Many-photon processes with the participation of squeezed light

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The description of an electromagnetic field of complicated statistics with noise, coherent, and squeezed features are discussed using the nondiagonal coherent-state representation of the density matrix and the normally ordered characteristic function derived therefrom. The quantity $\langle (a^\dagger)^n a^n \rangle$ which plays a central role in the theory of many-photon processes is found for this field. Particular cases with dominant coherent signal, noise, or squeezing are discussed. It is found that the ratios $\langle (a^\dagger)^n a^n \rangle / (\langle a^\dagger a \rangle)^n$ for the coherent, the chaotic, and the strongly squeezed vacuum states are 1, $n!$, and $(2n-1)!!$, respectively. The value of $\langle (a^\dagger)^n \rangle$ related to the anomalous coherence function is considered. It is found that the noise does not influence $\langle (a^\dagger)^n \rangle$.

I. INTRODUCTION

The possibility of two-quantum processes was discussed by Goeppert-Mayer as early as 1931,[1] long before the intensities of the light sources were high enough to observe such processes. Since the 1960's a considerable interest in many-photon processes can be seen.[2–11]

Many-photon processes give us a key to solve one of the principal problems of quantum optics—the measurement of the higher correlation functions of the radiation fields.[12] While the probability of a one-photon process depends on the intensity of light, the probability of many-photon processes depends strongly on the statistical properties of the optical field.

In the present paper we shall consider many-photon transitions in atoms. The radiation field is assumed to have complex statistical properties, including squeezing, noise, and coherent signal. Our motivation was to achieve a deeper understanding of experiments on many-photon processes where the statistics of light can be measured. On the other hand, multiphoton processes with the participation of a nonclassical electromagnetic field are of principal interest.

II. TRANSITION PROBABILITY

We consider an atom with a resonant absorption frequency ω_a and a quasimonochromatic field with ω_0 mean frequency and $\Delta\omega$ bandwidth. We assume that $\Delta\omega$ is small compared to the inverse lifetime of the excited state of the atom $\Delta\omega \ll \gamma$, that there is only a small detuning, i.e., $\omega_a \approx n\omega_0$, and that there is no intermediate level $\hbar\omega_i$ for which $\omega_i \approx n'\omega_0$, $n' < n$, n and n' being integers. Under these circumstances the transition probability in unit time equals[11]

$$W^{(n)} = \frac{G^{(n,n)}(\{0\},\{0\}) \, |\Phi(\{\omega_0\})|^2 \gamma}{[\gamma^2/4 + (\omega_a - n\omega_0)^2]} , \tag{2.1}$$

where $G^{(n,m)}(\{t\},\{t'\})$ is the (n,m)th-order normally ordered correlation function of the electromagnetic field[12,13] and $\Phi(\{\omega\})$ is a function introduced by Agarwal.[11]

$$G^{(n,n)}(\{0\},\{0\}) \sim \mathrm{Tr}[\rho (a^\dagger)^n a^n] \equiv G_{nn} , \tag{2.2}$$

where ρ is the density operator of the field.

G_{nn} describes the quantum-statistical properties of light. In a pure coherent state,

$$G_{nn} = (\langle a^\dagger a \rangle)^n = G_{11}^n , \tag{2.3}$$

while for a chaotic field,[11,13]

$$G_{nn} = n!(\langle a^\dagger a \rangle)^n = n! G_{11}^n . \tag{2.4}$$

III. STATISTICAL PROPERTIES OF LIGHT

The quantum statistics of the radiation field have recently been extensively investigated in connection with the squeezed states of light.[14–33] Experimentally detected squeezing has been first reported in Ref. 31.

A state is squeezed if either of the Hermitian field operators

$$X_+ = a + a^\dagger , \quad X_- = -i(a - a^\dagger) , \quad [X_+, X_-] = 2i \tag{3.1}$$

has a variance less than unity

$$\Delta X_+ < 1 \quad \text{or} \quad \Delta X_- < 1 . \tag{3.2}$$

According to the uncertainty relation

$$\Delta X_+ \Delta X_- \geq 1 , \tag{3.3}$$

where the equal sign stands for a pure squeezed state. Figure 1 shows the variances of X_+ and X_- for different quantum states: A is a coherent or vacuum state, B can be a chaotic or a photon number state. A and B are

