Higher-order effects in the large-angle coplanar symmetric He(e^-,2e^-)He$^+$ process: A modified-Glauber-approximation study

K. S. Baliyan and M. K. Srivastava
Department of Physics, University of Roorkee, Roorkee 247 667, Uttar Pradesh, India
(Received 15 January 1986)

The triple-differential cross sections for the ionization of helium by fast electrons in the large-angle coplanar symmetric energy-sharing case are studied by using the modified Glauber approximation. Comparison with experimental data indicates that the higher-order terms of the scattering amplitude contribute quite significantly to the cross section. They, however, make the results worse than those of the second-Born approximation.

The most sensitive test of any theory of single ionization by electron impact is provided by the measurement of triple-differential cross sections (TDCS) in (e,2e) coincidence experiments. In the experiments one measures the probability that in an ionizing event an incident electron with energy E_0 and momentum $\mathbf{k}_0$ leads to, in the final state, two electrons having energy (momentum) E_a ($\mathbf{k}_a$, direction θ_a,ϕ_a) and E_b ($\mathbf{k}_b$, direction θ_b,ϕ_b). Two types of kinematical arrangements have been widely used in the experiments. (i) The asymmetric coplanar Ehrhardt-type geometry[1] in which the magnitude of the momentum transfer $K=|\mathbf{k}_0-\mathbf{k}_a|$ is small and the energy sharing between the two outgoing electrons is highly asymmetric. This geometry is particularly sensitive to the scattering model used. The essential features of the experimental angular distribution of the slower electron b for fixed E_a, E_b, θ_a, and ϕ_a have been explained by using the second Born approximation.[2,3] The agreement with the experimental data further improves if higher-order terms of the scattering amplitude are included.[4,5] (ii) The coplanar symmetric energy-sharing geometry which is popular in (e,2e) spectroscopic studies, $E_a=E_b$, $\theta_a=\theta_b\simeq 45°$, $(\phi_a-\phi_b)=\pi$. Here the momentum transfer K is large, however, the residual ion essentially plays a spectator role. It is found that in this situation even the theories which are essentially first order in character lead to reasonable results.[6] However, for large θ_a ($=\theta_b$), it has been shown by Byron, Joachain, and Piraux,[7] and Pochat *et al.*[8] for hydrogen and helium, respectively, that the second-order Born term of the scattering amplitude becomes as important as and even more important than the first-order term. This is not surprising since for large angles of scattering the magnitude of the momentum transfer $\mathbf{K}_r=\mathbf{k}_0-\mathbf{k}_a-\mathbf{k}_b$ to the residual ion is large and it ceases to be just a spectator. They have also shown that under these conditions the second Born term is governed by the initial and final target states acting as intermediate states.

The aim of the present paper is to look for the importance of higher-order terms in the ionization process $e^-+\mathrm{He}(1^1S)\rightarrow\mathrm{He}^+(1s)+e^-+e^-$ for which experimental data exist at 200 eV incident energy in a coplanar symmetric geometry at large angles.[8] We have recently used the modified Glauber approximation[9] to include the contribution of the higher-order terms in the case of electron-impact ionization of hydrogen and helium in the coplanar asymmetric geometry.[4,5] The same procedure is being followed here. It should be pointed out that the Born series for $A(e,2e)A^+$ processes diverges term by term except for the first term.[10] The second-Born and higher-order calculations which have been carried out to analyze experimental data and lead to finite results use an approximate form for these terms. In this sense these higher order (beyond the first) contributions are at least partly empirical.

