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Plasma maser theory for magnetized plasma

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The plasma maser theory of Langmuir waves produced from interaction between ion-wave turbulence and electrons in magnetized plasma is presented. The most dominant destabilizing effect comes from the polarization term. The instability occurs even for the Maxwell electron distribution function. The importance of the magnetic field for the plasma maser theory is stressed.

I. INTRODUCTION

Since the prediction of the new mode-coupling process[1,2] (plasma maser), there has been much controversy about the process.[3,4] Almost all previous studies except Refs. 5 and 6 deal with unmagnetized plasma. The purpose of this paper is to study the growth rate of the Langmuir wave in the presence of ion-wave turbulence for magnetized plasma. The difference between unmagnetized and magnetized plasma is stressed. The most dominant plasma maser effect for magnetized plasma comes from the polarization mode-coupling term.

The effective dielectric constant of the Langmuir waves in the presence of ion-wave turbulence is obtained in Sec. II. The plasma maser effect of Langmuir waves by electrons scattered by ion waves is investigated in Sec. III. Discussions and conclusions are contained in Sec. IV.

II. FORMULATION

We consider a homogeneous magnetized plasma in the presence of an ion acoustic wave propagating along the external magnetic field with wave vector $\mathbf{k}=(0,0,k_{\parallel})$ (see Fig. 1). The steady ion acoustic turbulence is driven up by the relative drift between the ions.[7] Accordingly, it is safe to assume that the unperturbed electron distribution function is the Maxwellian

$$f_{0e}=(m/2\pi T)^{3/2}\exp[-m(v_\perp^2+v_\parallel^2)/2T] ;$$

here $\perp$ and $\parallel$ denote the components perpendicular and parallel to the magnetic field. The interaction of a test Langmuir wave (or Bernstein mode) with the ion acoustic waves in a plasma is governed by the Vlasov-Poisson equations,

$$\left[\frac{\partial}{\partial t}+\mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r}}-\frac{e}{m}\left[\mathbf{E}+\frac{1}{c}\mathbf{v}\times\mathbf{B}\right]\cdot\frac{\partial}{\partial \mathbf{v}}\right]f_e(\mathbf{r},\mathbf{v},t)=0 , \quad (1)$$

$$\nabla\cdot\mathbf{E}(\mathbf{r},t)=-4\pi e\int f_e(\mathbf{r},\mathbf{v},t)d\mathbf{v} , \quad (2)$$

where the notation is standard.

The plasma already contains low-frequency ion acoustic fluctuations. We, therefore, take the unperturbed electron distribution function F_{0e} as

$$F_{0e}=f_{0e}+\epsilon f_{1e}+\epsilon^2 f_{2e} , \quad (3a)$$

where f_{0e} is the space- and time-averaged part of the electron distribution function (Maxwellian) and f_{1e} and f_{2e} are the fluctuating parts due to the low-frequency ion acoustic fluctuations. ϵ is the ordering of the low-frequency turbulence.

The unperturbed electric and magnetic fields can be written as

$$\mathbf{E}_{0l}=\epsilon\mathbf{E}_l,\quad \mathbf{B}_{0l}=\mathbf{B}_0 , \quad (3b)$$

where $\mathbf{E}_l$ is the electric field of the ion acoustic wave. To order ϵ, we obtain, from Eq. (1),

$$\left[\frac{\partial}{\partial t}+\mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r}}-\frac{e}{m}\frac{\mathbf{v}\times\mathbf{B}_0}{c}\cdot\frac{\partial}{\partial \mathbf{v}}\right]f_{1e}(\mathbf{r},\mathbf{v},t)=\frac{e}{m}\mathbf{E}_l\cdot\frac{\partial}{\partial \mathbf{v}}f_{0e} . \quad (4)$$

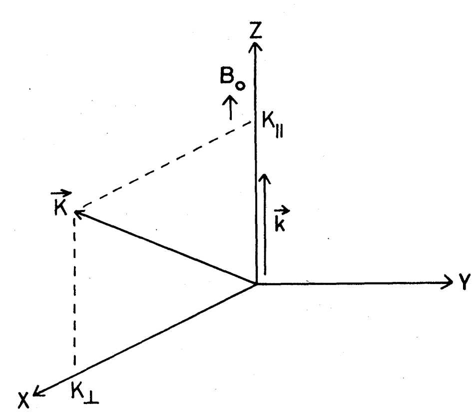

FIG. 1. Geometry of model: $\mathbf{K}=(K_\perp,0,K_\parallel)$ is the propagation vector of the Langmuir wave and $\mathbf{k}=(0,0,k_\parallel)$ is the propagation vector for the ion acoustic wave. B_0 is the external magnetic field in the z direction.

Taking a transform of the form

$$A(\mathbf{r},\mathbf{v},t)=\sum A(\mathbf{k},\mathbf{v},\omega)\exp[i(\mathbf{k}\cdot\mathbf{r}-\omega t)] ,$$

and Fourier-analyzing Eq. (4), we obtain

$$f_{1e}(k,\omega)=\frac{i\frac{e}{m}E_l(k,\omega)\frac{\partial}{\partial v_\parallel}f_{0e}}{\omega-k_\parallel v_\parallel+i0^+} , \tag{5}$$

where $i0^+$ represents the small imaginary part of ω, and ω and k are the frequency and wave number of the ion-wave fields.

We now perturb the steady state by a high-frequency electrostatic (ES) test wave field $\mu\delta\mathbf{E}_h$ ($\mu \ll \epsilon$). The total perturbed electric field and magnetic field and the electron distribution function are given, respectively, by

$$\delta\mathbf{E}=\mu\delta\mathbf{E}_h+\mu\epsilon\delta\mathbf{E}_{lh}+\mu\epsilon^2\Delta\mathbf{E} ,$$
$$\delta\mathbf{B}=0 , \tag{6}$$
$$\delta f=\mu\delta f_h+\mu\epsilon\delta f_{lh}+\mu\epsilon^2\Delta f ,$$

where $\delta\mathbf{E}_{lh},\Delta\mathbf{E}$ are the modulation electric fields and $\delta f_{lh},\Delta f$ are the electron distribution functions corresponding to the modulation fields.