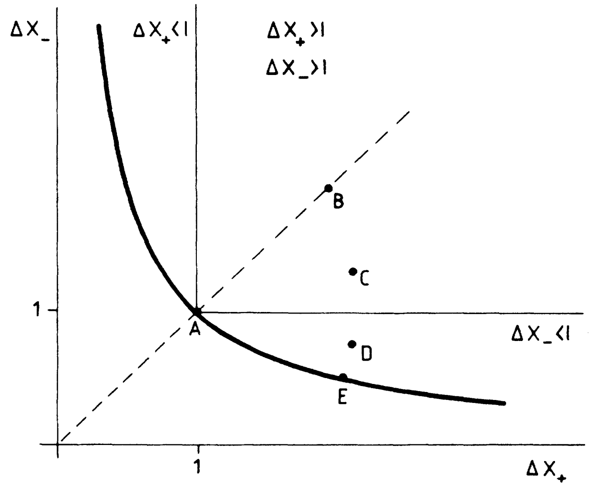

FIG. 1. Variances of the field quadratures X_+ and X_-. A is a coherent or vacuum state, B is a chaotic state, C has some squeezed features, D is squeezed but not a minimum-uncertainty state, and E is a squeezed minimum-uncertainty state.

unsqueezed states. C has some squeezed features but does not meet the condition (3.2) to be called a squeezed state. D is already squeezed but is not a minimum uncertainty state. An example of C and D is the superposition of noise and squeezing effects. E is a squeezed vacuum or a squeezed coherent state. A pure squeezed state can be obtained from a coherent (or vacuum) state by several lossless nonlinear optical processes (e.g., degenerate parametric generation, four-wave mixing with combination of the probe and conjugated modes,[14] or directly in an anisotropic case,[30] etc.) characterized by a Bogoliubov transformation

$$a_{\text{out}}=ua_{\text{in}}+va_{\text{in}}^{\dagger}\ ,\quad u^2-|v|^2=1\ . \tag{3.4}$$

In a squeezed state the P representation is inconvenient; thus, for the density operator ρ one should use the R representation

$$\rho=\frac{1}{\pi^2}\int d^2\alpha d^2\beta R(\beta^*,\alpha)|\beta\rangle\langle\alpha| \times\exp[-(|\beta|^2+|\alpha|^2)/2]\ . \tag{3.5}$$

As an example let us find $R(\beta^*,\alpha)$ for an initial vacuum state

$$\rho=|0\rangle\langle 0| \tag{3.6}$$

after a nonlinear process which realizes (3.4). It is convenient to find the normally ordered characteristic function first:

$$\chi(\eta)=\text{Tr}(\rho e^{\eta a^{\dagger}}e^{-\eta^* a})\ ,$$
$$\langle (a^{\dagger})^n a^m\rangle=\frac{\partial^n}{\partial\eta^n}\left[-\frac{\partial}{\partial\eta^*}\right]^m\chi(\eta)|_{\eta=\eta^*=0}\ . \tag{3.7}$$

Using (3.4), (3.6), and the well-known identity

$$e^A e^B=e^{A+B+\frac{1}{2}[A,B]}\ ,$$
if
$$[A,[A,B]]=[B,[A,B]]=0\ , \tag{3.8}$$

we find

$$\chi(\eta)=\exp[-|v|^2|\eta|^2+\tfrac{1}{2}(uv^*\eta^2+uv\eta^{*2})]\ . \tag{3.9}$$

The functions $R(\beta^*,\alpha)$ and $\chi(\eta)$ are connected by an integral transformation[30]

$$\chi(\eta)=\frac{1}{\pi}\int d^2\alpha\, R(\alpha^*,\alpha+\eta)e^{-|\alpha|^2-\alpha\eta^*}\ , \tag{3.10}$$

$$R(\beta^*,\alpha)=\frac{1}{\pi}e^{\beta^*\alpha}\int d^2\eta\,\chi(\eta)e^{-|\eta|^2-\eta\beta^*+\eta^*\alpha}\ . \tag{3.11}$$

Using (3.9) and (3.11) we have for a squeezed vacuum state

$$R(\beta^*,\alpha)=\frac{1}{u}\exp\left[\frac{1}{2u}(v^*\alpha^2+v\beta^{*2})\right]\ . \tag{3.12}$$

