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A correlated-emission laser gyroscope

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It is shown that correlated-emission noise quenching can be achieved in a ring laser via a spatially modulated active medium. A gyroscope based on such a correlated-emission laser has a potential sensitivity superseding the usual quantum limit.

In this Rapid Communication we propose and develop the theory for a new class of correlated-emission laser[1] (CEL) gyroscope[2] which is well suited to the precision measurement of ultrasmall (effective)[3] rotation rates. The envisioned system would have a potential sensitivity superseding the usual quantum limit[4] and would avoid the problem of "locking." Such a device would be ideal for experiments allowing long measurement times but not requiring a rapid response. Examples of problem areas which would profitably use this type of high-precision probe span the spectrum from general relativity and cosmology[5] to solid-state and atomic physics.[6]

The basis for the present work is the observation that spontaneous-emission (vacuum fluctuation) noise can be suppressed in the relative phase angle between the two counterpropagating modes of a ring laser.[7] In previous work[1] such considerations were developed with the laser gravity-wave detector in mind. To this end, it was shown that such spontaneous-emission noise quenching is achieved in the quantum-beat and Hanle-effect lasers as discussed in Ref. 1. In a later publication[8] it was shown that the sensitivity of a gravity-wave detector based on these lasers is potentially improved beyond the usual quantum limit. In the present paper these considerations are extended to include ring-laser interferometry.

Hence, a CEL scheme is developed which is particularly appropriate to the ring laser in which the two modes are characterized by different directions of propagation, but may have essentially the same frequency. It is shown that spontaneous emission noise in the relative phase angle between these two laser modes may be eliminated by preparing a gain medium which is modulated in space, as in Fig. 1(a). Such a modulation could be produced by, for example, positioning the lasing atoms or by selectively exciting thin "wafers" to yield a hologramlike[9] modulation in the active medium. Such a "holographic laser" provides another technique for producing a correlated-spontaneous-emission device which supplements the previous examples of quantum-beat and Hanle-effect lasers.[1]

Next, an idealized[10] laser gyro in which the atomic medium is frozen[11] (no thermal motion) is analyzed. The mirrors are characterized by a given reflectance[12] (no absorption) and problems[13] like $1/f$ (flicker-floor) noise are ignored. The thrust of the present analysis is to show that, in principle, a new class of high-precision ring-laser probe is possible. Clearly the extent to which these considerations will carry over to laboratory practice is an open question. However, the study of such simple models provides us with a deeper insight into the quantum limits of gyro operation and (hopefully) points the way to possible improvements in future generation systems.

Motivated by the above considerations, we next consider the problem of noise quenching in a CEL ring laser. The equation of motion for the density matrix $\rho(\hat{a}_1,\hat{a}_1^\dagger,\hat{a}_2,\hat{a}_2^\dagger,t)$ of the two modes of the ring resonator is given[14] in terms

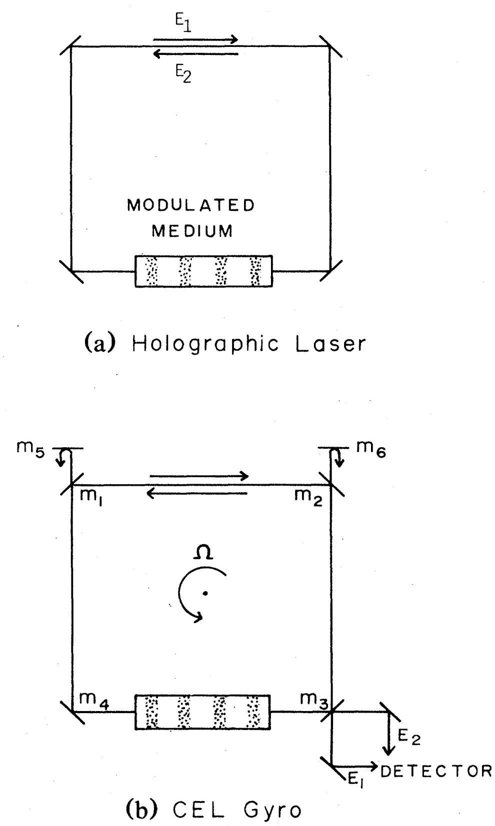

FIG. 1. (a) CEL ring laser in which correlation is produced by striated or modulated gain medium. (b) CEL laser gyroscope in which light is extracted from mirrors m_1 and m_2 and reinjected by mirrors m_5 and m_6. This leads to an enhanced gyroscope sensitivity.