We now linearize the Vlasov equation (1) and obtain

$$\left[\frac{\partial}{\partial t}+\mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r}}-\frac{e}{m}\left[\epsilon\mathbf{E}_l(\mathbf{r},t)+\frac{\mathbf{v}\times\mathbf{B}_0}{c}\right]\cdot\frac{\partial}{\partial\mathbf{v}}\right]\delta f(\mathbf{r},\mathbf{v},t)$$
$$-\frac{e}{m}\delta\mathbf{E}_h(\mathbf{r},t)\cdot\frac{\partial}{\partial\mathbf{v}}(f_{0e}+\epsilon f_{1e}+\epsilon^2 f_{2e})=0 . \tag{7}$$

To orders μ, $\mu\epsilon$, and $\mu\epsilon^2$, we obtain from the above equation

$$P\delta f_h-\frac{e}{m}\delta\mathbf{E}_h\cdot\frac{\partial}{\partial\mathbf{v}}f_{0e}=0 , \tag{8}$$

$$P\delta f_{lh}-\frac{e}{m}\mathbf{E}_l\cdot\frac{\partial}{\partial\mathbf{v}}\delta f_h-\frac{e}{m}\delta\mathbf{E}_h\cdot\frac{\partial}{\partial\mathbf{v}}f_{1e}$$
$$-\frac{e}{m}\delta\mathbf{E}_{lh}\cdot\frac{\partial}{\partial\mathbf{v}}f_{0e}=0 , \tag{9}$$

$$P\Delta f-\frac{e}{m}\left\langle\mathbf{E}_l\cdot\frac{\partial}{\partial\mathbf{v}}\delta f_{lh}\right\rangle-\frac{e}{m}\delta\mathbf{E}_h\cdot\frac{\partial}{\partial\mathbf{v}}f_{2e}$$
$$-\frac{e}{m}\left\langle\delta\mathbf{E}_{lh}\cdot\frac{\partial}{\partial\mathbf{v}}f_{1e}\right\rangle-\frac{e}{m}\Delta\mathbf{E}\cdot\frac{\partial}{\partial\mathbf{v}}f_{0e}=0 , \tag{10}$$

where

$$P\equiv\frac{\partial}{\partial t}+\mathbf{v}\cdot\frac{\partial}{\partial\mathbf{r}}-\frac{e}{mc}\mathbf{v}\times\mathbf{B}_0\cdot\frac{\partial}{\partial\mathbf{v}} .$$

In our detailed considerations, we will assume for magnetized plasma

$$k_e>K_\parallel>k_\parallel$$
and (11)
$$(\mathbf{K}\cdot\mathbf{k})/|\mathbf{K}|\ |\mathbf{k}|<1 .$$

Instead of Eq. (11), DuBois and Pesme[4] assume for unmagnetized plasma

$$k_e>|\mathbf{k}|>|\mathbf{K}|$$
and (11′)
$$(\mathbf{K}\cdot\mathbf{k})/|\mathbf{K}|\ |\mathbf{k}|\simeq 1 .$$

Accordingly, both assumptions are not equivalent. In comparing the results from both studies [Eq. (C3)], we take Eq. (11) for Eq. (54) and Eq. (11′) for Eq. (C1), respectively.

Making use of the above equations and the Poisson equation, we obtain, after a lengthy but straightforward calculation, the effective dielectric constant of the high-frequency ES wave [$\varepsilon_h(\mathbf{K},\Omega)$] in the presence of the low-frequency ion-wave turbulence. The result is

$$\varepsilon_h(\mathbf{K},\Omega)=\varepsilon_0(\mathbf{K},\Omega)+\varepsilon_d(\mathbf{K},\Omega)+\varepsilon_p(\mathbf{K},\Omega) , \tag{12}$$

where $\varepsilon_0(\mathbf{K},\Omega)$ is the linear part and is given by

$$\epsilon_0(\mathbf{K},\Omega)=1+\left[\frac{\omega_{pe}}{K}\right]^2\sum_{n=-\infty}^{\infty}\int\frac{J_n^2(K_\perp v_\perp/\Omega_e)}{\Omega-K_\parallel v_\parallel-n\Omega_e}\left[\frac{n\Omega_e}{v_\perp}\frac{\partial}{\partial v_\perp}+K_\parallel\frac{\partial}{\partial v_\parallel}\right]f_{0e}d\mathbf{v} , \tag{13}$$

$\varepsilon_d(\mathbf{K},\Omega)$ is the direct mode-coupling term

$$\epsilon_d(\mathbf{K},\Omega)=-\left[\frac{\omega_{pe}}{K}\right]^2\left[\frac{e}{m}\right]^2\sum_{\mathbf{k}}|E_l(\mathbf{k},\omega)|^2\sum_{a,s,n}\int\frac{J_a^2(K_\perp v_\perp/\Omega_e)}{\Omega-K_\parallel v_\parallel-a\Omega_e}\frac{\partial}{\partial v_\parallel}\frac{J_s^2(K_\perp v_\perp/\Omega_e)}{(\Omega-\omega-K_\parallel' v_\parallel-s\Omega_e)}$$
$$\times\left[\frac{\partial}{\partial v_\parallel}\frac{J_n^2(K_\perp v_\perp/\Omega_e)}{\Omega-K_\parallel v_\parallel-n\Omega_e}\left[\frac{n\Omega_e}{v_\perp}\frac{\partial}{\partial v_\perp}+K_\parallel\frac{\partial}{\partial v_\parallel}\right]f_{0e}\right.$$
$$\left.+\left[\frac{s\Omega_e}{v_\perp}\frac{\partial}{\partial v_\perp}+K_\parallel\frac{\partial}{\partial v_\parallel}\right]\frac{1}{k_\parallel v_\parallel-\omega+i0^+}\frac{\partial}{\partial v_\parallel}f_{0e}\right]d\mathbf{v} , \tag{14}$$

$\varepsilon_p(\mathbf{K},\Omega)$ gives the most dominant mode-coupling term (polarization term),

$$\varepsilon_p(\mathbf{K},\Omega)=\left\{\left[\frac{\omega_{pe}}{K}\right]^2\left[\frac{e}{m}\right]^2\sum_{\mathbf{k}}|E_l(\mathbf{k},\omega)|^2\sum_{a,s,n}\int\frac{J_a^2(K_\perp v_\perp/\Omega_e)}{\Omega-K_{||}v_{||}-a\Omega_e}\right.$$
$$\times\left[\frac{\partial}{\partial v_{||}}\frac{J_s^2(K_\perp v_\perp/\Omega_e)}{(\Omega-\omega-K'_{||}v_{||}-s\Omega_e)}\left[\frac{s\Omega_e}{v_\perp}\frac{\partial}{\partial v_\perp}+K'_{||}\frac{\partial}{\partial v_{||}}\right]f_{0e}\right.$$
$$\left.\left.+\left[\frac{a\Omega_e}{v_\perp}\frac{\partial}{\partial v_\perp}+K'_{||}\frac{\partial}{\partial v_{||}}\right]\frac{1}{\omega-k_{||}v_{||}+i0^+}\frac{\partial}{\partial v_{||}}f_{0e}\right]d\mathbf{v}\right\}$$
$$\times\left\{\frac{\omega_{pe}^2}{|\mathbf{K}-\mathbf{k}|^2\varepsilon_0(\mathbf{K}-\mathbf{k},\Omega-\omega)}\int\frac{J_s^2(K_\perp v_\perp/\Omega_e)}{(\Omega-\omega-K'_{||}v_{||}-s\Omega_e)}\left[\frac{\partial}{\partial v_{||}}\frac{J_n^2(K_\perp v_\perp/\Omega_e)}{\Omega-K_{||}v_{||}-n\Omega_e}\left[\frac{n\Omega_e}{v_\perp}\frac{\partial}{\partial v_\perp}+K_{||}\frac{\partial}{\partial v_{||}}\right]f_{0e}\right.\right.$$
$$\left.\left.+\left[\frac{s\Omega_e}{v_\perp}\frac{\partial}{\partial v_\perp}+K_{||}\frac{\partial}{\partial v_{||}}\right]\frac{1}{k_{||}v_{||}-\omega+i0^+}\frac{\partial}{\partial v_{||}}f_{0e}\right]d\mathbf{v}\right\}. \quad (15)$$

Here, $\mathbf{K}=(K_\perp,0,K_{||})$ and $K'_{||}=K_{||}-k_{||}$. J_a,J_s,J_n are the Bessel functions, and ω_{pe} and Ω_e are the electron plasma frequency and gyrofrequency, respectively. Note that Eq. (12) agrees with Eq. (12) in Ref. 8 if we keep $a=s=n=0$ terms of Bessel functions and put $J_0=1$. Accordingly, Eq. (12) is an extension of the unmagnetized analysis into the magnetized plasma.

