å.[21] Spectra were recorded on DEF-392 x-ray film. The spectra were subsequently put into digital form (intensity transmissivity as a function of position) using an accurate, computer-controlled digital microdensitometer.[22] It was found that the Lyman-α widths were slightly reduced and the signal-to-background ratio improved by removing the developed emulsion from one side of the film. A print of a typical film and a microdensitometer scan over the Lyman-α–Balmer-β region is shown in Fig. 2. A number of such scans across the film were made, with the microdensitometer viewing a region of the film 0.5 mm high by 10 μm wide. Each scan covered about 8.5 mm of film. For each scan, the dispersion function of the spectrometer may be written as

$$n\lambda_0 = \frac{2d}{\gamma}\left(1-\frac{4d^2}{n^2}\frac{\delta}{\lambda^2}\right)\left(\frac{\cos\alpha}{1-\beta\sin\alpha}\right)\sin\left(\frac{x-x_0}{2R}\right), \quad (1)$$

where n is the order of diffraction, λ_0 is the rest-frame wavelength of the transition studied, and d is the spacing between the diffracting planes of the crystal. The angle between the direction of observation and the axis of the ion beam is $90° - \alpha$, where α is small. R is the Rowland-circle radius (15 cm), x is the distance along the film, and x_0 is a constant. The term involving δ accounts for refraction of x rays by the diffracting crystal, the refractive index of the crystal being $\mu = 1 - \delta$.

The scans were analyzed with the assumption that the only Balmer-β components present in the spectrum were the strong $2p\,^2P_{1/2}$-$4d\,^2D_{3/2}$ and $2p\,^2P_{3/2}$-$4d\,^2D_{5/2}$ components. Peak centroids were found by least-squares fitting to Gaussian profiles. The dispersion relation (1) was then inverted and the fitted centroids for these two Balmer components in different scans were used to fit the parameters $(2d/\gamma)$ and x_0. The dispersion relation was then used with these values for the fit parameters and the fitted centroids for the Lyman-α peaks to derive wave-

TABLE I. Values assumed for the Balmer-β components, and Lyman-α wavelengths.

Transition	Theoretical wavelength (Å)	This experiment (Å)
$2p\,^2P_{1/2}$–$4d\,^2D_{3/2}$	7.087807[a]	
$2s\,^2S_{1/2}$–$4p\,^2P_{3/2}$	7.090160[a]	
$2p\,^2P_{1/2}$–$4s\,^2S_{1/2}$	7.098244[a]	
$2s\,^2S_{1/2}$–$4p\,^2P_{1/2}$	7.100926[a]	
$1s\,^2S_{1/2}$–$2p\,^2P_{3/2}$	1.778016[b]	1.77815 ± 0.00019
	1.778044[a]	
$1s\,^2S_{1/2}$–$2p\,^2P_{1/2}$	1.783442[b]	1.78364 ± 0.00019
	1.783469[a]	
$2p\,^2P_{3/2}$–$4d\,^2D_{5/2}$	7.171197[a]	
$2p\,^2P_{3/2}$–$4d\,^2D_{3/2}$	7.174808[a]	
$2p\,^2P_{3/2}$–$4s\,^2S_{1/2}$	7.188503[a]	

[a] G. W. Erickson, J. Phys. Chem. Ref. Data **6**, 831 (1977).

[b] P. J. Mohr, At. Data Nucl. Data Tables **29**, 435 (1983).

lengths for the Lyman-α transitions. The value of δ/λ^2 is calculated to be 2.11×10^{-6} Å^{-2} (see Ref. 23).

The results obtained for the Lyman-α wavelengths are, for $1s\,^2S_{1/2}$–$2p\,^2P_{3/2}$, 1.77815 ± 0.00019 Å and, for $1s\,^2S_{1/2}$–$2p\,^2P_{1/2}$, 1.78364 ± 0.00019 Å. The quoted error includes the quadrature sum of contributions from the statistical error in the centroid determination (0.000047 Å for the $1s\,^2S_{1/2}$–$2p\,2P_{3/2}$ transition and 0.00011 Å for the $1s\,^2S_{1/2}$–$2p\,^2P_{1/2}$ transition), 0.000037 Å from the uncertainty in the parameters $2d/\gamma$ and x_0 used in the dispersion relation, 0.000027 Å from an estimated 10% uncertainty in the refractive-index correction. A contribution of 0.000032 Å has been added linearly to account for possible shifts of the Balmer-β components due to perturbation of the $n=4$ levels by electric fields experienced by the Fe^{25+} ions in the carbon targets. This possible source of error arises because the observed radiation is emitted in part by ions decaying inside the carbon target. It is known that fields of order 6×10^8 V/cm may be experienced by ions moving through solid targets.[24] Such a field is estimated to lead to a shift of the most sensitive $n=4$ levels of ~1000 cm^{-1}. Further systematic errors can arise due to the neglect of the weaker Balmer-β peaks. A theoretical estimate of this effect was made, as well as a preliminary computer analysis of several scans including all the Balmer-β peaks, from which we concluded that neglect of the weaker Balmer-β peaks in our analysis did not lead to shifts greater than 0.000078 Å for $1s\,^2S_{1/2}$–$2p\,^2P_{3//2}$ or 0.000035 Å for $1s\,^2S_{1/2}$–$2p\,^2P_{1/2}$. These contributions are therefore added linearly to give the final quoted errors. Table I gives the values assumed for the Balmer-β components, and Lyman-α wavelengths obtained. Table II presents values of the Lamb-shift contribution to the $1s$–$2p$ transitions and the $2p_{1/2,3/2}$ fine structure derived from the measured Lyman-α wavelengths.