of the annihilation (creation) operators $\hat{a}_j$ $(\hat{a}_j^\dagger)$, $j=1,2$, by

$$\dot{\rho}(\hat{a}_1,\hat{a}_1^\dagger,\hat{a}_2,\hat{a}_2^\dagger)=-i(\Omega_1-\nu_1)[\hat{a}_1^\dagger\hat{a}_1,\rho]-i(\Omega_2-\nu_2)[\hat{a}_2^\dagger\hat{a}_2,\rho]$$

$$-\frac{\alpha_0}{2}\int_{-l/2}^{l/2}n(x)[\hat{A}(x)\hat{A}^\dagger(x)\rho+\rho\hat{A}(x)\hat{A}^\dagger(x)-2\hat{A}^\dagger(x)\rho\hat{A}(x)]+\mathcal{L}_1\rho+\mathcal{L}_2\rho\ , \quad (1)$$

where the above the laser modes have "bare-cavity" eigenfrequencies Ω_1 and Ω_2, operating frequencies ν_1 and ν_2, α_0 represents the gain per atom, and $n(x)$ is the density of lasing atoms. The field operator $\hat{A}(x)$, written in terms of the resonator normal-mode function $u_j(x)$ is

$$\hat{A}(x)=\hat{a}_1\exp(-i\nu_1 t)u_1(x)+\hat{a}_2\exp(-i\nu_2 t)u_2(x)\ . \quad (2)$$

Finally, the jth mode loses energy at rate γ_j as described by the Liouville operator

$$\mathcal{L}_j\rho=-\tfrac{1}{2}\gamma_j(a_j^\dagger a_j\rho+\rho a_j^\dagger a_j-2a_j\rho a_j^\dagger)\ .$$

It is convenient to summarize the information contained in Eq. (1) via the quantum Langevin equation,[14] that is,

$$\dot{\hat{a}}_1=-i(\Omega_1-\nu_1)\hat{a}_1+\tfrac{1}{2}\alpha_1\hat{a}_1+\tfrac{1}{2}\alpha_{12}\hat{a}_2e^{-i\Phi}-\tfrac{1}{2}\gamma_1\hat{a}_1+\hat{F}_1, \quad (3a)$$

$$\dot{\hat{a}}_2=-i(\Omega_1-\nu_2)a_2+\tfrac{1}{2}\alpha_2a_2+\tfrac{1}{2}\alpha_{21}\hat{a}_1e^{i\Phi}-\tfrac{1}{2}\gamma_2\hat{a}_2+\hat{F}_2\ . \quad (3b)$$

The Langevin noise operators appearing in Eqs. (3a) and (3b) are defined by

$$\langle\hat{F}_i^\dagger(t)\hat{F}_j(t')\rangle=2D_{ij}\delta(t-t')\ , \quad (4)$$

where the diffusion coefficients are

$$D_{ij}=\frac{1}{4}\begin{pmatrix}\alpha_1+\alpha_1^* & (\alpha_{21}^*+\alpha_{12})e^{-i\Phi}\\ (\alpha_{12}+\alpha_{21}^*)e^{i\Phi} & \alpha_2+\alpha_2^*\end{pmatrix}\ . \quad (5)$$

The phase angle Φ is given by $(\nu_1-\nu_2)t$. For the present discussion we will consider the gain coefficients to be equal, $\alpha_1=\alpha_2=\alpha$.

The cross-coupling coefficients α_{12} and α_{21} depend upon the spatial distribution of the gain medium. For example, when the active medium is spread uniformly over the region $-l/2\le x\le l/2$ we find $\alpha_{12}=\alpha_{21}=0$. However, if we consider a modulated gain medium such that the active atomic medium is distributed in a sequence of thin wafers, having relative weights A_n, according to the expression

$$n(x)=\begin{cases}n_0\sum_n A_n\delta(x-n\lambda/2), & -\frac{l}{2}\le x\le\frac{l}{2}\ ,\\ 0, \text{ otherwise}\ ,\end{cases} \quad (6)$$

as in Fig. 1(a), we find that $\alpha_{12}=\alpha_{21}=\alpha$. That is, we have a strong correlation between modes 1 and 2. Note in particular that the cross-coupling diffusion coefficients D_{12} and D_{21} are nonvanishing in the case of the modulated gain medium but are zero when the lasing medium is uniformly pumped. As was shown in Ref. 1, finite D_{12} can lead to a quenching of spontaneous-emission fluctuations in the relative phase angle. We next consider the application of such a CEL to the laser gyroscope.