III. PLASMA MASER OF LANGMUIR WAVES

There are two different modes in Eq. (13): the Langmuir wave $(\Omega\simeq\omega_{pe}K_{||}/K)$ and the Bernstein mode $(\Omega\simeq n\Omega_e)$. In the following, we mainly consider the Langmuir wave [$n=0$ in Eq. (13)]. The magnetic field is assumed to be strong, so that $\Omega_e>\omega_{pe}$.

The linear dispersion relation of the Langmuir wave for magnetized plasma reduces to

$$\varepsilon_0(\mathbf{K},\Omega)=0=1+\left[\frac{\omega_{pe}}{K}\right]^2\int\frac{J_0^2}{\Omega-K_{||}v_{||}}K_{||}\frac{\partial}{\partial v_{||}}f_{0e}d\mathbf{v}\ ; \quad (16)$$

here, $f_{0e}=(m/2\pi T)^{3/2}\exp[-m(v_\perp^2+v_{||}^2)/2T]$. Equation (16) reduces to

$$1+\left[\frac{\omega_{pe}}{K}\right]^2\exp(-\beta)I_0(\beta)\frac{m}{T}\times\left[1+\left[\frac{m}{2T}\right]^{1/2}\frac{\Omega}{K_{||}}Z\left[\frac{\Omega}{K_{||}v_e}\right]\right]=0\ , \quad (17)$$

where $\beta=K_\perp^2T/m\Omega_e^2$, Z is the plasma dispersion function, and I_0 is the modified Bessel function. For $\Omega/K_{||}v_e\gg1$, $\varepsilon_0(K,\Omega)$ reduces to

$$\varepsilon_0(\mathbf{K},\Omega)=1-\left[\frac{\omega_{pe}}{\Omega}\right]^2\left[\frac{K_{||}}{K}\right]^2\left[1+\left[\frac{K_{||}v_e}{\Omega}\right]^2\right]\times\exp(-\beta)I_0(\beta)\ . \quad (18)$$

Accordingly, we obtain the linear dispersion relation of Langmuir wave for magnetized plasma

$$\Omega\approx\omega_{pe}\frac{K_{||}}{K}[\exp(-\beta)I_0(\beta)]^{1/2}\ . \quad (19)$$

There are several competing processes which coexist with the plasma maser effect. The range of validity of wave numbers $\mathbf{K}$ and $\mathbf{k}$ are summarized in Appendix A. Next, we calculate the growth rate of the Langmuir wave through the plasma maser interaction. The growth rate originates from two different processes: the direct-coupling term [Eq. (14)] and the polarization term [Eq. (15)]. We obtain the growth rate for these cases.

Case A. Growth rate from the direct-coupling term. We keep the $n=s=a=0$ term in Eq. (14). Bearing in mind the fact that the plasma maser effect comes from the condition $\omega=k_{||}v_{||}$, Eq. (14) reduces to

$$\varepsilon_d(\mathbf{K},\Omega)=\left[\frac{\omega_{pe}}{K}\right]^2\left[\frac{e}{m}\right]^2a\sum_{\mathbf{k}}|E_l(\mathbf{k},\omega)|^2\int\frac{1}{\Omega-K_{||}v_{||}}\frac{\partial}{\partial v_{||}}\frac{1}{\Omega-\omega-K'_{||}v_{||}}K_{||}\frac{\partial}{\partial v_{||}}\frac{1}{\omega-k_{||}v_{||}-i0^+}\frac{\partial}{\partial v_{||}}f_{0e}(v_{||})dv_{||}\ , \quad (20)$$

with

$$a=\int_0^\infty J_0^4(K_\perp v_\perp/\Omega_e)f_{0e}(v_\perp)2\pi v_\perp dv_\perp \ . \quad (21)$$

Here $f_{0e}(v_\parallel)=(m/2\pi T)^{1/2}\exp(-mv_\parallel^2/2T)$ and $f_{0e}(v_\perp)=(m/2\pi T)\exp(-mv_\perp^2/2T)$, respectively. The Langmuir-wave resonances are not important for the plasma maser effect. Partial integration of Eq. (20) then leads to

$$\left[\frac{\omega_{pe}}{K}\right]^2\left[\frac{e}{m}\right]^2 a\sum_{\mathbf{k}}|E_l(\mathbf{k},\omega)|^2K_\parallel^2 \times\int\frac{(3K_\parallel-k_\parallel)}{\Omega^4}\frac{1}{\omega-k_\parallel v_\parallel-i0^+}\frac{\partial}{\partial v_\parallel}f_{0e}(v_\parallel)dv_\parallel \ . \quad (22)$$

In deriving Eq. (22), we have replaced $(\Omega-K_\parallel v_\parallel)^{-3}$ by Ω^{-3}.

Equation (22) can be written as

$$\left[\frac{\omega_{pe}}{K}\right]^2\left[\frac{e}{m}\right]^2 a\sum_{\mathbf{k}}|E_l(\mathbf{k},\omega)|^2\frac{K_\parallel^2(3K_\parallel-k_\parallel)}{\Omega^4}\frac{m}{Tk_\parallel} \times\left[1+\left[\frac{m}{2T}\right]^{1/2}\frac{\omega}{k_\parallel}Z^*\left[\frac{\omega}{k_\parallel v_e}\right]\right], \quad (23)$$

where $Z(z)$ is the plasma dispersion function, and $*$ denotes the complex conjugate. Under the small-argument limit $(\omega/k_\parallel v_e \ll 1)$, the imaginary part of Eq. (23) becomes

$$-3\left[\frac{\pi m}{M}\right]^{1/2}\left[\frac{K_\parallel}{K}\right]^2\left[\frac{\omega_{pe}}{\Omega}\right]^4 a\sum_{\mathbf{k}}W_s\frac{K_\parallel|k_\parallel|}{k_e^2} \ ; \quad (24)$$

here, $W_s=[\,|E_l(\mathbf{k},\omega)|^2/4\pi NT](k_e/k)^2$ is the normalized ion-wave energy and k_e is the electron Debye wave number. In deriving Eq. (24), we have used the following relation:

$$\mathrm{Im}Z^*\left[\frac{\omega}{k_\parallel v_e}\right]=-\pi^{1/2}\mathrm{sgn}(k_\parallel) \ , \quad (25)$$

where Im is the imaginary part of the relevant term and $\mathrm{sgn}(k_\parallel)$ is the usual signum function. The growth rate of the Langmuir wave due to the direct mode-coupling process (γ_d) is

$$\gamma_d=-\mathrm{Im}\varepsilon_d(\mathbf{K},\Omega)\Big/\frac{\partial}{\partial\Omega}\mathrm{Re}\varepsilon_0(\mathbf{K},\Omega) \ ; \quad (26)$$

here Re is the real part of the relevant term. From Eq. (18), we get

$$\frac{\partial}{\partial\Omega}\mathrm{Re}\varepsilon_0(\mathbf{K},\Omega)\approx\frac{2\omega_{pe}^2}{\Omega^3}\left[\frac{K_\parallel}{K}\right]^2\exp(-\beta)I_0(\beta) \ . \quad (18')$$

