

Macroscopic variables of the macroscopic system and the theory of measurement

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Macroscopic variables of the macroscopic system lose the quantum fluctuations in the macroscopic limit. A separate Hilbert space corresponds to each c -number value of these macroscopic variables. The measuring process in quantum mechanics consists of recording these c -number values and amounts to determination of the Hilbert space to which the system belongs. These observations provide a solution to the problem of measurement and lead to predictions concerning the c -number equations of motion of the macroscopic variables.

I. INTRODUCTION

The problem of measurement in the theory of quantum mechanics has a long history¹ and is certainly one of the most important questions of theoretical physics. There are various aspects of the theory of measurement but in this paper we concentrate on the following question: If the system of the object plus the macroscopic apparatus is described by the usual rule of the quantum mechanics (i.e., by the Schrödinger equation), then does the interaction between the object and the apparatus bring the pure state (of the object) into the mixed state (the reduction of the wave packet)? This is, of course, the central problem of the theory of measurement.

We discuss in the following the same subject from a novel point of view and give the answer in the affirmative. The discussion is based on the method of functional integration. It enables us to study quite easily the phase coherence or incoherence for the macroscopic system and thereby elucidate the structure of the Hilbert space of the macroscopic system. The results of these investigations, presented in Sec. II, are that the macroscopic variables of the macroscopic system have no fluctuations and a different Hilbert space corresponds to different values of the macroscopic variables. The macroscopic system is characterized here by the ideal limit where the number of degrees of freedom N becomes infinite. We are familiar with the notion of the different Hilbert spaces for different values of the macroscopic variables in various phenomena; ferromagnet, superconductor, chiral dynamics in particle physics, etc. To measure an object is, in our terminology, to construct a list of the fluctuationless numerical values (i.e., c -number values) of the macroscopic variables and this fact brings the process of measurement to the same level of the Newtonian mechanics. We can also say that the measurement determines the Hilbert space to which the system belongs.² The limit $N \rightarrow \infty$ answers at the same time the objection raised on the basis of the unitarity problem.³

In our theory the transition from the pure state to the mixed state occurs by the interaction of the object with the macroscopic apparatus. The loss of information takes place during this transition, and it is due to the complete destruction of the phase coherence between the states with

different values of the macroscopic variables. The transition time Δt is estimated in the following to be proportional to $1/N$.

It is desirable to have a solution to the problem of measurement within the framework of the quantum theory without recourse to the theory which has independent and different bases or assumptions, such as the theory of the irreversible thermodynamics.⁴ Otherwise the quantum theory will not be a closed theory. Our theory satisfies the above criterion.

We also have predictions which can, in principle, be checked. Several examples of the equations of motion of the macroscopic variables are presented at the end. There is no fluctuation to these equations (i.e., they are c -number equations) even though they undergo quantum corrections.

The discussions in the following are given in terms of the second-quantized field theory by introducing the bosonic field operator $\phi(\mathbf{x})$ and its conjugate momentum field operator $\pi(\mathbf{x}) = (\hbar/i)\partial/\partial\phi(\mathbf{x})$ to represent the degrees of freedom of the detector system. It is more general and easier than the quantum theory of an N -particle system. The macroscopic nature is represented by the limit $V = \infty$ with N/V fixed where V is the volume of the detector system.

II. MACROSCOPIC VARIABLES OF THE MACROSCOPIC SYSTEM AND THE STRUCTURE OF THE HILBERT SPACE

Extensive and intensive variables are associated with any macroscopic system. In the limit $V = \infty$ there do not exist, in the Hilbert space the operators corresponding to the extensive variables since its matrix elements are infinite. We divide the intensive variables into two classes. Class I consists of operators denoted by $A_i = A_i[\phi, \pi]$, $i = 1, 2, \dots$ which involve averaging of the local variables over the macroscopic region. Examples are

$$\int d^3x \phi(\mathbf{x})/V, \quad \int d^3x \pi(\mathbf{x})/V, \\ \int d^3x \int d^3y \phi(\mathbf{x})C(\mathbf{x}-\mathbf{y})\phi(\mathbf{y})/V,$$

