Entropy and time(s)

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A connection between entropy and parametrized time is established within the context of parametrized relativistic quantum mechanics. By recognizing the existence of three operationally distinct, and physically distinguishable, forms of time, it is shown that parametrized relativistic quantum theories have both an arrow of (parametrized) time and an equivalence between (coordinate) time and space in the relativistic sense.

INTRODUCTION

Cramer[1] has recently compared the standard Copenhagen interpretation (CI) of quantum mechanics with an alternative interpretation he calls the transactional interpretation (TI). His is just one of several interpretations that have been discussed in the recent literature. These interpretations include hidden variable,[2,3] semiclassical,[4–7] collapse,[8–10] advanced-action,[6,11–13] and nonlocal[14] interpretations as identified by Cramer.[1] Many of the differences between Cramer's TI and other interpretations, including the standard CI, can be traced to the role time plays in establishing correlations between spatially separated parts of a system. In another recent article,[15] Wheeler has described time as one of four great remaining mysteries. It is part of an ideal continuum that he suggests is only an approximation of nature. Within the continuum, time serves two apparently incompatible functions: it is an irreversible arrow that many modern scientists have tried to link to the second law of thermodynamics,[16,17] and it is a reversible coordinate in the Einsteinian sense. Does time serve all of these functions, or are separate concepts being inappropriately labeled by the word "time?" Time, it seems, is not understood.

The basic flaw in understanding time is a confusion stemming from treating "t" as if it is the same physical quantity in every context in which it is used. An analogy can readily be drawn.[18] With the advent of relativity, a question arose about the equivalence of gravitational and inertial mass. Is Newton's mass the same as Einstein's mass? Only by assuming the masses could be different was a deeper insight into the concept of mass attained. Should a similar treatment be accorded to time?

Time has been treated as both a parameter and a coordinate.[19] Within the context of parametrized relativistic quantum mechanics[20] (PRQM), it has been shown that "time" can be used in three operationally distinct, and physically distinguishable, forms: Galilean time, Minkowski time, and historical time. Minkowski time is the temporal coordinate of a space-time four vector. Galilean time and historical time are evolution (ordering) parameters in nonrelativistic quantum mechanics (NRQM) and PRQM, respectively. These distinctions make it possible to have an arrow of (parametrized) time and an equivalence between (coordinate) time and space. The coordinate role has already been dealt with,[20] but the role of parametrized time as an arrow has not. This paper shows how parametrized time can be linked to the second law of thermodynamics. The link is established by deriving Boltzmann's H theorem within the context of PRQM. Identification of this connection is fundamental to acquiring a correct understanding of time in the continuum. A proper understanding of how continuum time is used may be an important step on the road to unraveling Wheeler's mysteries and deciding upon a correct interpretation of quantum mechanics.

THE ARROW OF HISTORICAL TIME

A link between time and thermodynamics is usually made through Boltzmann's H theorem[21–23] or, equivalently, the temporal rate of entropy change. An analogous theorem can be derived within the context of PRQM. The derivation will make use of two assumptions: the principle of detailed balance and a master equation.

The principle of detailed balance asserts that the transition probabilities for the transition from state r to s, and the inverse transition from state s to r, are equal; thus

$$W_{rs}=W_{sr} \ . \tag{1}$$

Let $P_r(\tau)$ be the probability that an isolated system is in state r at historical time τ. The rate of change of P_r with respect to τ increases when systems in other states make transitions to state r, and decreases when systems in state r make transitions to other states; thus

$$\frac{dP_r}{d\tau}=\sum_s P_s W_{sr}-\sum_s P_r W_{rs} \ . \tag{2}$$

Equation (2) is the PRQM analog of the usual master equation.[22,23] Both Eqs. (1) and (2) are viewed as working hypotheses subject to verification. This is the same status accorded to their analogs in NRQM.

The principle of detailed balance is usually presented as a theorem based on the Hermitian property of transition probabilities calculated to lowest order in Galilean time-dependent perturbation theory. Blokhintsev[24] has pointed out that NRQM transition probabilities are not, in general, Hermitian; they are unitary. A similar analysis, making use of unitary operators,[25] applies to PRQM.

Limitations on the validity of the NRQM master equation have also been noted. Kittel,[22] for example, remarked that superposition of accessible states could affect the validity of the master equation. Consequently, we restrict the status of Eqs. (1) and (2) to plausible, working hypotheses. This status is sufficient for our present purposes, though the range of validity of Eqs. (1) and (2) is a worthwhile subject for future study. Recent work by Horwitz *et al.*[26,27] is a step in this direction.

Combining Eqs. (1) and (2) yields a master equation with the form

$$\frac{dP_r}{d\tau}=\sum_s W_{rs}(P_s-P_r)\ . \tag{3}$$

Carrying our analogy with NRQM further, we need to relate quantum probabilities with thermodynamic quantities. Horwitz *et al.*[26] have made these connections within the context of PRQM. Denoting Boltzmann's constant by k_B, entropy S in PRQM is defined as

$$S=-k_B\sum_r P_r \ln P_r\ . \tag{4}$$

Equation (4) is an extension of the usual definition. Given a grand canonical ensemble, entropy in the PRQM context differs from the usual NRQM entropy by the addition of a mass potential which is analogous to chemical potential. Further details are presented in Ref. 26.

An arrow of historical time is constructed using the procedure presented by Kittel[22] and Feynman.[21] Differentiating S with respect to τ, using probability conservation, and rewriting the double summation yields

$$\frac{dS}{d\tau}=-\frac{k_B}{2}\sum_r\sum_s W_{rs}(P_s-P_r)(\ln P_r-\ln P_s)\geq 0\ . \tag{5}$$

The inequality is valid because each term in the double summation is negative. Noting that $S=-k_B H$ defines Boltzmann's H, Eq. (5) shows that $dH/d\tau\leq 0$, which is Boltzmann's H theorem.

Equation (5) shows that entropy can only be constant or increase with respect to a monotonically increasing historical time. A similar result has been obtained by Horwitz *et al.*[27] using an entirely different approach to the proof of Boltzmann's H theorem. No restrictions are imposed on Minkowski (coordinate) time by Eq. (5). Thus, within the range of validity of working hypotheses, PRQM is a theory with both an arrow of historical time and a Minkowski time in the relativistic sense.

By recognizing the existence of a unique coordinate (Minkowski) time and a unique parametrized (historical) time, there is no longer a conflict between a universal direction of time, and a time which may proceed as readily from future to past as from past to future. The two times are different: the former is a parameter, and the latter is a coordinate. A general procedure for physically distinguishing between the two times is described in Ref. 20.

More can be gained by recognizing a connection between entropy and historical time than just reconciling the apparent incompatibilities of continuum time. Cramer's transactional interpretation[1] of quantum mechanics, particularly his atemporal mechanisms, acquire a dimension within which they can evolve. As Cramer noted, many of the difficulties of the Copenhagen interpretation are tied to a locality assumption and time asymmetry. His transactional interpretation seems to avoid these difficulties by allowing nonlocality and introducing atemporal mechanisms via the offer and confirmation waves. How can the evolution of Cramer's offer and confirmation waves, an atemporal phenomenon, be described mathematically? One now obvious answer is that offer and confirmation waves evolve within the historical time domain, while Cramer's advanced and retarded waves are linked to Minkowski time. These distinctions remove ambiguities in the properties associated with what should be, and apparently are, two entirely different times.

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