Inserting Eqs. (18′) and (24) into Eq. (26), we obtain

$$\frac{\gamma_d(\mathbf{K},\Omega)}{\omega_{pe}}=\frac{3a}{2}\left[\frac{\pi m}{M}\right]^{1/2}[\exp(-\beta)I_0(\beta)]^{-3/2} \times\sum_{\mathbf{k}}W_s\frac{K|k_\parallel|}{k_e^2} \ . \quad (27)$$

Equation (27) is a slight modification of the previous result[1,9] obtained for unmagnetized plasma.

Case B. Growth rate from the polarization term. The dominant contribution for the Langmuir wave occurs from the $n=s=a=0$ term in Eq. (15) which is justified by the assumption $\Omega_e>\omega_{pe}$. The neglected terms are smaller by a factor $(\omega_{pe}/\Omega_e)^2$. Equation (15) reduces to

$$\varepsilon_p(\mathbf{K},\Omega)=\left[\frac{\omega_{pe}}{K}\right]^2\left[\frac{e}{m}\right]^2 \times\sum_{\mathbf{k}}|E_l(\mathbf{k},\omega)|^2\frac{\omega_{pe}^2(K_\parallel-k_\parallel)K_\parallel}{|\mathbf{K}-\mathbf{k}|^2\varepsilon_0(\mathbf{K}-\mathbf{k},\Omega-\omega)}S \ , \quad (28)$$

with

$$S=\int_{-\infty}^{\infty}\frac{1}{\Omega-K_\parallel v_\parallel}\frac{\partial}{\partial v_\parallel}\left[\frac{a}{\Omega-\omega-(K_\parallel-k_\parallel)v_\parallel}+\frac{b}{\omega-k_\parallel v_\parallel+i0^+}\right]\frac{\partial}{\partial v_\parallel}f_{0e}(v_\parallel)dv_\parallel \times\int_{-\infty}^{\infty}\frac{1}{\Omega-\omega-(K_\parallel-k_\parallel)v_\parallel}\frac{\partial}{\partial v_\parallel}\left[\frac{a}{\Omega-K_\parallel v_\parallel}+\frac{b}{k_\parallel v_\parallel-\omega+i0^+}\right]\frac{\partial}{\partial v_\parallel}f_{0e}(v_\parallel)dv_\parallel \ ; \quad (29)$$

here,

$$b=\int_0^\infty J_0^2(K_\perp v_\perp/\Omega_e)f_{0e}(v_\perp)2\pi v_\perp dv_\perp=\exp(-\beta)I_0(\beta) \ . \quad (30)$$

It is instructive to rewrite Eq. (29) into

$$S=\int_{-\infty}^{\infty}\frac{1}{\Omega-K_\parallel v_\parallel}\frac{\partial}{\partial v_\parallel}\left[\frac{b}{\Delta\Omega-\Delta K v_\parallel}+\frac{b}{\omega-k_\parallel v_\parallel+i0^+}+\frac{a-b}{\Delta\Omega-\Delta K v_\parallel}\right]\frac{\partial}{\partial v_\parallel}f_{0e}(v_\parallel)dv_\parallel \times\int_{-\infty}^{\infty}\frac{1}{\Delta\Omega-\Delta K v_\parallel}\frac{\partial}{\partial v_\parallel}\left[\frac{b}{\Omega-K_\parallel v_\parallel}+\frac{b}{k_\parallel v_\parallel-\omega+i0^+}+\frac{a-b}{\Omega-K_\parallel v_\parallel}\right]\frac{\partial}{\partial v_\parallel}f_{0e}(v_\parallel)dv_\parallel=(A+B+C)(D+E+F) \ , \quad (31)$$

with

$$A=\int_{-\infty}^{\infty}\frac{1}{\Omega-K_{\parallel}v_{\parallel}}\frac{\partial}{\partial v_{\parallel}}\frac{b}{\Delta\Omega-\Delta K v_{\parallel}}\frac{\partial}{\partial v_{\parallel}}f_{0e}(v_{\parallel})dv_{\parallel}\ , \tag{32}$$

$$B=\int_{-\infty}^{\infty}\frac{1}{\Omega-K_{\parallel}v_{\parallel}}\frac{\partial}{\partial v_{\parallel}}\frac{b}{\omega-k_{\parallel}v_{\parallel}+i0^{+}}\frac{\partial}{\partial v_{\parallel}}f_{0e}(v_{\parallel})dv_{\parallel}\ , \tag{33}$$

$$C=\int_{-\infty}^{\infty}\frac{1}{\Omega-K_{\parallel}v_{\parallel}}\frac{\partial}{\partial v_{\parallel}}\frac{a-b}{\Delta\Omega-\Delta K v_{\parallel}}\frac{\partial}{\partial v_{\parallel}}f_{0e}(v_{\parallel})dv_{\parallel}\ , \tag{34}$$

$$D=\int_{-\infty}^{\infty}\frac{1}{\Delta\Omega-\Delta K v_{\parallel}}\frac{\partial}{\partial v_{\parallel}}\frac{b}{\Omega-K_{\parallel}v_{\parallel}}\frac{\partial}{\partial v_{\parallel}}f_{0e}(v_{\parallel})dv_{\parallel}\ , \tag{35}$$

$$E=\int_{-\infty}^{\infty}\frac{1}{\Delta\Omega-\Delta K v_{\parallel}}\frac{\partial}{\partial v_{\parallel}}\frac{b}{k_{\parallel}v_{\parallel}-\omega+i0^{+}}\times\frac{\partial}{\partial v_{\parallel}}f_{0e}(v_{\parallel})dv_{\parallel}\ , \tag{36}$$

$$F=\int_{-\infty}^{\infty}\frac{1}{\Delta\Omega-\Delta K v_{\parallel}}\frac{\partial}{\partial v_{\parallel}}\frac{a-b}{\Omega-K_{\parallel}v_{\parallel}}\frac{\partial}{\partial v_{\parallel}}f_{0e}(v_{\parallel})dv_{\parallel}\ , \tag{37}$$

where $\Delta\Omega=\Omega-\omega$ and $\Delta K=K_{\parallel}-k_{\parallel}$. Now, it is easy to show

$$A+B\propto\int_{-\infty}^{\infty}\frac{1}{(\Omega-K_{\parallel}v_{\parallel})(\Delta\Omega-\Delta K v_{\parallel})(\omega-k_{\parallel}v_{\parallel}+i0^{+})}\times\frac{\partial}{\partial v_{\parallel}}f_{0e}(v_{\parallel})dv_{\parallel}\ , \tag{38}$$

and

$$D+E\propto\int_{-\infty}^{\infty}\frac{1}{(\Delta\Omega-\Delta K v_{\parallel})(\Omega-K_{\parallel}v_{\parallel})(k_{\parallel}v_{\parallel}-\omega+i0^{+})}\times\frac{\partial}{\partial v_{\parallel}}f_{0e}(v_{\parallel})dv_{\parallel}\ . \tag{39}$$

Accordingly, $\mathrm{Im}[(A+B)(D+E)]=0$ because $A+B\propto(D+E)^{*}$; here, $*$ represents the complex conjugate. Thus, the imaginary part of Eq. (31) reduces to

$$\mathrm{Im}S=\mathrm{Im}B\times\mathrm{Re}F+\mathrm{Im}E\times\mathrm{Re}C\ . \tag{40}$$

In deriving Eq. (40), we have used the condition for the plasma maser $\mathrm{Im}A=\mathrm{Im}C=\mathrm{Im}D=\mathrm{Im}F=0$.