etc., where $C(\mathbf{x})$ is some c -number function. The integration does not necessarily extend over the whole region but can be limited to the smaller region as long as it is macroscopic compared with the atomic scale. [The above operators are intensive since the definition of the intensive operator is that if $\phi(\mathbf{x})$ and $\pi(\mathbf{x})$ become independent of \mathbf{x} it has a finite limit when $V = \infty$.] Class II includes the rest of the operators, $B_i = B_i[\phi, \pi]$, $i = 1, 2, \dots$, especially the local operators such as $\phi(\mathbf{x}), \pi(\mathbf{x})$ or the operator $\int d^3y \phi(\mathbf{x})C(\mathbf{x}-\mathbf{y})\phi(\mathbf{y})$, etc. The components of class-I operators should be subtracted from the class-II operators of course: $\phi(\mathbf{x})$ is, for example, actually $\phi(\mathbf{x}) - \int d^3x \phi(\mathbf{x})/V$. Now we state the theorem.

Theorem. For a macroscopic system, all the intensive variables of class I lose fluctuations.

This is nothing but the law of large number and is proved by the stationary-phase method. Let the Schrödinger wave function of the system at the time t_0 be given. It is actually a wave functional $\langle \phi | \Psi \rangle_{t_0} \equiv \Psi_{t_0}[\phi]$ in our case. Then the $\Psi_t[\phi]$ for later time t is given in terms of the well-known functional path integral formula⁵ by

$$\Psi_t[\phi] = \int \int [d\phi'] \exp \left[i \int_{t_0}^t L(t') dt' \right] \Psi_{t_0}[\phi_0][d\phi_0]. \quad (1)$$

The Schrödinger operator $\phi(\mathbf{x})$ has been replaced by the integration variable $\phi(\mathbf{x}, t)$, and $L(t') = L[\phi(t'), \dot{\phi}(t')]$

($\dot{\phi} \equiv d\phi/dt$) is the Lagrangian of the system (the argument \mathbf{x} of ϕ will be suppressed unless it causes confusion). The functional integration $\int [d\phi']$ is performed for $\phi(\mathbf{x}, t')$ with $t_0 < t' < t$ but at $t' = t$ or t_0 , $\phi'(t')$ has a fixed value $\phi(t) \equiv \phi$ or $\phi(t_0) \equiv \phi_0$. We write Eq. (1) as

$$\Psi_t[\phi] = \int K_{t, t_0}(\phi, \phi_0) \Psi_{t_0}[\phi_0][d\phi_0]. \quad (2)$$

The evolution kernel K is rewritten as a sum of the terms each of which is a contribution to K under the condition that the arbitrarily chosen set of the class-I operator A_i ($i = 1, 2, \dots$) have a given value $a_i(t')$ for $t_0 < t' < t$. Thus we use an identical transformation as

$$K_{t, t_0}(\phi, \phi_0) = \int \int [da][d\phi'] \exp \left[i \int_{t_0}^t L dt' \right] \times \prod_i \prod_{(t_0 \leq t' \leq t)} \delta(A_i[\phi'(t')] - a_i(t')), \quad (3)$$

where in A_i , π is written in terms of ϕ and $\dot{\phi}$: $A_i[\phi, t] \equiv A_i[\phi, \pi(\phi, \dot{\phi})]$, and we introduced

$$\int [da] \equiv \int_{-\infty}^{\infty} \cdots \int \prod_i \prod_{(t_0 \leq t' \leq t)} da_i(t').$$