CONCLUSION

We have demonstrated a novel technique for the measurement of the ground-state Lamb shift in hydrogenic ions by making a simultaneous observation of the Lyman-α and Balmer-β transitions in the fourth and first diffraction orders, for the hydrogenic ion Fe^{25+}. From our data we obtain values for the wavelengths of the Lyman-α transitions with an accuracy of 107 ppm. From these we can extract the ground-state Lamb shift with an accuracy of 17%, and a value for the $2p_{1/2,3/2}$ fine-structure splitting with an accuracy of 3%. The results obtained are in reasonable agreement with theory and with previous measurements. The accuracy of the present experiment is limited by the statistical fluctuations in the signal, and by the resolution obtained. Additional uncertainties arise from lack of knowledge of the spectral profiles, including possible weak satellite contamination, uncertainty in the refractive index correction, and the effect of electric fields inside the target. We can envisage experimental improvements which should result in wavelength accuracies at the ppm level for hydrogenic ions in the range $Z=20$ to $Z=40$, and the technique should also

TABLE II. Values of the Lamb-shift contribution to the $1s$-$2p$ transitions and the $2p_{1/2,3/2}$ fine structure derived from the measured Lyman-α wavelengths.

		Experiment (cm^{-1})	Theory[a] (cm^{-1})	Other measurements[b,c,d]
Lamb shift	($1s$–$2p_{3/2}$)	36000 ± 6000	31892 ± 21	27400 ± 4800[b]
	($1s$–$2p_{1/2}$)	38400 ± 6000	32160 ± 21	33300 ± 5600[b]
Fine structure	$2p_{1/2,3/2}$	173000 ± 5300	171104 ± 1	176600 ± 3200[b]
				173400 ± 3200[c]
				170200 ± 1600[d]

[a] P. J. Mohr, At. Data Nucl. Data Tables **29**, 435 (1983); W. R. Johnson and G. Soff, *ibid.* **33**, 406 (1985). The values given for the $1s$-$2p$ Lamb shifts are for the *transitions*, as given in Table I of each reference.

[b] J. P. Briand *et al.*, Phys. Rev. Lett. **50**, 832 (1983).

[c] C. J. Hailey *et al.*, J. Phys. B **18**, 1443 (1985).

[d] J. P. Briand *et al.*, Z. Phys. A **318**, 1 (1984).

be applicable to the much higher-Z hydrogenic ions ($Z \sim 80-90$) which will become available in the next few years with the new generation of high-energy heavy-ion accelerators.

ACKNOWLEDGMENTS

This experiment is one of a series which have been carred out at GSI, GANIL, and the Lawrence Berkeley Laboratory (LBL). We wish to acknowledge the help and participation of P. H. Mokler, E. C. Finch, D. Lecler, and J. P. Grandin in the experiments at GSI and GANIL, and also the contributions of B. P. Duval, H. A. Klein, and J. Laursen in earlier, related experiments. J.D.S., A.F.M., J.M.L., and S.D.R. would like to gratefully acknowledge support from E. W. J. Mitchell and Science and Engineering Research Council (SERC) Grants No. GR-C-00040 and No. GR-D-42955. G.C.C., D.D.D., and P.D.E. worked under the auspices of the U.S. Department of Energy under Contract No. W-7405-ENG-48. We are also grateful to B. L. Henke, H. Yamada, and C. J. Hailey for clarifying discussions regarding the calculation of refractive indices, and to H. Yamada for performing an independent calculation of δ. We would also like to thank the operators of the LBL superHILAC who did an excellent job in providing us with the iron beams for this experiment.

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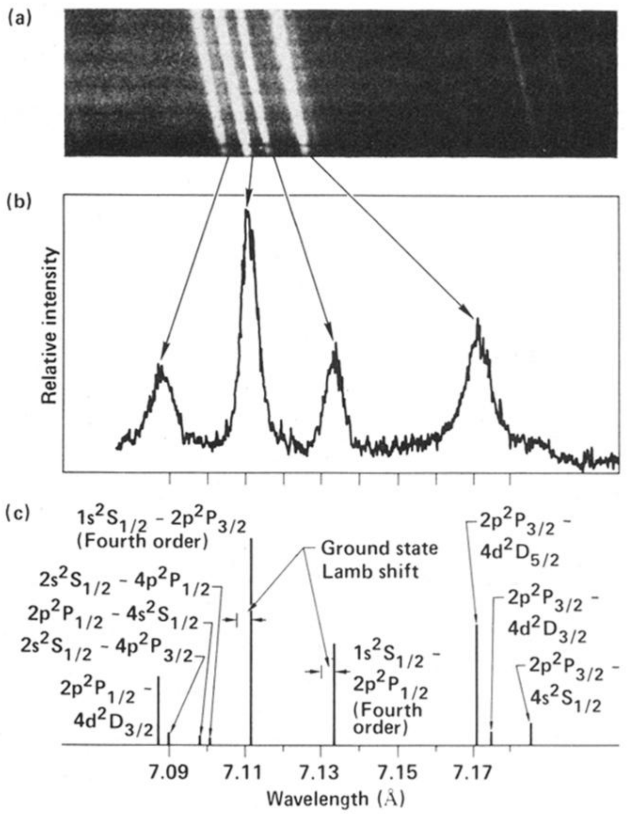
[24] S. Datz *et al.*, Phys. Rev. Lett. **40**, 843 (1978).



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