The scattering amplitude f_{MG} in the modified Glauber approximation is given by[9]

$$f_{\mathrm{MG}}=f_G+f_{B2}-f_{G2}\ , \tag{1}$$

where f_{B2} and f_{G2} are, respectively, the second-order Born and Glauber amplitudes and f_G is the full Glauber amplitude. The amplitude f_G in Eq. (1) is evaluated by following the method of Roy, Das, and Sil.[11] The second-Born amplitude f_{B2} is taken to be just the contribution of the initial ($1\,^1S$) state acting as an intermediate state. This state has been shown to dominate the direct second-Born amplitude in the large-θ region by Pochat *et al.*[8] The procedure for evaluating f_{G2} has been given elsewhere in detail.[5] Here we outline the main steps. It is given by (see Ref. 12)

$$f_{G2}=\frac{i}{\pi k_0}\int\frac{d\mathbf{p}}{p^2|\mathbf{K}-\mathbf{p}|^2}\times\langle\Phi_f(\mathbf{r}_1,\mathbf{r}_2)\,|\,B(\mathbf{p})B(\mathbf{K}-\mathbf{p})\,|\,\Phi_i(\mathbf{r}_1,\mathbf{r}_2)\rangle\ , \tag{2}$$

where

$$B(\mathbf{q})=2-e^{i\mathbf{q}\cdot\mathbf{b}_1}-e^{i\mathbf{q}\cdot\mathbf{b}_2} \tag{3}$$

and $\mathbf{b}_1,\mathbf{b}_2$ are, respectively, the projections of the position vectors $\mathbf{r}_1$ and $\mathbf{r}_2$ of the two initially bound electrons onto the plane perpendicular to the direction of the Glauber path integral; Φ_i and Φ_f are, respectively, the wave functions of the initial and final states of the target and $\mathbf{p}$ is a two-dimensional vector in the plane of $\mathbf{b}_1$ and $\mathbf{b}_2$. For the initial state of He we have chosen the analytical fit to the Hartree-Fock wave function given by[13]

$$\Phi_i(\mathbf{r}_1,\mathbf{r}_2)=u(\mathbf{r}_1)u(\mathbf{r}_2) \tag{4}$$

with

$$u(\mathbf{r})=\gamma_1 e^{-\alpha_1 r}+\gamma_2 e^{-\alpha_2 r} ,$$
$$\alpha_1=1.41, \quad \alpha_2=2.61 , \tag{5}$$
$$\gamma_1=0.73485, \quad \gamma_2=0.58715 .$$

The final-state wave function Φ_f is taken to be the symmetrized product of the He^+ ground-state wave function v for the bound electron with the continuum Coulomb wave function $\phi_{\mathbf{k}_b}$ [orthogonalized to ground-state orbital $u(\mathbf{r})$] for the ejected electron with momentum $\mathbf{k}_b$,

$$\Phi_f(\mathbf{r}_1,\mathbf{r}_2)=\frac{1}{\sqrt{2}}[\phi_{\mathbf{k}_b}(\mathbf{r}_1)v(\mathbf{r}_2)+v(\mathbf{r}_1)\phi_{\mathbf{k}_b}(\mathbf{r}_2)] , \tag{6}$$

where

$$v(\mathbf{r})=\left[\frac{\lambda'^3}{\pi}\right]^{1/2} e^{-\lambda' r}, \quad \lambda'=2 , \tag{7}$$

$$\phi_{\mathbf{k}_b}(\mathbf{r})=\psi^{(-)}_{\mathbf{k}_b}(\mathbf{r})-\langle u(\mathbf{r}') \,|\, \psi^{(-)}_{\mathbf{k}_b}(\mathbf{r}')\rangle u(\mathbf{r})$$
$$=\psi^{(-)}_{\mathbf{k}_b}(\mathbf{r})-T_1 u(\mathbf{r}) , \tag{8}$$

and

$$\psi^{(-)}_{\mathbf{k}_b}(\mathbf{r})=(2\pi)^{-3/2}e^{\pi\delta/2}\Gamma(1+i\delta)e^{i\mathbf{k}_b\cdot\mathbf{r}}$$
$$\times {}_1F_1(-i\delta,1,-ik_b r-i\mathbf{k}_b\cdot\mathbf{r}), \quad \delta=1/k_b . \tag{9}$$