Neglecting the irrelevant constant factor, we rewrite Eq. (3) as

$$K = \int \int \int [da][d\phi'][dJ] \exp \left[-iV \sum_i \int_{t_0}^t dt' J_i(t') a_i(t') \right] \exp \left[i \int_{t_0}^t L dt' + iV \sum_i \int_{t_0}^t dt' J_i(t') A_i[\phi'(t')] \right],$$

where $\int [dJ]$ is similarly defined as $\int [da]$. The functional integration $\int [d\phi']$ gives a factor $\exp(iW[J, \phi, \phi_0])$. Since W is a macroscopic quantity, we write $W = V\omega[J, \phi, \phi_0]$. Thus K is given by

$$K = \int \int [da][dJ] \exp \left[iV\omega[J, \phi, \phi_0] - iV \sum_i \int_{t_0}^t dt' J_i(t') a_i(t') \right]. \quad (4)$$

The integral over J is dominated in the limit $V = \infty$ by the region where the phase becomes stationary. We assume that there are S distinct such regions. (In case there are continuously many such regions the summation in the following formulas is replaced by the integration.) Thus K becomes

$$K = \sum_{s=1}^S \int [da] \exp(iV\Gamma^s[a, \phi, \phi_0]) \bar{C}_{\phi, \phi_0}^s[a], \quad (5)$$

where

$$\Gamma^s[a, \phi, \phi_0] \equiv \omega[J^s, \phi, \phi_0] - \sum_i \int_{t_0}^t dt' J_i^s(t') a_i(t'), \quad (6)$$

which is a Legendre transform of ω , and J^s is expressed in terms of $a_i(t)$ by solving

$$\frac{\partial \omega[J^s, \phi, \phi_0]}{\partial J_i^s(t)} = a_i(t). \quad (7)$$

Equations (6) and (7) define Γ as a Legendre transform of ω and Γ is called the effective action (density) in particle physics. The factor $\bar{C}_{\phi, \phi_0}[a]$ is of the order $(1/\sqrt{V})^P$. Here P is the number of the relevant integration variables.

The integration over a is again dominated by the stationary phase. Let S' be the number of distinct regions satisfying the stationarity condition for each s . Then K is given by

$$K = \sum_{s=1}^S \sum_{s'=1}^{S'} \exp(iV\Gamma^s[a^{s, s'}, \phi, \phi_0]) C_{\phi, \phi_0}(a^{s, s'}) \quad (8)$$

with another factor $C^s(a^{s,s'})$, and $a^{s,s'}$ is determined by

$$\frac{\partial \Gamma^s[a^{s,s'}, \phi, \phi_0]}{\partial a_i^{s,s'}(t)} = 0, \quad s = 1, \dots, S, \quad s' = 1, \dots, S'. \quad (9)$$

This is the c -number equation of motion for $a_i(t)$. It is in general a nonlocal equation in space and time. We call each solution of Eq. (9) a trajectory. To different s and/or s' correspond different trajectories since the equations of motion are different. The trajectory depends on the boundary conditions at t and t_0 so that the trajectory is written as $a^{(s)}(\phi, \phi_0)$ where (s) represents the set s and s' . Now we point out several points.

(1) The wave functional $\Psi_t[\phi]$ is a sum of terms each of which has an infinite phase. The phase difference of an arbitrary pair of terms of different trajectories is infinite in general. This is obvious by Eqs. (2) and (8):

$$\Psi_t[\phi] = \sum_{(s)} \int [d\phi_0] \Psi^{(s)}[\phi, \phi_0], \quad (10)$$

$$\Psi^{(s)}[\phi, \phi_0] \equiv \exp\{iV\Gamma^s[a^{(s)}(\phi, \phi_0)]\} C_{\phi, \phi_0}[a^{(s)}(\phi, \phi_0)].$$

The value of $\Gamma^s[a^{(s)}(\phi, \phi_0)]$ depends on (s) , ϕ , and ϕ_0 , i.e., on the trajectory.