One can ascertain directly that this state is a pure state ($\rho^2=\rho$) by finding out that R satisfies the relation

$$\int d^2\gamma\, R(\beta^*,\gamma)R(\gamma^*,\alpha)e^{-|\gamma|^2}=R(\beta^*,\alpha)\ , \tag{3.13}$$

and also that for real v's

$$\Delta X_{\pm}=u\pm v\ ,\quad \Delta X_+\Delta X_-=1\ . \tag{3.14}$$

This means that during a lossless nonlinear process characterized by (3.4) we go, in Fig. 1, from a vacuum state A to a pure squeezed state E along the minimum-uncertainty hyperbola.

In a real situation there are factors which lead to noise: (1) the initial state before the squeezing process may be already noisy, (2) the nonlinear optical processes, e.g., four-wave mixing are connected with loss and noise, and (3) after the squeezing process any optical processing may inject noise into the field. The resulting electromagnetic field has complicated statistics with noise, squeezing aspects, and coherent signal.

The noisy initial state can be modeled in the P representation by the weight function of the input density operator

$$P_0(\alpha)=\frac{1}{\pi N_0}\exp(-|\alpha-\alpha_0|^2/N_0)\ , \tag{3.15}$$

describing a superposition of a coherent signal (α_0) with noise (N_0).

The loss and noise effects during the nonlinear process with interaction length L can be accounted instead of (3.4) by[30]

$$a_{\text{out}}=u(L)a_{\text{in}}+v(L)a_{\text{in}}^{\dagger}+F(L)\ , \tag{3.16}$$

where

$$F(x)=\int_0^x dx'[Q(x')u(x-x')+Q^{\dagger}(x')v(x-x')]\ . \tag{3.17}$$

$Q(x)$ is a noise operator satisfying the usual commutative relation for noise sources

$$[Q(x),Q^{\dagger}(x')]=2\gamma\delta(x-x') , \tag{3.18}$$

γ is the loss coefficient and

$$[u(x)]^2-|v(x)|^2=e^{-2\gamma x} . \tag{3.19}$$

The injection of additional noise after squeezing can be described by a transformation

$$\tilde{a}_{\rm out}=pa_{\rm out}+qb \quad p^2+q^2=1 , \tag{3.20}$$

where the mode characterized by the annihilation operator b is in a chaotic state.

Using the expressions (3.10) and (3.11), (3.15) and (3.16), and (3.20) one can find directly that any combination of the above-mentioned factors (1), (2), and (3) leads to the next form of R and χ for the output field

$$R(\beta^*,\alpha)=(m^2-|s|^2)^{1/2} \times\exp\{\beta^*\alpha-m(\alpha-W)(\beta^*-W^*) \times+\tfrac{1}{2}[s^*(\alpha-W)^2+s(\beta^*-W^*)^2]\} , \tag{3.21}$$

$$\chi(\eta)=\exp(-M|\eta|^2+\tfrac{1}{2}[S^*\eta^2+S\eta^{*2}]+\eta W^*-\eta^* W) , \tag{3.22}$$

where $W=|W|e^{i\psi}$ is the coherent signal, and m and s or M and S are related to the noncoherent part of the field,

$$M=\frac{m}{m^2-|s|^2}-1 , \quad S=\frac{s}{m^2-|s|^2}\equiv|S|e^{i\phi} . \tag{3.23}$$

For the vacuum $M=S=W=0$, in the pure coherent state $M=S=0$ and for the chaotic field $W=S=0$. The state is a pure squeezed state if

$$M=Q\equiv\tfrac{1}{2}[(4|S|^2+1)^{1/2}-1] , \quad S=e^{i\phi}\cosh r\sinh r , \tag{3.24}$$

r being the squeezing parameters. If $\phi=0$, then

$$\Delta X_{\pm}=e^{\pm}2r . \tag{3.25}$$

The mixed state emerging from a superposition of a pure squeezed state with coherent signal W and a chaotic field is described by (3.21) and (3.22) if

$$M=Q+N , \tag{3.26}$$

where N is the noise photon number.