The final expression for f_{G2} is given by

$$f_{G2}=i\frac{\sqrt{2}}{\pi k_0}\int\frac{d\mathbf{p}}{p^2\,|\mathbf{K}-\mathbf{p}\,|^2}\{[4D(0)-2D(\mathbf{p})-2D(\mathbf{K}-\mathbf{p})+D(\mathbf{K})][E_1(0)-T_1E_2(0)]$$
$$+[D(\mathbf{K}-\mathbf{p})-2D(0)][E_1(\mathbf{p})-T_1E_2(\mathbf{p})]$$
$$+[D(\mathbf{p})-2D(0)][E_1(\mathbf{K}-\mathbf{p})-T_1E_2(\mathbf{K}-\mathbf{p})]+D(0)[E_1(\mathbf{K})-T_1E_2(\mathbf{K})]\} , \tag{10}$$

with

$$D(\mathbf{p})=16(2\pi)^{1/2}\sum_{i=1}^{2}\gamma_i\frac{\alpha_i+2}{[(\alpha_i+2)^2+p^2]^2} , \tag{11}$$

$$E_1(\mathbf{p})=\sqrt{(2/\pi)}e^{\pi\delta/2}\Gamma(1-i\delta)$$
$$\times\sum_{i=1}^{2}\gamma_i\frac{\partial}{\partial\alpha_i}[[A_i(\mathbf{p})-B_i(\mathbf{p})]^{-i\delta}A_i(\mathbf{p})^{i\delta-1}] , \tag{12}$$

$$E_2(\mathbf{p})=8\pi\sum_{i=1}^{2}\sum_{j=1}^{2}\gamma_i\gamma_j\frac{\alpha_i+\alpha_j}{[(\alpha_i+\alpha_j)^2+p^2]^2} \tag{13}$$

where

$$A_i(\mathbf{p})=\alpha_i^2+k_b^2+p^2+2\mathbf{p}\cdot\mathbf{k}_b , \tag{14}$$

$$B_i(\mathbf{p})=2(k_b^2+i\alpha_i k_b-\mathbf{p}\cdot\mathbf{k}_b) . \tag{15}$$

The integration over $\mathbf{p}$ in Eq. (10) is performed numerically.

In Fig. 1, TDCS, given by

$$\frac{d^3\sigma}{dE_a d\Omega_a d\Omega_b}=\frac{k_a k_b}{k_0}\,|f\,|^2 , \tag{16}$$

are plotted as a function of θ $(=\theta_a=\theta_b)$ at an incident electron energy of 200 eV. It contains our MG results along with those in the first-Born approximation. They are compared with the experimental data and the second-Born results of Pochat *et al.*[8] Since the experimental data is relative all the results have been normalized to the