(2) Let us prepare at $t=t_0$ a Schrödinger wave functional $\Psi_{t_0}[\phi_0]$ with a well-defined finite phase. It will decompose into a sum of terms with infinite phase difference after an elapse of the time Δt . The order of magnitude of Δt is estimated as follows. $\Gamma[a]$ always has the form $\int_{t_0}^t dt' \gamma[a, t']$ as is known from the perturbation theory. Therefore, for small $\Delta t = t - t_0$ we write $V\Gamma = V\Delta t(1/\Delta t) \int_{t_0}^t \gamma dt' \equiv V\Delta t \bar{\gamma}$, where $\bar{\gamma}$ is the average of the action density. The validity of the dominance of the stationary phase is ensured if

$$\Delta t \gg (V\bar{\gamma})^{-1}.$$

For large V , the wave functional assumes the form (10) instantaneously.

(3) There does not exist any finite operator that connects different trajectories. To show this, we first observe that any class-II operator can be represented by $f[\phi, \pi]$ with f some functional of ϕ and π . Then we choose a pair of different trajectories $a^{(s)}(\phi, \phi_0)$, the one with $(s), \phi_0$, and the other with $(\bar{s}), \bar{\phi}_0$ (ϕ is the same for both terms). Consider the following matrix element:

$$\begin{aligned} & \left[\Psi_t^{*(s)}[\phi, \phi_0], f \left[\phi, \frac{\hbar}{i} \frac{\partial}{\partial \phi} \right] \Psi_t^{(\bar{s})}[\phi, \bar{\phi}_0] \right] \\ & \equiv \int [d\phi] C_{\phi, \phi_0}^{*s} \exp\{-iV\Gamma^s[a^{(s)}(\phi, \phi_0)]\} f \left[\phi, \frac{\hbar}{i} \frac{\partial}{\partial \phi} \right] \\ & \quad \times \exp\{iV\Gamma^{\bar{s}}[a^{(\bar{s})}(\phi, \bar{\phi}_0)]\} C_{\phi, \bar{\phi}_0}^{\bar{s}}, \end{aligned} \quad (11)$$

where Eq. (10) has been used. If $f[\phi, \phi_0]$ is a finite operator when $V = \infty$, Eq. (11) vanishes in the limit $V \rightarrow \infty$. It is of the order $(1/\sqrt{V})^Q$ where Q is the number of the degrees of freedom of ϕ relevant in the integration. (We

have assumed that there exists a stationary phase of $\exp(-iV\Delta\Gamma)$ where $\Delta\Gamma = \Gamma^s[a^{(s)}(\phi, \phi_0)] - \Gamma^{\bar{s}}[a^{(\bar{s})}(\phi, \bar{\phi}_0)]$. If there is not, Eq. (11) vanishes even faster. $C^{(s)}$ or $C^{(\bar{s})}$ also contains some powers of $1/\sqrt{V}$, but it is related to the normalization of $\Psi^{(s)}$ or $\Psi^{(\bar{s})}$ and has nothing to do with the orthogonality relation or with the off-diagonal matrix elements of $f[\phi, \pi]$.) We conclude that $\Psi_t^{(s)}[\phi]$ constitutes a Hilbert space for each (s) : to each different (s) there corresponds a different Hilbert space.

The same conclusion can also be derived by the observation given in (4) below. We next have to clarify the operators acting on each Hilbert space.

(4) For this purpose let us study the operators of class I. Since any fluctuation of the class-I operators disappears and $a_i(t)$ obeys the deterministic equation (9), they lose operator character and become c -numbers. For each value of $a_i(t)$ there is associated a Hilbert space since for different a_i the value of Γ will be different. We note here an important fact that the operator which is canonically conjugate to the class-I operator is an extensive operator. [For example, $\int d^3x \pi(\mathbf{x})$ is conjugate to $\int d^3x \phi(\mathbf{x})/V$.] Since any extensive operator ceases to exist in the limit, *all the quantum degrees of freedom corresponding to class I are lost*. It is the class-II operators that operate in the Hilbert space specified by the specific value of $a_i(t)$. The Hilbert space will change at every instant of time, even if we are following the same trajectory of $a_i(t)$.

Note that the Hilbert spaces with different trajectories are entirely disconnected: In order to make a finite shift to a_i we need a momentum operator conjugate to A_i , but it does not exist.