The basis for the laser gyroscope is, of course, the Sagnac frequency difference between the two counterpropagating modes. In notation of Ref. 1, we now have $\Omega_1-\Omega_2=S\Omega$, where Ω is the (effective) rotation rate and S is the gyroscope scale factor $(4A/\lambda p)$ in which A is the area enclosed by the ring, p is the ring perimeter, and λ is the reduced wavelength. Again we emphasize that the effective rotation rate may be due to many effects other than physical rotation, e.g., magnetic gravity, Fizeau-type effects, the Faraday effect, etc.

Consider next the uncertainty in the frequency determination of an ordinary laser due to spontaneous emission. This is given[4] by $\Delta\nu=\gamma\sqrt{\hbar\nu/Pt_m}$, where γ is the cavity decay rate, P is the laser power, t_m the measurement time, and $\hbar\nu$ the photon energy. Hence, the minimum detectable rotation rate is found, by equating the Sagnac frequency difference to the uncertainty in laser frequency,[15] to be

$$\Omega^{\min}\cong S^{-1}\gamma\left(\frac{\hbar\nu}{Pt_m}\right)^{1/2} \quad \text{(conventional laser gyroscope quantum limit)}\ . \quad (7)$$

Proceeding toward the CEL analog of Eq. (7) we note that the essential ingredients in conventional gyroscope operation are gain, loss, and mode coupling due to backscattering. The analysis of the ring-laser problem including the backscattering problem was first given by Arnowitz and Collins[16] and has been repeated and applied in many subsequent publications. Extending the CEL dynamics as given by Eqs. (3a) and (3b) to include the effects of controlled backscattering,[17] as in Fig. 1(b), and rewriting the equations of motion in terms of amplitude and phase variables defined by $a_i=\rho_i\exp(-i\theta_j)$, we have our working CEL-gyroscope equations

$$2\dot{\rho}_1=\alpha_1\rho_1+\alpha_{12}\rho_2\cos\psi-\gamma_1\rho_1+\gamma_{12}\rho_2\cos(\psi+\phi)\ , \quad (8a)$$

$$2\dot{\rho}_2=\alpha_2\rho_2+\alpha_{21}\rho_1\cos\psi-\gamma_2\rho_2+\gamma_{21}\rho_1\cos(\psi+\phi)\ , \quad (8b)$$

$$\dot{\psi}=\Omega_1-\Omega_2-\tfrac{1}{2}\left(\alpha_{12}\frac{\rho_2}{\rho_1}+\alpha_{12}\frac{\rho_1}{\rho_2}\right)\sin\psi$$

$$-\tfrac{1}{2}\left(\gamma_{12}\frac{\rho_2}{\rho_1}+\gamma_{21}\frac{\rho_1}{\rho_2}\right)\sin(\psi+\phi)+\mathcal{F}(t)\ , \quad (8c)$$

where the relative phase angle ψ is defined as

$$\psi=(\nu_1-\nu_2)t+\theta_1-\theta_2\ , \quad (9)$$

and the loss rate and backscatter cross-coupling rate [as indicated in Fig. 1(b)] are given by

$$\gamma_1=\gamma_2=\gamma=\frac{c}{p}(1-r^2)\ , \quad (10a)$$

$$\gamma_{12}=\gamma_{21}=\gamma_c=\frac{c}{p}t^2r_c\ . \quad (10b)$$

In the above c is the speed of light, p is the perimeter of the ring, while r and t are the field reflectance and the transmittance of mirrors 1 and 2, and r_c is the reflectance of mirrors 5 and 6 of Fig. 1(b). The extra phase ϕ accumulated in the backscattering depends on the position of the external mirrors 5 and 6. Finally, the noise source in Eq. (8c) is obtained from Eqs. (3a) and (3b) and is defined by