The expression for G_{nn} can easily be obtained using the well-known generating function of the Hermite polynomials. From (3.7) and (3.22) we find

$$G_{nn}=\langle a^{\dagger n}a^n\rangle =(n!)^2\sum_{k=0}^{n}\left|H_k\left[\frac{iW}{\sqrt{2S}}\right]\right|^2\frac{|S|^k M^{n-k}}{2^k(k!)^2(n-k)!} . \tag{3.27}$$

Expression (3.27) gives G_{nn} in a rather cumbersome way, which can be simplified only for small n's

$$G_{11}=M+|W|^2 , \tag{3.28}$$

$$G_{22}=2M^2+4M|W|^2+|W|^4+|S|^2+W^2S^*+W^{*2}S, \tag{3.29}$$

$$G_{33}=6M^3+18M^2|W|^2+9M|W|^4+9M|S|^2 +9M(SW^{*2}+S^*W^2)+9|W|^2|S|^2 +3|W|^2(SW^{*2}+S^*W^2)+|W|^6 . \tag{3.30}$$

Measuring G_{nn} for different n's one can find the values of W, S, and M characterizing the complex statistics of light.

The measurement of $\Delta X_{\pm}$ can also give useful information on M and S, for in the rather general case characterized by $\chi(\eta)$ of Eq. (3.22) we have

$$\Delta X_{\pm}^2=1+2M\pm S\pm S^* , \tag{3.31}$$

that is,

$${\rm Re}\,S=\tfrac{1}{4}(\Delta X_+^2-\Delta X_-^2) , \tag{3.32}$$

$$M=\tfrac{1}{4}(\Delta X_+^2+\Delta X_-^2)-\tfrac{1}{2} . \tag{3.33}$$

IV. DISCUSSION OF PARTICULAR CASES

To discuss G_{nn} for higher n let us consider some particular cases.

(a) $|W|^2 \gg M$, $|W|^2 \gg S$, i.e., the coherent signal is much stronger than the noise and the squeezing effects. Under these circumstances G_{nn} is close to its coherent value given by (2.3)

$$G_{nn}\approx|W|^{2n}+\frac{n(n-1)}{2}|W|^{2n-4}(SW^{*2}+S^*W^2) +n^2|W|^{2n-2}M . \tag{4.1}$$

From (4.1), (3.23), and (3.28) we have

$$\frac{G_{nn}-G_{11}^n}{G_{11}^{n-1}}\approx n(n-1)[M+|S|\cos(2\psi-\phi)] . \tag{4.2}$$

For example, if $n=2$,

$$G_{22}-G_{11}^2=2|W|^2[M+|S|\cos(2\psi-\phi)]+M^2+|S|^2 , \tag{4.3}$$

the second and third terms being much smaller than the first one.

(b) $M\gg|W|^2$, $M\gg|S|$, i.e., the noise prevails over both coherent signal and squeezing. In this case,

$$G_{nn}\approx n!M^n[1+nM^{-1}|W|^2 +\frac{n(n-1)}{4M^2}(|W|^4+W^2S^* +W^{*2}S+|S|^2)] , \tag{4.4}$$

and

$$\frac{G_{nn}-n!G_{11}^n}{n!G_{11}^n} \approx \frac{n(n-1)}{4M^2}[\,|S|^2+2|W|^2|S| \times\cos(2\psi-\phi)-|W|^4] \;. \quad (4.5)$$

(c) $|S| \gg N$, $|S| \gg |W|^2$, i.e., the main feature of the state is squeezing:

$$G_{nn} \approx (n!)^2 \sum_{l=0}^{[n/2]} \frac{|S|^{2l-1}Q^{n-2l-1}}{2^{2l}(n-2l)!(l!)^2} \times[Q|S|+2lQ|W|^2\cos(2\psi-\phi) +(n-2l)|S|(N+|W|^2)] \;, \quad (4.6)$$

where $[k+\frac{1}{2}]=[k]=k$.