(5) The Hamiltonian of the system which determines the infinitesimal time development does not exist in the limit since it is an extensive variable. The usual unitary operator describing the time evolution for finite time interval cannot be defined accordingly: *the time development is not given by a unitary transformation*. This can be seen in another way: we have in the theory two kinds of variables which show c -number time evolution (class-I variables) and q -number unitary time development. In such a case it is impossible to construct a unitary operator which affects the time evolution of the whole system. This resolves the unitarity problem and at the same time violates the time reversal invariance—characteristic property of the measuring process. It is also impossible to construct an antiunitary operator to achieve the time inversion.

It is interesting to note that the disappearance of the Hamiltonian and the validity of the c -number equation of motion (9) are ensured at the same time (i.e., $V = \infty$). This is a consistent trading of the equation of motion.

(6) We have to know how to calculate the time evolution of operators in each Hilbert space. The problem is the same as, for example, the ferromagnetic case. There are several familiar methods to develop the quantum theory in the presence of c -number nonzero magnetization density which belongs to class I. In our case it is more convenient to evaluate Green's function

$$G_J \equiv \langle \phi(\mathbf{x}_1, t_1) \phi(\mathbf{x}_2, t_2) \cdots \phi(\mathbf{x}_n, t_n) \rangle_J$$

in the Heisenberg representation, where the source term

$V \sum_i J_i(t) A_i(t)$ is added to the Hamiltonian. As a function of J , G_J is multivalued and when the limit $J_i(t) \rightarrow 0$ ($J_i(t) = -\partial\Gamma[a]/\partial a_i(t)$) is taken we have to select the correct branch where the corresponding trajectory $a_i(t)$ defines the relevant Hilbert space we are discussing. The actual construction of the Green's function can be performed if the system is defined.

(7) An example can be given for the quantum mechanics of an N -particle system of equal mass with $N = \infty$. Let the coordinates of each particle be \mathbf{X}_i ($i=1-N$) then the center-of-mass coordinates $\mathbf{X} = \sum_{i=1}^N \mathbf{x}_i/N$ is a class-I operator and becomes the c -number \mathbf{x}_c when $N = \infty$. The canonically conjugate variable is the total momentum $\mathbf{P} = \sum_{i=1}^N \mathbf{p}_i$ which is of the order N , and when $N = \infty$ it does not exist as an operator in the Hilbert space. Thus, the above variables are eliminated from the quantum degrees of freedom. Instead the value \mathbf{x}_c defines a Hilbert space where the relative coordinates and the relative momenta act as well-defined operators. They belong to the class-II operators.

III. MEASUREMENT OF THE OBJECT

The measuring apparatus is a macroscopic system which interacts with the object. The measurement is performed by reading off the values of the macroscopic variables such as the position of the needle, the grain density, or the current density, etc. They all correspond to the operator involving the average over the region of macroscopic size (which is assumed to be infinite compared with the atomic size), i.e., they belong to class I. The above consideration comes from the unavoidable uncertainties involved in the observation which is macroscopic in principle. There is another important issue here—the class-I operators have no fluctuation and hence lie on the same level as the dynamical variables in Newtonian mechanics. We recall here the standard two criteria of the measuring theory.

$$\langle \phi | \Psi \rangle_t = \sum_k c_k(t) | \lambda_k \rangle \int [d\phi'] \exp \left[i \int_{t_0}^t dt' (L[\phi] + L^I[\lambda_k, \phi]) \right] \langle \phi_0 | \Psi \rangle_{t_0} [d\phi_0].$$

For each k we get a different equation of motion for $a(t)$, $\partial\Gamma[\lambda_k, a]/\partial a(t) = 0$, so that there is unique correspondence between k and the solution a_k , and hence, the state of the apparatus [criterion (i)]. There is an infinite phase difference $\exp\{iV(\Gamma[\lambda_k, a_k] - \Gamma[\lambda_{k'}, a_{k'}])\}$ if $k \neq k'$, so that there does not exist any finite operator that connects k and k' [criterion (ii)]. In the usual experiment the c -number value $a^B(t)$ of $a(t)$ is recorded before the interaction L^I (which is t independent usually). It selects one Hilbert space associated with that $a(t)$. After the interaction $a(t)$ is also measured to get $a^A(t)$, and it is the difference $a^A(t) - a^B(t)$ that gives the information on λ_k . Whether $a^A(t)$ is clearly separated from $a^B(t)$ or not depends on the efficiency of the detector, and amplifying devices are introduced which should be included in the Hamiltonian of the apparatus.