Now, it is straightforward to show

$$\mathrm{Im}B=-\frac{bK_{\parallel}}{\Omega^{2}k_{\parallel}}\frac{m}{T}\left[\frac{\pi m}{2T}\right]^{1/2}\frac{\omega}{k_{\parallel}}\mathrm{sgn}(k_{\parallel})\ , \tag{41}$$

$$\mathrm{Re}F=\frac{(a-b)(K_{\parallel}-k_{\parallel})(3K_{\parallel}-2k_{\parallel})}{\Omega^{4}}\ , \tag{42}$$

$$\mathrm{Im}E=-\frac{b(K_{\parallel}-k_{\parallel})}{\Omega^{2}k_{\parallel}}\frac{m}{T}\left[\frac{m\pi}{M}\right]^{1/2}\mathrm{sgn}(k_{\parallel})\ , \tag{43}$$

and

$$\mathrm{Re}C=\frac{(a-b)K_{\parallel}(3K_{\parallel}-k_{\parallel})}{\Omega^{4}}\ . \tag{44}$$

In obtaining Eqs. (41) and (43), we have used the condition for the plasma maser $\omega=k_{\parallel}v_{\parallel}$. On substituting Eqs. (41)—(44) into Eq. (40), we obtain

$$\mathrm{Im}S=\frac{b(b-a)K_{\parallel}(K_{\parallel}-k_{\parallel})(6K_{\parallel}-3k_{\parallel})}{\Omega^{6}k_{\parallel}}\frac{m}{T}\times\left[\frac{\pi m}{M}\right]^{1/2}\mathrm{sgn}(k_{\parallel})\ . \tag{45}$$

Thus, we get the imaginary part of the polarization term

$$\begin{aligned}\mathrm{Im}\varepsilon_{p}(\mathbf{K},\Omega)&=\left[\frac{\omega_{pe}}{K}\right]^{2}\left[\frac{e}{m}\right]^{2}\sum_{\mathbf{k}}\frac{|E_{l}(\mathbf{k},\omega)|^{2}\omega_{pe}^{2}K_{\parallel}(K_{\parallel}-k_{\parallel})}{|\mathbf{K}-\mathbf{k}|^{2}\varepsilon_{0}(\mathbf{K}-\mathbf{k},\Omega-\omega)}\mathrm{Im}S\\&=3b(b-a)\left[\frac{\pi m}{M}\right]^{1/2}\left[\frac{K_{\parallel}}{K}\right]^{2}\sum_{\mathbf{k}}\frac{|E_{l}(\mathbf{k},\omega)|^{2}}{4\pi NT}\frac{(K_{\parallel}-k_{\parallel})^{2}}{|\mathbf{K}-\mathbf{k}|^{2}}\frac{(2K_{\parallel}-k_{\parallel})}{|k_{\parallel}|}\varepsilon_{0}(\mathbf{K}-\mathbf{k},\Omega-\omega)^{-1}\left[\frac{\omega_{pe}}{\Omega}\right]^{6}.\end{aligned} \tag{46}$$

From Eq. (18), we obtain

$$\varepsilon_{0}(\mathbf{K}-\mathbf{k},\Omega-\omega)=1-\frac{\omega_{pe}^{2}}{(\Omega-\omega)^{2}}\frac{(K_{\parallel}-k_{\parallel})^{2}}{|\mathbf{K}-\mathbf{k}|^{2}}\left[1+\frac{(K_{\parallel}-k_{\parallel})^{2}v_{e}^{2}}{(\Omega-\omega)^{2}}\right]\exp(-\beta)I_{0}(\beta)\ . \tag{47}$$

For $|\mathbf{K}|>k_{\parallel}$ and $\Omega>\omega$, we expand $\varepsilon_{0}(\mathbf{K}-\mathbf{k},\Omega-\omega)$ and find

$$\begin{aligned}\varepsilon_{0}(\mathbf{K}-\mathbf{k},\Omega-\omega)\approx&-\left[\frac{\omega_{pe}}{\Omega}\right]^{2}\left[\frac{K_{\parallel}}{K}\right]^{2}\left[\frac{2\omega}{\Omega}-\frac{2k_{\parallel}}{K_{\parallel}}\left[\frac{K_{\perp}}{K}\right]^{2}\right]\exp(-\beta)I_{0}(\beta)\\&+\left[\frac{\omega_{pe}}{\Omega}\right]^{2}\left[\frac{K_{\parallel}}{K}\right]^{2}\left[\frac{K_{\parallel}}{k_{e}}\right]^{2}\frac{2k_{\parallel}}{K_{\parallel}}\exp(-\beta)I_{0}(\beta)\ .\end{aligned} \tag{48}$$

In obtaining Eq. (48), we have used the linear dispersion relation $\varepsilon_0(\mathbf{K},\Omega)=0$ [Eq. (18)]. Accordingly, under the following conditions,

$$\left[\frac{M}{m}\right]^{1/2}\frac{k_e}{K_{\parallel}}\left[\frac{K_{\perp}}{K}\right]^2>1$$

and (49)

$$\left[\frac{K_{\perp}}{K}\right]^2\left[\frac{k_e}{K_{\parallel}}\right]^2>1 .$$

Equation (48) reduces to

$$\varepsilon_0(\mathbf{K}-\mathbf{k},\Omega-\omega)\approx\left[\frac{\omega_{pe}}{\Omega}\right]^2\left[\frac{K_{\parallel}}{K}\right]^2\left[\frac{K_{\perp}}{K}\right]^2\frac{2k_{\parallel}}{K_{\parallel}}\times\exp(-\beta)I_0(\beta) . \tag{50}$$

To the lowest order in the small parameter $k_{\parallel}/|\mathbf{K}|$, we get

$$\frac{(K_{\parallel}-k_{\parallel})^2}{|\mathbf{K}-\mathbf{k}|^2}\approx\left[\frac{K_{\parallel}}{K}\right]^2 . \tag{51}$$

On substituting Eqs. (50) and (51) into Eq. (46), we obtain

$$\mathrm{Im}\varepsilon_p(\mathbf{K},\Omega)=\left[\frac{K_{\parallel}}{K_{\perp}}\right]^2\left[\frac{\omega_{pe}}{\Omega}\right]^4\frac{3(a-b)}{2}\left[\frac{\pi m}{M}\right]^{1/2}\times\sum_{\mathbf{k}}W_s\frac{|k_{\parallel}|K_{\parallel}}{k_e^2} . \tag{52}$$

In obtaining Eq. (52), we have used Eq. (30).