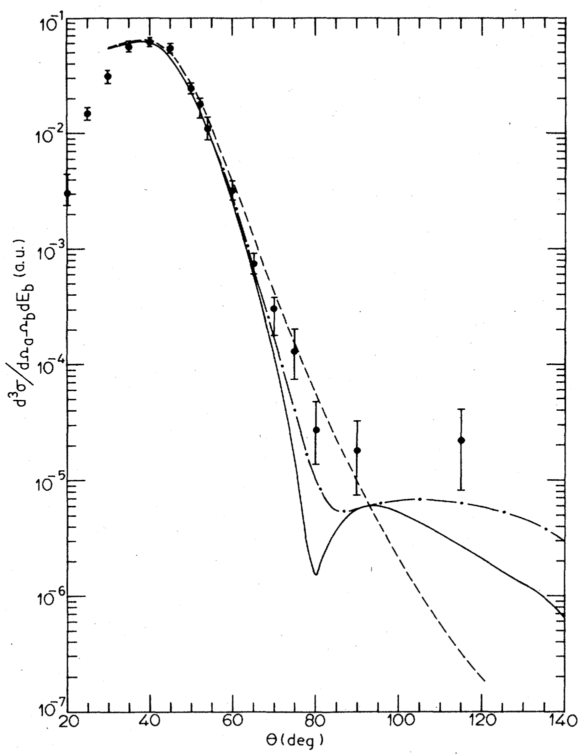

FIG. 1. Triple-differential cross section (in a.u.) for the electron-impact ionization of helium for the case of coplanar symmetric energy-sharing geometry with $E_0=200$ eV, $E_a=E_b=87.7$ eV as a function of θ $(=\theta_a=\theta_b)$. The dashed curve corresponds to the first-Born approximation, the dash-dotted curve to the second-Born results of Ref. 8, and the continuous curve to the present MG results. All the results are normalized to the second-Born result at $\theta=40°$. The experimental data, shown by ⌶, are those of Pochat *et al.*[8]

second-Born result at 40°. The $B2$ and MG results along with the experimental data begin to differ significantly from the first-Born results at $\theta \simeq 70°$. In the region from about 70° to 100°, the amplitudes f_{B1}, f_{B2}, and f_{G2} are comparable to each other. The shallow dip in the $B2$ results and the sharp one in the MG is the result of their mutual interference cancellation. Beyond $\theta \simeq 100°$, f_{B1} contributes very little and the results are dominated by the second- and higher-order contributions. The latter appear to decrease the cross section and move it away from the data at $\theta = 115°$ relative to the second-Born results of Pochat *et al.*[8] The MG results are thus worse than the second-Born ones. All these results should, however, be regarded as an evidence of the dominant contribution of the second- and higher-order terms which have only been estimated. Their evaluation needs to be improved. The final state acting as an intermediate state in the second-Born term has already been shown to contribute significantly.[7] The contribution of other intermediate states is also important but it needs to be calculated without recourse to the average-excitation-energy approximation and closure which is not valid in the present situation. An improvement in the evaluation of the higher-order terms ($n > 2$) over what is done here is likewise needed.

The work was supported by the Indian Space Research Organization (ISRO). One of us (K.S.B.) would like to thank the ISRO for financial support.

[1]H. Ehrhardt, M. Schulz, T. Tekaat, and K. Willmann, Phys. Rev. Lett. **22**, 89 (1969).
[2]F. W. Byron, Jr., C. J. Joachain, and B. Piraux, J. Phys. B **13**, L673 (1980); **15**, L293 (1982).
[3]H. Ehrhardt, M. Fischer, K. Jung, F. W. Byron, Jr., C. J. Joachain, and B. Piraux, Phys. Rev. Lett. **48**, 1807 (1982).
[4]K. S. Baliyan and M. K. Srivastava, Phys. Rev. A **32**, 3098 (1985); **33**, 2155 (1986).
[5]K. S. Baliyan and M. K. Srivastava, J. Phys. B **19**, 3603 (1986).
[6]I. E. McCarthy and E. Weigold, Phys. Rep. **27**, 275 (1976).
[7]F. W. Byron, Jr., C. J. Joachain, and B. Piraux, J. Phys. B **16**, L769 (1983).
[8]A. Pochat, R. J. Tweed, J. Peresse, C. J. Joachain, B. Piraux, and F. W. Byron, Jr., J. Phys. B **16**, L775 (1983).
[9]F. W. Byron, Jr. and C. J. Joachain, J. Phys. B **8**, L284 (1975); T. T. Gien, *ibid.* **9**, 3203 (1976).
[10]Yu. V. Popov, J. Phys. B **14**, 2449 (1981); Yu. V. Popov, I. Bang, and J. J. Benayoun, *ibid.* **14**, 4637 (1981).
[11]A. C. Roy, A. K. Das, and N. C. Sil, Phys. Rev. A **28**, 181 (1983).
[12]A. C. Yates, Chem. Phys. Lett. **25**, 480 (1974).
[13]F. W. Byron, Jr. and C. J. Joachain, Phys. Rev. **146**, 1 (1966).