The reason why a small number of degrees of freedom

(i) Let the eigenstate of the operator Λ , of the object be $|\lambda_k\rangle$ with the eigenvalue λ_k and let the state of the total system before interaction be

$$|\Psi\rangle = \left[\sum_k c_k | \lambda_k \rangle \right] |\psi\rangle, \quad (12)$$

where $|\lambda_k\rangle$ refers to the object and $|\psi\rangle$ to the apparatus. Then, after the interaction between the object and the apparatus

$$|\Psi\rangle = \sum_k c_k | \lambda_k \rangle |\psi_k\rangle \equiv \sum_k c_k | k \rangle, \quad (13)$$

where $|\psi_k\rangle$ is the characteristic state of the apparatus specific to k .

(ii) For different k and k' , the phases of the states $|k\rangle$ and $|k'\rangle$ are incoherent. In other words, there does not exist any operator with a finite and nonzero matrix element between $|k\rangle$ and $|k'\rangle$.

We show in the following that in our case these are indeed satisfied in a natural way. In order to observe Λ we first choose the apparatus whose interaction with the object is given by the interaction Hamiltonian $H^I[\Lambda, \phi, \pi]$ or the interaction Lagrangian $L^I[\Lambda, \phi, \dot{\phi}]$.

A macroscopic variable of the apparatus chosen in the measuring process is written as A which belongs to class I. It is actually c -number $a(t)$, and its equation of motion is obtained by repeating the same arguments as before. In Eq. (1) we have only to replace $L \rightarrow L + L^I + L^0$, where L^0 is the Lagrangian of the object and $\Psi_{t_0}[\phi_0] \rightarrow |\Psi\rangle$ of Eq. (12). The functional integrals include the integrals over the degrees of freedom of the object. But for the moment we neglect them and assume as usual that the coefficients c_k 's are given functions of t . This neglects some quantum effects but will not be crucial for our discussions. Then we have

(even one degree) can produce a macroscopic shift of the apparatus is clear in our case: each contribution to the phase from the individual degree of freedom of the apparatus adds up to make the total phase stationary.

As an example, consider the measurement of the position \mathbf{y} of the charged particle. We take as A the grain density operator $A_{\mathbf{x}_0} = \int_{\mathbf{x}_0} \rho(\mathbf{x}) d^3x/V$, where the integration extends over the macroscopic region V around \mathbf{x}_0 . L^I is given by the Coulomb interaction $L^I = -e \int d^3x |\mathbf{y} - \mathbf{x}|^{-2} c(\mathbf{x})$ where $c(\mathbf{x})$ is the charge density of the apparatus, \mathbf{y} is the position of the object particle, and e is the charge of the particle. $A_{\mathbf{x}_0}$ will have a c -number finite value near $\mathbf{x}_0 = \mathbf{y}$.

The arguments have been given above for the pure state $|\psi\rangle$ but they also hold if it is a mixed state.

IV. EXAMPLES OF THE DETERMINISTIC EQUATION OF THE CLASS-I VARIABLES

Finally, some examples of Eq. (9) are given within the framework of the quantum mechanics of an N -particle system. We derive in particular the equation of motion of the center-of-mass coordinate of the macroscopic object (with finite macroscopic extent in space). These are the predictions of our theory which can, in principle, be tested.