In the squeezed vacuum state $N=|W|^2=0$, $|S|^2=Q(1+Q)$, $G_{11}=Q$, and

$$G_{nn} \approx (n!)^2 Q^n \sum_{l=0}^{[n/2]} \frac{(1+1/Q)^l}{2^{2l}(n-2l)!(l!)^2} \;. \quad (4.7)$$

For a slightly squeezed vacuum ($Q \ll 1$)

$$G_{nn} \approx \frac{(n!)^2}{[n/2]!)^2 2^{2[n/2]}} (\langle a^\dagger a\rangle)^{(n-[n/2])} \;, \quad (4.8)$$

while in a strongly squeezed vacuum ($Q \gg 1$)

$$G_{nn} \approx (2n-1)!!(\langle a^\dagger a\rangle)^n \;. \quad (4.9)$$

From (2.3), (2.4), and (4.9) we can see that in the expression for G_{nn} the factors before $(\langle a^\dagger a\rangle)^n$ for the coherent, the chaotic, and the strongly squeezed vacuum states are 1, $n!$, and $(2n-1)!!$, respectively. It means that the ratio $\langle (a^\dagger)^n a^n\rangle/(\langle a^\dagger a\rangle)^n$ in the case of the squeezed vacuum increases even in comparison to the Gaussian noise. This is quite remarkable if one remembers that the $(n!)$ factor in (2.4) is explained[8] by the presence of more irregularities in the intensity distribution in the case of the chaotic field compared to the pure coherent one.

V. ANOMALOUS COHERENCE FUNCTION

So far we have dealt with diagonal correlation functions G_{nn}. Recently, the importance of the off-diagonal correlation functions or anomalous coherence functions[34] has been pointed out.[35] There are several possibilities to measure the anomalous coherence function. In Ref. 34 it was suggested that the measurement of the intensity correlations of the superposition of a field under consideration and a coherent field would yield information on the anomalous coherence function of the original field. Agarwal pointed out that mixing of a field with its phase-conjugated replica provides a convenient way to measure the anomalous coherence function.[35]

In this section we shall investigate G_{n0}, which is in fact an equal time anomalous coherence function. For the general case with noise, coherent signal, and squeezing, from (3.7) and (3.22) using the Hermite polynomial generating function, we have

$$G_{n0}=\langle a^{\dagger n}\rangle = i^n (S^*)^{n/2} 2^{-n/2} H_n \left[\frac{-iW^*}{\sqrt{2S^*}}\right] \;. \quad (5.1)$$

A remarkable result of (5.1) is that the noise does not influence G_{n0} at all. For small n's

$$G_{10}=W^* \;,$$
$$G_{20}=S^*+G_{10'}^2 \;, \quad (5.2)$$
$$G_{30}=3W^*S^*+G_{10}^3 \;.$$

It can be seen from (5.2) that for a squeezed vacuum the second-order anomalous coherence function G_{20} gives us S^* immediately. If $|W|^2 \gg |S|$, we have

$$G_{n0} \approx G_{10}^n + \frac{n(n-1)}{2} G_{10}^{n-2} S^* \;. \quad (5.3)$$

On the contrary, in the case of squeezed chaotic field (or squeezed vacuum),

$$G_{n0}=(n-1)!!(S^*)^{n/2} \;, \quad n=2l>0$$
$$G_{n0}=0 \;, \quad n=2l+1 \quad (5.4)$$

while for an unsqueezed chaotic field all G_{n0}'s vanish.

It is to be noted that the mean values and the variances of the observables X_+ and X_- are closely related to the off-diagonal coherence functions

$$G_{01}=\tfrac{1}{2}(\langle X_+\rangle + i\langle X_-\rangle) \;,$$
$$G_{10}=\tfrac{1}{2}(\langle X_-\rangle - i\langle X_-\rangle) \;, \quad (5.5)$$

and

$$\Delta X_\pm^2 = 1+2G_{11} \pm (G_{20}+G_{02}) - 2G_{10}G_{01} \mp (G_{10}^2+G_{01}^2) \;. \quad (5.6)$$

VI. CONCLUSIONS

In the absence of noise a number of nonlinear optical processes, especially if an intracavity arrangement is used, are expected to produce squeezing. In a real process the emerging field has chaotic, coherent, and squeezing features. Using a density-operator–characteristic-function approach, we have shown here how these features affect the quantity $\langle (a^\dagger)^n a^n\rangle$ which can be measured experimentally, e.g., by multiphoton processes. Such measurements together with measuring the mean values and variances of the field quadratures X_+ and X_- can give us the value of M, W, and S needed to describe an electromagnetic field of complex statistics. According to our results the probability of a multiphoton process for a strongly squeezed vacuum state is higher than for a coherent or a chaotic field.

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