$$\langle \mathcal{F}_\psi^*(t)\mathcal{F}_\psi(t')\rangle = 2D(\psi)\delta(t-t') \ , \tag{11a}$$

where, in the physically interesting case that $\rho_1=\rho_2=\rho$, the phase diffusion rate is given by

$$D(\psi)=\frac{\alpha}{4\rho^2}(1-\cos\psi) \ . \tag{11b}$$

In this case and choosing $\phi=\pi$, our basic working equation (8c) becomes

$$\dot{\psi}=S\Omega-(\alpha-\gamma_c)\sin\psi+\mathcal{F}(\psi) \ . \tag{12}$$

We now divide ψ into its average value ψ_0 and fluctuations $\delta\psi$ about this value. Since $S\Omega \ll \alpha-\gamma_c$ we have $\psi \ll 1$, so that $\sin\psi \cong \psi$, and Eq. (12) yields the average phase

$$\psi_0=\frac{S\Omega}{\alpha-\gamma_c}, \text{ when } t_m \gtrsim (\alpha-\gamma_c)^{-1} \ , \tag{13}$$

where t_m is the measurement time. Note that when the backscattering mirrors 5 and 6 are removed, $\gamma_{12}=\gamma_c$ is zero. Hence, we see from Eq. (13) that the effect of controlled backscattering is to *increase* the size of the signal phase. This is in marked contrast to a conventional laser-gyroscope operation where backscattering is a nuisance.

Likewise, Eq. (12) implies that fluctuations $\delta\psi$ about ψ_0 are given by

$$\delta\psi=\int_0^t \exp[-(\alpha-\gamma_c)(t-t')]\mathcal{F}(t')dt' \ , \tag{14}$$

and the mean-square fluctuation about ψ_0 is then found from Eqs. (14) and (11b) to be

$$\langle \delta\psi^2\rangle \cong \frac{2\alpha/\bar{n}}{\alpha-\gamma_c}\{1-\exp[-(\alpha-\gamma_c)t]\}(1-\cos\psi_0) \ . \tag{15}$$

Inserting Eq. (13) into Eq. (15), noting that $\psi_0 \ll 1$, and taking $t=t_m > 1/(\alpha-\gamma_c)$, the rms fluctuations are found to be of order

$$\delta\psi \simeq \left(\frac{S\Omega}{\alpha-\gamma_c}\right)\left(\frac{\alpha}{\bar{n}(\alpha-\gamma_c)}\right)^{1/2} \ . \tag{16}$$

Hence, for small rotation rates (such that $S\Omega \ll \alpha-\gamma_c$) this spontaneous-emission enduced error (16) is negligible compared to "shot noise" which goes as $1/\bar{n}^{1/2}$.

As noted above, the limiting source of noise is now shot noise. The phase error is then $\delta\psi \sim 1/\bar{n}^{1/2}$, where $\bar{n}$ is the average photon number detected in time t_m, that is, $\bar{n}=P_d t_m/\hbar\nu$, where P_d is the laser power at the detector. Equating the shot-noise error $\delta\psi$ to the signal ψ_0 as given by Eq. (6) and solving for the minimum detectable rotation rate, we find

$$\Omega^{\min} \simeq \left(\frac{\alpha-\gamma_c}{\gamma}\right) S^{-1}\gamma\left(\frac{\hbar\nu}{P_d t_m}\right)^{1/2} \ . \tag{17}$$

Finally, we note that the power P_d is not the total emitted power P but only $(\gamma_d/\gamma)P$ since the detector port M_3 extracts only a fraction γ_d/γ of the total emitted power. Furthermore, $\alpha \cong \gamma_c+\gamma_d$, so that the prefactor $(\alpha-\gamma_c)/\gamma$ in Eq. (17) is just γ_d/γ, which taken together with the fact that $P_d=(\gamma_d/\gamma)P$, leads to our final result

$$\Omega^{\min} \cong \varepsilon S^{-1}\gamma\left(\frac{\hbar\nu}{P t_m}\right)^{1/2} \ , \tag{18}$$

where $\varepsilon = 1/\sqrt{\gamma t_m}$, where we have used Eq. (13) to write $t_m^{-1} \sim (\alpha-\gamma_c)=\gamma_d$.