Let the N -particle system of mass m with the mutual interaction potential $V(\mathbf{x}_i - \mathbf{x}_j)$ be in the external potential $W(\mathbf{x}_i)$ produced by the source at the origin. Denoting the center of mass by \mathbf{X} and the relative coordinate by \mathbf{x}' the Lagrangian is given by

$$\begin{aligned} L &= \sum_i \frac{m}{2} \dot{\mathbf{x}}_i^2 - \sum_{\substack{i,j \\ (i>j)}} V(\mathbf{x}_i - \mathbf{x}_j) - \sum_i W(\mathbf{x}_i) \\ &= \frac{M}{2} \dot{\mathbf{x}}^2 - \sum_i W(\mathbf{X} + \mathbf{x}'_i) - L^{\text{rel}}, \end{aligned}$$

where $M = mN$, and L^{rel} refers to the relative coordinates only. We assume here $|\mathbf{X}| \gg |\mathbf{x}'_i|$ and expand W , using $\sum_i \mathbf{x}'_i = 0$, as

$$\begin{aligned} \sum_i W(\mathbf{X} + \mathbf{x}'_i) &= NW(\mathbf{X}) \\ &+ \frac{1}{2} \sum_{i,k,l} (\partial^2 W / \partial X^k \partial X^l) x_i^k x_i^l + \dots \end{aligned}$$

This is inserted into the functional integral formula

$$K \equiv \int \int [dX][dx'] \exp \left[i \int L dt \right]$$

and perform the \mathbf{x}'_i integration. We define

$$\begin{aligned} \langle x_i^k x_i^l \rangle &\equiv \frac{\int [dx'_i] x_i^k x_i^l \exp \left[i \int L^{\text{rel}} dt \right]}{\int [dx'_i] \exp \left[i \int L^{\text{rel}} dt \right]} \\ &= \frac{1}{3} \langle \mathbf{x}'_i \cdot \mathbf{x}'_i \rangle \delta^{kl} \\ &= \frac{1}{3} \sum_i \frac{\langle \mathbf{x}'_i \cdot \mathbf{x}'_i \rangle}{N} \delta^{kl} \\ &\equiv \frac{1}{3} \langle \bar{\mathbf{x}}'^2 \rangle \delta^{kl}, \end{aligned}$$

where $\langle \bar{\mathbf{x}}'^2 \rangle$ is the mean-square radius of the system. Then

$$K = \int [dX] \exp \left[i \int L^{\text{eff}} dt \right],$$

where

$$L^{\text{eff}} = \frac{M}{2} \dot{\mathbf{x}}^2 - NW(\mathbf{X}) - \frac{N}{6} \langle \bar{\mathbf{x}}'^2 \rangle \Delta W(\mathbf{X})$$

and $\Delta \equiv \nabla^2$. For $N \rightarrow \infty$ we get a deterministic equation of motion for \mathbf{X} :

$$\frac{M}{2} \ddot{\mathbf{X}} + N \nabla W(\mathbf{X}) + \frac{N}{6} \langle \bar{\mathbf{x}}'^2 \rangle \nabla \Delta W(\mathbf{X}) = 0. \quad (14)$$

The quantum effects are continued in $\langle \bar{\mathbf{x}}'^2 \rangle$. In the absence of the external potential it reduces to $\ddot{\mathbf{X}} = 0$ which has the same form as the Heisenberg equation of motion $\ddot{\mathbf{X}} = (i/\hbar)^2 [H, [H, \mathbf{X}]] = 0$ but our equation is a c -number equation and \mathbf{X} does not show any quantum diffusion.