The growth rate of the Langmuir wave due to the polarization term (γ_p) is

$$\gamma_p=-\mathrm{Im}\varepsilon_p(\mathbf{K},\Omega)\Big/\frac{\partial}{\partial\Omega}\mathrm{Re}\varepsilon_0(\mathbf{K},\Omega) . \tag{53}$$

Using Eqs. (18′) and (52), we get

$$\frac{\gamma_p(\mathbf{K},\Omega)}{\omega_{pe}}=\tfrac{3}{4}(b-a)\left[\frac{\pi m}{M}\right]^{1/2}[\exp(-\beta)I_0(\beta)]^{-3/2}\times\left[\frac{K}{K_{\perp}}\right]^2\sum_{\mathbf{k}}W_s\frac{|k_{\parallel}|K}{k_e^2} . \tag{54}$$

Note that $b>a$ by definition [Eqs. (21) and (30)]. Thus, $\gamma_p(\mathbf{K},\Omega)>0$ even for the Maxwell distribution function of electrons.

It is straightforward to compare the magnitudes of the two growth rates, Eqs. (27) and (54):

$$\frac{\gamma_p(\mathbf{K},\Omega)}{\gamma_d(\mathbf{K},\Omega)}\approx\frac{(b-a)}{2a}\left[\frac{K}{K_{\perp}}\right]^2 . \tag{55}$$

For $k_e/K_{\parallel}\approx(M/m)^{1/2}$, Eq. (49) gives

$$\frac{M}{m}\gtrsim(K/K_{\perp})^2>1 . \tag{56}$$

Furthermore, by definition, $b>a$. Thus, the maximum value of Eq. (55) reduces to

$$\max[\gamma_p(\mathbf{K},\Omega)/\gamma_d(\mathbf{K},\Omega)]>1 . \tag{57}$$

Accordingly, we can conclude that the polarization term gives the main destabilizing effect in the plasma maser theory for magnetized plasma. Note that the contribution from the polarization term [Eq. (54)] vanishes for unmagnetized plasma because $b=a$. Unfortunately, both the recent erratum[10] for Ref. 8 and Eq. (35) in Ref. 4 overlooked this point. Accordingly, we can conclude that the most dominant destabilizing plasma maser effect originates from the polarization mode-coupling term for the magnetized plasma (for details, see Appendix B).

In this paper, we have considered only the Langmuir waves ($n=0$) in Eq. (13). By the same method, we can study the generation of the ES Bernstein mode [$\Omega\sim n\Omega_e$ in Eq. (13)] through the plasma maser effect. It is easy to show that the most dominant destabilizing term for the Bernstein mode comes also from the polarization term [Eq. (15)]. We have assumed a steady ion-wave turbulence throughout this paper. For a growing ion-wave fluctuation, an additional damping term is pointed out[4] (for details, see Sec. IV and Appendix C).

IV. DISCUSSION AND CONCLUSIONS

There is some experimental evidence which shows the energy up-conversion from low-frequency ion density fluctuations to high-frequency Langmuir waves. In the presence of a coherent low-frequency ion-wave pump field, the turbulent electrostatic bursts with frequency ω_{pe} were observed in laboratory experiments.[11,12] The similar experimental results were reported earlier.[13] The high-frequency Langmuir waves are also observed in a Z-pinch plasma.[14] In space plasma physics, the Langmuir-wave emissions are modulated by the low-frequency ion density fluctuations.[15] In all of the above experiments, the amplitudes of the low-frequency waves are much larger than those of the high-frequency Langmuir waves. It is, hence, tempting to assume that the Langmuir emissions are modulated by the low-frequency turbulences through the plasma maser effect considered in this paper.

Here, we comment on the relation between the plasma maser effect and clumps[16] in plasma turbulence. The plasma maser effect comes from the resonant electrons. The resonant electrons are known to form clumps in phase space. Accordingly, the plasma maser effect may belong to the clumps effect in plasma.

In almost all previous studies, the steady turbulent state of ion acoustic waves is assumed. Accordingly, the calculation does not permit the background electron distribution to evolve by plateau formation;[17] hence, the energy transferred from the ion acoustic waves by resonant interaction must go into unstable Langmuir waves. This raises the question of the behavior of the system when the ion acoustic turbulence and background electron distribution function are not fixed by external agents, but are free to evolve self-consistently. As an answer to the above question, there are now two papers (for details, see Nambu[18] and Isakov *et al.*[19]). The first one[18] writes the induction [$\mathbf{D}(\mathbf{x},t)$] in the form

$$\mathbf{D}(\mathbf{x},t)=\int_{-\infty}^{\mathbf{x}} d\mathbf{x}' \int_{-\infty}^{t} dt' \varepsilon_h(\mathbf{x}-\mathbf{x}',t-t',(t+t')/2) \times \delta\mathbf{E}_h(\mathbf{x}',t') , \quad (58)$$

and obtains the growth rate of the Langmuir wave in the presence of the nonsteady ion acoustic wave turbulence as follows:

$$\gamma(\mathbf{K},\Omega(t),t) = -\mathrm{Im}\varepsilon_h(\mathbf{K},\Omega(t),t) \Big/ \frac{\partial}{\partial\Omega(t)} \mathrm{Re}\varepsilon_0(\mathbf{K},\Omega(t),t) . \quad (59)$$

Thus, the growth rate of the Langmuir wave obtained in this paper is basically valid even if we permit the background electron distribution to evolve the plateau formation.[17] The second one[19] assumes the induction in the form

$$\mathbf{D}(\mathbf{x},t)=\int_{-\infty}^{\mathbf{x}} d\mathbf{x}' \int_{-\infty}^{t} dt' \varepsilon_h(\mathbf{x}-\mathbf{x}',t-t',t')\delta\mathbf{E}_h(\mathbf{x}',t') . \quad (60)$$

Then, they obtain the growth rate

$$\gamma(\mathbf{K},\Omega(t),t)=-\left[\mathrm{Im}\varepsilon_h(\mathbf{K},\Omega(t),t)+\frac{\partial^2\varepsilon_0(\mathbf{K},\Omega(t),t)}{2\partial\Omega\,\partial t}\right] \Big/ \frac{\partial}{\partial\Omega(t)}\mathrm{Re}\varepsilon_0(\mathbf{K},\Omega(t),t) . \quad (61)$$

If we substitute Eqs. (16), (24), and (52) into Eq. (61), we get

$$\frac{\gamma(\mathbf{K},\Omega(t),t)}{\omega_{pe}} = \tfrac{3}{2}(\pi m/M)^{1/2}[\exp(-\beta)I_0(\beta)]^{-3/2} \times \sum_{\mathbf{k}} W_s \frac{K\,|k_{\parallel}|}{k_e^2} \times [a-b+(b-a)/2(K/K_{\perp})^2] . \quad (62)$$

Equation (62) vanishes for unmagnetized plasma ($b=a$). For magnetized plasma ($b>a$), the growth rate vanishes only for $K_{\perp}=K_{\parallel}$. Accordingly, we can safely say that the up-conversion from the ion-sound waves to the Langmuir wave occurs for magnetized plasma even for the nonstationary background electron distribution function due to plateau formation.