Thus, for our idealized model, gyroscope sensitivity is improved by the factor ε. We emphasize that the present calculation ignores mirror losses due to absorption γ_a. When such losses are included in the simplest models, the factor ε is governed by $\gamma_a/(\gamma_a+\gamma)$. In conclusion, we see that (1) the CEL gyroscope has, in principle, a sensitivity superseding the conventional quantum limit, and (2) this device is not hampered by the usual "deal-band" locking problem associated with low rotation rates. The application of the present considerations to real-world experiments will be discussed elsewhere.

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[1]M. Scully, Phys. Rev. Lett. **55**, 2802 (1985).

[2]For an excellent introduction to the subject, see F. Arnowitz, in *Laser Application,* edited by M. Ross (Academic, New York, 1971), p. 172. A very complete account of the theory of ring-laser gyroscope operation is given in L. Menegozzi and W. Lamb, Phys. Rev. A **8**, 2103 (1973). A more recent review is given by W. Chow *et al.*, Rev. Mod. Phys. **57**, 61 (1985).

[3]A Faraday-active element in a ring cavity constitutes an example of an effective rotation rate since the counterpropagating waves have different velocities.

[4]The problem of spontaneous-emission noise in a ring laser is discussed by S. Ezekiel, SPIE J. **487**, 13 (1984); see also J. Gea-Banacloche, M. Scully, and D. Anderson, Opt. Commun. **57**, 67 (1986).

[5]The laser gyroscope as an experimental probe for magnetic gravity and preferred frame cosmological effects is discussed in M. Scully, M. Zubairy, and M. Haugen, Phys. Rev. A **24**, 2009 (1981). See also, R. O'Connell, Phys. Today **38**, No. 2, 104 (1985).

[6]The application of the laser gyroscope to parity-violation experiments is discussed by J. Elliot and J. Small, SPIE J. **487**, 128 (1984).

[7]In this case it is noted that two-level atoms will suffice, whereas three levels were necessary in Ref. 1.

[8]M. Scully and J. Gea-Banacloche, Phys. Rev. A **34**, 4043 (1986).

[9]When two waves interfere in a block of film, the silver atoms are often said to be deposited out along "planes" of constructive interference. Such regions of excited atoms could lead to the type of modulation which I have in mind. In addition to this type of holographic spatial interference, one can readily envision laying down thin films and separating these active thin-film regions by a precise number of optical wavelengths.

[10]It is important to emphasize that present generation laser gyroscopes are bounded by the spontaneous-emission quantum limit. Fluctuations in mirror positions cancel since the two counterpropagating modes share a common optical path.

[11]However, it should be noted that radiative decay of the atomic medium is allowed.

[12]The absorption and reflectance of modern high-quality laser-gyroscope mirrors is largely at the discretion of the experimenter. However, it should be noted that there will always be absorption losses and that these are not included in the present discussion. Future work will discuss this important extention of the present simplified model.

[13]The $1/f$ noise is associated with stochastic variations in the laser plasma, etc.

[14]The present Langevin equations are similar in form to those of Ref. 1. However, for a general discussion of quantum Langevian equations, see M. Lax, in *Statistical Physics, Phase Transitions, and Superfluidity,* edited by M. Chretien *et al.* (Gordon and Breach, New York, 1966), Vol. 2; H. Haken, *Handbuch der Physik* (Springer-Verlag, New York, 1970); M. Sargent, III, M. O. Scully, and W. E. Lamb, Jr., *Laser Physics* (Addision-Wesley, Reading, MA, 1974).

[15]We note that this laser gyroscope limit also applies to passive devices.

[16]F. Arnowitz and R. Collins, Appl. Phys. Lett. **9**, 55 (1966).

[17]A small amount of light tE_1 (tE_2) is transmitted by mirror m_1 (m_2) and is reflected by m_5 (m_6) so that an amount $t_c^2 r_1 E$ ($t_c^2 r_2 E$) reenters the ring cavity through m_1 (m_2). In this way we are taking a little light from E_1 (E_2), phase shifting it, and reinjecting it into E_2 (E_1). All of this takes place at a rate c/p. A detailed analysis supporting this heuristic argument has been carried out and will be published elsewhere.