When we have two macroscopic bodies with particle coordinates \mathbf{x}_i ($i = 1-N$) or \mathbf{y}_j ($j = 1-N'$) and the potential $V(\mathbf{x}_i - \mathbf{y}_j)$, we introduce the center-of-mass and relative coordinates of each body as $\mathbf{x}_i = \mathbf{X} + \mathbf{x}'_i$, $\mathbf{y}_j = \mathbf{Y} + \mathbf{y}'_j$. Define $\mathbf{R}_{\pm} = \mathbf{X} \pm \mathbf{Y}$ and assume $|\mathbf{R}_{-}| > |\mathbf{x}'_i|, |\mathbf{y}'_j|$. Then we get two kinds of c -number equations in the limit $N, N' \rightarrow \infty$,

$$\begin{aligned} \ddot{\mathbf{R}}_{+} &= 0, \\ \mu \ddot{\mathbf{R}}_{-} + NN' \nabla V(\mathbf{R}_{-}) \\ &+ \frac{NN'}{6} (\langle \bar{\mathbf{x}}'^2 \rangle + \langle \bar{\mathbf{y}}'^2 \rangle) \nabla \Delta V(\mathbf{R}_{-}) = 0, \end{aligned} \quad (15)$$

where $\mu^{-1} = (Nm)^{-1} + (N'm)^{-1}$ and $\langle \rangle$ is the average in the relative coordinates of each body. Equation (15) may represent the equation of motion of the system of two large molecules or nuclei. The detailed discussions of Eq. (14) or (15), including numerical estimates, will be given in a separate paper.

V. DISCUSSIONS

We have seen that the essential ingredients that cause the transition from the pure state to the mixed state is the structure of the Hilbert space inherent in the macroscopic system. More precisely it is the complete loss of the phase coherence between the states with different values of the macroscopic variables—the class-I intensive variables. Note that they involve the averaging over the region of macroscopic size. The entropy defined by the expectation value of the negative of the logarithm of the density matrix increases during this transition time Δt which has been estimated to be $1/\sqrt{\gamma}V$ in point (2) of Sec. II. When $V \rightarrow \infty$ the entropy change becomes discontinuous.

It seems that our measuring theory is a purely objective one. The process of the observation is finally a subjective event and the discussions should include the chain of systems, object—apparatus—amplifying systems—our brain. In order to complete our investigations we have only to include the apparatus and all the macroscopic devices following the apparatus in our macroscopic system. However, the reduction of the wave packet occurs definitely at the first junction through the interaction between the object and the apparatus. The role of the subsequent chain of macroscopic systems is to transfer (or read off) the c -number values of the macroscopic variables of the apparatus to new values of the macroscopic variables of another macroscopic system. The interaction between two macroscopic systems is easily arranged in such a way that the transcription from the first system to the second can be performed without changing the state of the first system. This part may be discussed on the classical level since any class-I operator has no fluctuation and the quantum effects on the c -number values will be negligibly

small [see Eq. (14) or (15), for example]. We emphasize again that in our theory the process of measurement is on the same level as the Newtonian mechanics. It is clear from the above observations that the reduction takes place whether we look at the apparatus or not.

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¹For examples see the following book: *Foundation of Quantum Mechanics*, Proceedings of the International School of Physics "Enrico Fermi," Course IL, edited by B. d'Espagnat (Academic, New York, 1971).

²Recent papers by Machida and Namiki introduce many Hilbert spaces which avoid the unitarity problem. However, our mechanism in the following is quite different from theirs since they superpose Hilbert spaces in the measuring process: S. Machida and M. Namiki, *Prog. Theor. Phys.* **63**, 1457 (1980); **63**, 1833 (1980). See also H. Everett III, *Rev. Mod. Phys.* **29**, 454 (1957).

³W. H. Furry, *Boulder Lectures in Theoretical Physics* (University of Colorado Press, Boulder, CO, 1966), Vol. 8A; E. P. Wigner, *Am. J. Phys.* **31**, 6 (1963); A. Fine, *Phys. Rev. D* **2**, 2783 (1970).

⁴A. Daneri, A. Loinger, and G. M. Prosperi, *Nucl. Phys.* **33**, 297 (1962).

⁵R. P. Feynman, *Rev. Mod. Phys.* **20**, 367 (1948); *Phys. Rev.* **74**, 1430 (1948). See also the textbook by N. N. Bogoliubov and D. V. Shirkov, *Introduction to the Theory of Quantized Fields* (Interscience, New York, 1959).