Here, we clarify the physical mechanism[20] of the plasma maser instability. A system contains both the low-frequency resonant field (E_l) and the high-frequency nonresonant field (δE_h). A nonresonant field δE_h causes particles to oscillate with a frequency Ω^* different from the field frequency Ω because of the Doppler effect, i.e., $\Omega^*=\Omega-Kv$. It is common to assume that a high-frequency nonresonant field on the average performs no work. This assumption is correct only if we are talking about the average work over the period. However, when there is a resonant field E_l, the average must be calculated slightly more accurately. The electrons experience the acceleration or deceleration due to the resonant interaction with the low-frequency resonant field. Thus, the period $2\pi/\Omega^*$ increases or decreases due to the Doppler effect. Accordingly, electrons can exchange energy also with the high-frequency nonresonant field in addition to the low-frequency resonant field. In other words, the modulation fields cause high-frequency dissipative nonlinear forces,[9] which is the origin of the plasma maser instability.

In conclusion, we have obtained the effective dielectric constant of the Langmuir wave in the presence of the ion-wave turbulence for magnetized plasma. The analysis predicts that, in contrast to the unmagnetized plasma, the most dominant destabilizing plasma maser effect comes from the polarization mode-coupling term [Eq. (54)]. The growth of the Langmuir wave occurs even for the Maxwell distribution function of electrons.

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APPENDIX A

There are two competing processes which potentially give the same order contribution as that of the plasma maser. The first one is the nonlinear Landau resonance.[21] The condition is $\Omega(\mathbf{K})\pm\omega(\mathbf{k})=(\mathbf{K}\pm\mathbf{k})\cdot\mathbf{v}$. This gives the resonance velocity $v_{\parallel}\sim\Omega/(K_{\parallel}\pm k_{\parallel})\sim\omega_{pe}/(K_{\parallel}\pm k_{\parallel})$ for $\Omega\gg\omega$. Accordingly, for $K_{\parallel}<k_e$ and $k_{\parallel}<k_e$, the resonance velocity is much larger than the electron thermal velocity ($v_{\parallel}\gg v_e$). Thus, we can rule out the nonlinear Landau interaction under the condition

$$k_{\parallel}<k_e . \quad (A1)$$

The second one is the resonant decay interaction.[22] The conditions are $\Omega(\mathbf{K})\pm\omega(\mathbf{k})=\Omega'(\mathbf{K}')$ and $\mathbf{K}\pm\mathbf{k}=\mathbf{K}'$. The three-wave decay instability is kinematically possible under the conditions

$$\omega_{pe}\frac{K_{\parallel}}{K}[\exp(-\beta)I_0(\beta)]^{1/2} = \omega_{pe}\frac{K'_{\parallel}}{K'}[\exp(-\beta)I_0(\beta)]^{1/2}+k_{\parallel}c_s , \quad (A2)$$

$$K_{\parallel}=K'_{\parallel}+k_{\parallel} , \quad (A3)$$

$$K_{\perp}=K'_{\perp} . \quad (A4)$$

In deriving Eq. (A2), we use Eq. (19) and $\omega=k_{\parallel}c_s$. Furthermore, to identify the plasma wave as the high-frequency mode, the following condition is necessary:

$$\omega_{pe}\frac{K_{\parallel}}{K}[\exp(-\beta)I_0(\beta)]^{1/2}>\omega_{pi}\ . \tag{A5}$$

Here, ω_{pi} is the ion plasma frequency. If we substitute Eqs. (A3) and (A4) into (A2), and assume $k_e>K_{\parallel}>k_{\parallel}$, we get

$$K\simeq k_e(M/m)^{1/2}[\exp(-\beta)I_0(\beta)]^{1/2}\ . \tag{A6}$$

Equation (A6) with $k_e>K_{\parallel}$ does not coexist with the necessary condition, Eq. (A5). Thus, we can conclude that, at least for $k_e>K_{\parallel}>k_{\parallel}$ considered in this paper, we can rule out the two competing processes: nonlinear Landau resonance and the resonant three-wave decay process. We assume that the above conditions are satisfied throughout this paper.

APPENDIX B

According to Eq. (4) in Ref. 4, the polarization mode-coupling term $[\varepsilon_p(\mathbf{K},\Omega)]$ for the unmagnetized plasma reduces to

$$\varepsilon_p(\mathbf{K},\Omega)=\sum_{\mathbf{k}}\left[\frac{4\pi e^3}{m^2K\,|\mathbf{K}-\mathbf{k}|}\right]^2\frac{|\phi(\mathbf{k},\omega)|^2}{\varepsilon_0(\mathbf{K}-\mathbf{k},\Omega-\omega)}\times(A+B)(C+D)\ , \tag{B1}$$

with

$$A=\int d\mathbf{v}\frac{1}{\Omega-\mathbf{K}\cdot\mathbf{v}}\left[(\mathbf{K}-\mathbf{k})\cdot\frac{\partial}{\partial\mathbf{v}}\left[\frac{1}{\omega-\mathbf{k}\cdot\mathbf{v}+i0^+}\mathbf{k}\cdot\frac{\partial}{\partial\mathbf{v}}f_{0e}\right]\right]\ , \tag{B2}$$

$$B=\int d\mathbf{v}\frac{1}{\Omega-\mathbf{K}\cdot\mathbf{v}}\left[\mathbf{k}\cdot\frac{\partial}{\partial\mathbf{v}}\left[\frac{1}{\Omega-\omega-(\mathbf{K}-\mathbf{k})\cdot\mathbf{v}}(\mathbf{K}-\mathbf{k})\cdot\frac{\partial}{\partial\mathbf{v}}f_{0e}\right]\right]\ , \tag{B3}$$

$$C=\int d\mathbf{v}\frac{1}{\Omega-\omega-(\mathbf{K}-\mathbf{k})\cdot\mathbf{v}}\left[\mathbf{k}\cdot\frac{\partial}{\partial\mathbf{v}}\left[\frac{1}{\Omega-\mathbf{K}\cdot\mathbf{v}}\mathbf{K}\cdot\frac{\partial}{\partial\mathbf{v}}f_{0e}\right]\right]\ , \tag{B4}$$

$$D=\int d\mathbf{v}\frac{1}{\Omega-\omega-(\mathbf{K}-\mathbf{k})\cdot\mathbf{v}}\left[\mathbf{K}\cdot\frac{\partial}{\partial\mathbf{v}}\left[\frac{1}{\mathbf{k}\cdot\mathbf{v}-\omega+i0^+}\mathbf{k}\cdot\frac{\partial}{\partial\mathbf{v}}f_{0e}\right]\right]\ , \tag{B5}$$

where $|\phi(\mathbf{k},\omega)|^2$ is the turbulent potential fluctuation of the ion waves. $\varepsilon_0(\mathbf{K}-\mathbf{k},\Omega-\omega)$ is the linear dielectric function of the electrostatic waves. $\mathbf{K},\Omega$ and $\mathbf{k},\omega$ are the wave number and the frequency of the Langmuir and ion-wave fields. The $+i0^+$ in the denominator of Eqs. (B2) and (B5) is the small imaginary part.

It is instructive to rewrite the polarization term into

$$(A+B)(C+D)=(a+c)(b+d)\ , \tag{B6}$$

where

$$a=\int d\mathbf{v}\frac{1}{\Omega-\mathbf{K}\cdot\mathbf{v}}(\mathbf{K}-\mathbf{k})\cdot\frac{\partial}{\partial\mathbf{v}}\left[\left[\frac{1}{\omega-\mathbf{k}\cdot\mathbf{v}+i0^+}+\frac{1}{\Omega-\omega-(\mathbf{K}-\mathbf{k})\cdot\mathbf{v}}\right]\mathbf{k}\cdot\frac{\partial}{\partial\mathbf{v}}f_{0e}\right]\ , \tag{B7}$$

$$b=\int d\mathbf{v}\frac{1}{\Omega-\omega-(\mathbf{K}-\mathbf{k})\cdot\mathbf{v}}\mathbf{K}\cdot\frac{\partial}{\partial\mathbf{v}}\left[\left[\frac{1}{\mathbf{k}\cdot\mathbf{v}-\omega+i0^+}+\frac{1}{\Omega-\mathbf{K}\cdot\mathbf{v}}\right]\mathbf{k}\cdot\frac{\partial}{\partial\mathbf{v}}f_{0e}\right]\ , \tag{B8}$$

$$c=\int d\mathbf{v}\frac{1}{\Omega-\mathbf{K}\cdot\mathbf{v}}\left[\mathbf{k}\cdot\frac{\partial}{\partial\mathbf{v}}\left[\frac{1}{\Omega-\omega-(\mathbf{K}-\mathbf{k})\cdot\mathbf{v}}(\mathbf{K}-\mathbf{k})\cdot\frac{\partial}{\partial\mathbf{v}}f_{0e}\right]-(\mathbf{K}-\mathbf{k})\cdot\frac{\partial}{\partial\mathbf{v}}\left[\frac{1}{\Omega-\omega-(\mathbf{K}-\mathbf{k})\cdot\mathbf{v}}\mathbf{k}\cdot\frac{\partial}{\partial\mathbf{v}}f_{0e}\right]\right]\ , \tag{B9}$$

$$d=\int d\mathbf{v}\frac{1}{\Omega-\omega-(\mathbf{K}-\mathbf{k})\cdot\mathbf{v}}\left[\mathbf{k}\cdot\frac{\partial}{\partial\mathbf{v}}\left[\frac{1}{\Omega-\mathbf{K}\cdot\mathbf{v}}\mathbf{K}\cdot\frac{\partial}{\partial\mathbf{v}}f_{0e}\right]-\mathbf{K}\cdot\frac{\partial}{\partial\mathbf{v}}\left[\frac{1}{\Omega-\mathbf{K}\cdot\mathbf{v}}\mathbf{k}\cdot\frac{\partial}{\partial\mathbf{v}}f_{0e}\right]\right]\ . \tag{B10}$$

Now, it is easy to show that $a=-b^*$; here $*$ represents the complex conjugate. Thus, $\mathrm{Im}ab=0$; here Im stands for the imaginary part. Accordingly, the imaginary part of the polarization term comes from $\mathrm{Im}(ad+bc)=\mathrm{Im}a\times\mathrm{Re}d+\mathrm{Im}b\times\mathrm{Re}c=\mathrm{Im}a\times\mathrm{Re}(d+c)$; here Re shows the real part. After a straightforward calculation, we get

$$\mathrm{Re}c=-\mathrm{Re}d=\int d\mathbf{v}\frac{[k^2K^2-(\mathbf{k}\cdot\mathbf{K})^2]f_{0e}}{(\Omega-\mathbf{K}\cdot\mathbf{v})^2[\Omega-\omega-(\mathbf{K}-\mathbf{k})\cdot\mathbf{v}]^2}\ . \tag{B11}$$

Thus, $\mathrm{Im}(ad+bc)=0$ exactly for the unmagnetized plasma. Accordingly, Eq. (35) in Ref. 4 vanishes identically for the unmagnetized plasma.

APPENDIX C

DuBois and Pesme[4] consider the nonresonant parametric interaction caused by the nonsteady ion-sound turbulence. They obtain an additional damping term

$$\frac{\delta\gamma^{\rm NR}(\mathbf{K},\Omega)}{\omega_{pe}} \simeq \frac{-1}{9}\frac{e^2}{m^2K^2\omega_{pe}^4}\int d^3k I_k \frac{\tilde{\nu}_k^s}{\omega_{pe}}\frac{(\mathbf{k}\cdot\mathbf{K})^2 k_e^8}{k^8}\,, \tag{C1}$$

where I_k and $\tilde{\nu}_k^s$ are the ion-sound spectrum and the ion-sound growth rate, respectively. Accordingly, the growth rate of the Langmuir waves in the presence of the ion-sound turbulences is written as a sum of three terms:

$$\gamma(\mathbf{K},\Omega)=\gamma^{\rm TSW}(\mathbf{K},\Omega)+\gamma_p(\mathbf{K},\Omega,B_0\neq 0)+\delta\gamma^{\rm NR}(\mathbf{K},\Omega)\,, \tag{C2}$$

where $\gamma^{\rm TSW}(\mathbf{K},\Omega)$ is obtained in Ref. 1, $\gamma_p(\mathbf{K},\Omega,B_0\neq 0)$ is Eq. (54) in this paper, and $\delta\gamma^{\rm NR}(\mathbf{K},\Omega)$ is given by Eq. (C1).

Next, the ratio of the two growth rates reduces to

$$\frac{|\delta\gamma^{\rm NR}(\mathbf{K},\Omega)|}{\gamma_p(\mathbf{K},\Omega,B_0\neq 0)} \simeq \frac{1}{10}\frac{k_*}{K}\left[\frac{k_e}{k_s}\right]^4 \frac{m}{M}\frac{\tilde{\nu}_k^s}{\nu^e}\,, \tag{C3}$$

where m and M are masses for the electron and the ion. k_*, k_s, and ν^e are defined in Ref. 4. The ion-sound growth rate $\tilde{\nu}_k^s \propto \mathrm{Im}(\varepsilon_k^l+\varepsilon_k^N)$; here ε_k^l and ε_k^N are the linear and nonlinear dielectric functions for the ion-sound wave.[23] Accordingly, for steady ion-sound turbulences, $\tilde{\nu}_k^s=0$ and the damping term $\delta\gamma^{\rm NR}$ vanishes. Furthermore, if we assume a quasisteady ion-sound turbulence, Eq. (C3) is smaller than unity because $\tilde{\nu}_k^s \ll \nu^e$. Thus, $\delta\gamma^{\rm NR}$ gives an additional secondary damping effect for the plasma maser.

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