Analytical treatment of free-electron laser in the long-pulse limit

G. Dattoli and A. Renieri
Comitato Nazionale per la Ricerca e per lo Sviluppo della Energia Nucleare, e delle Energie Alternative, Centro Ricerche Energia Frascati, CP-65, 00044 Frascati, Italy

A. Torre
Istituto Nazionale di Fisica Nucleare, Napoli, Italy and Dipartimento di Fisica, Universitá di Salerno, Salerno, Italy

Juan C. Gallardo
Quantum Institute, University of California, Santa Barbara, California 93106
(Received 26 September 1986)

We discuss a method to obtain analytical solutions of the evolution equation of the optical signal in long-pulse free-electron-laser oscillators in the low-gain and small-signal regime. Supermodes are identified with the eigenstates, harmonic-oscillator orthonormal functions, of a non-Hermitian Hamiltonian. Inhomogeneous broadening effects due to energy spread and emittance are included. The space-time characteristics of the laser field are obtained with a significant reduction in the computer time.

I. INTRODUCTION

The concept of supermode[1] (SM) has been introduced to treat the pulse propagation in a free-electron laser[2,3] (FEL) and to get around the rather complicated situation of dealing with a large number of coupled longitudinal modes in this inherently multimode problem.[4] The SM's can be qualitatively understood as those configurations of longitudinal modes which remain invariant in shape after each cavity round trip, although they can grow in amplitude and change the overall phase. They are rigorously defined as an overcomplete orthonormal set of eigenfunctions of a Volterra-type integro-differential equation.[1]

The use of SM's has greatly simplified the numerical difficulties involved in the analysis of the optical signal evolution in an FEL operating with rf accelerating devices[3,4] and they have been particularly useful in elucidating many features of the experimental results, i.e., the structure of the spectrum and the dependence of the small-signal gain on the cavity detuning.[1,5] More recently, SM's have been utilized successfully to analyze the data of the les Anneaux de Collisons de l'Accélérateur Linéaire d'Orsay FEL storage-ring experiment.[6,7]

Elleaume[7] has proposed an interesting method to find analytical solutions for the SM evolution in the long electron bunch limit. He has shown that the FEL pulse-propagation equation reduces to a Schrödinger-type equation with a non-Hermitian harmonic-oscillator Hamiltonian. As a consequence SM's have been identified with the harmonic-oscillator orthonormal functions.

In this paper we will analyze the same problem discussed in Ref. 7 using an algebraic approach that has the advantage of being quite general and allows us to study the dynamics of an FEL operating with a long electron bunch by means of the well-established methods of quantum optics, i.e., those employed in the analysis of the evolution of Glauber and Yuen two-photon states.[8]

Our starting point will be the integrodifferential equation defining SM's introduced in Ref. 1 which will be solved analytically in the long bunch limit. We also include in the analysis the effects of inhomogeneous broadening due to the energy spread and emittance of the electron beam.

II. GENERAL PROCEDURE

Before going into the technical details we will clarify the meaning of "long electron bunch limit." The e beam produced by a rf accelerator is characterized by a series of bunches with an rms longitudinal length σ_z. Due to the different speed of the radiation and electron pulses, the former will advance a net slippage distance $\Delta = N\lambda$ at the end of the undulator; N and λ are the number of periods of the undulator and the operating wavelength, respectively. The fundamental lengths of the problem, Δ and σ_z, define the dimensionless coupling parameter $\mu_c = \Delta/\sigma_z$, which is a measure of the relative slippage of the laser pulse with respect to the electron pulse, and also determine the number of longitudinal modes[3] coupled by the FEL interaction. The long electron bunch limit refers to a situation where $\mu_c \ll 1$.

The equation defining the space-time evolution of the optical electric field $E(z,t)$ for an FEL operating in the low-gain and small-signal regime with a bunched e beam is given by[1]

$$2T_c\frac{\partial E(z,t)}{\partial t}+\left\{\gamma_T+ig_0\Theta\left[\nu_0-\pi\left[\frac{\Delta\omega}{\omega}\right]_0^{-1}\right]\right\}E(z,t)+\Delta\Theta g_0\frac{\partial E(z,t)}{\partial z}$$
$$=-i\frac{g_0(2\pi)^{3/2}}{\mu_c\Delta^2}\int_0^{\Delta}d\eta\,\eta e^{i\nu_0\eta/\Delta}E(z+\eta,t)\int_{z+\eta}^{z+\Delta}dz'f(z')\ , \quad (1)$$

where

$$g_0=2\pi\frac{L_w\lambda}{\gamma\Sigma_L}\frac{I}{I_0}\frac{K^2}{(1+K^2)^{3/2}}\left[\frac{\Delta\omega}{\omega}\right]_0^{-2}$$

is the small signal coefficient, Σ_L is the laser cross section, $(\Delta\omega/\omega)_0\approx 1/2N_w$ is the homogeneous linewidth, $K=(e/mc^2)(B_w/k_w)$ is the undulator parameter, I_0 is the Alfvèn current, I the peak current, and finally ν_0 is the resonance parameter. We have also used γ_T to denote the cavity losses, $\Theta=2c\delta t/g_0\Delta$ the delay parameter, δt is the time difference between the electron pulse-pulse period and the cavity round-trip T_c, and $f(z)$ is the electron distribution.

The above equation is valid in a large time scale compared with the cavity round-trip period T_c; time t is $t=nT_c$ where n is an integer denoting the nth cavity round trip of the laser pulse. The left-hand side (lhs) of Eq. (1) accounts for the "free-propagation," while the right-hand side (rhs) describes the laser—electron-beam interaction and thus all the information connected with the slippage of the radiation pulse and its concurrent lethargy. Equation (1) translates mathematically the intuitive idea of the lethargic behavior, i.e., the laser pulse is pushed back toward the trailing edge of the e bunch by the FEL interaction resulting in a slowing-down oscillation period of the laser pulse inside of the optical cavity.

Analytical solutions of Eq. (1) can be found in two limiting cases corresponding to short and long e-bunch configurations.[2] In this note we will be interested in the latter case. The assumption is that the interaction is mainly centered around the maximum of the e-bunch distribution and therefore it is insensitive to the slippage and to the lethargy. Expanding the rhs of Eq. (1) up to the second order in Δ and assuming that the laser pulse is centered about the maximum of the electron-beam distribution, we obtain

$$\frac{\partial\tilde{E}(\zeta,\tau)}{\partial\tau}=\left\{G_1(\nu_0)\left[1+\frac{\mu_c^2}{8}\left[\frac{G_2(\nu_0)}{G_1(\nu_0)}\right]^2\right]-\frac{\gamma_T}{g_0}\right\}\tilde{E}(\zeta,\tau)$$
$$+\mu_c[G_3(\nu_0)-\Theta]\frac{\partial\tilde{E}(\zeta,\tau)}{\partial\zeta}+\frac{1}{2}\left\{\mu_c^2G_4(\nu_0)\frac{\partial^2}{\partial\zeta^2}-G_1(\nu_0)\zeta^2\right\}\tilde{E}(\zeta,\tau)\ , \quad (2)$$

where we have defined

$$E(\zeta,t)=\tilde{E}(\zeta,\tau)\exp\left\{-i\Theta\left[\nu_0-\pi\left[\frac{\Delta\omega}{\omega}\right]_0^{-1}\right]\tau\right\}\ ,$$
$$\zeta=\frac{z}{\sigma_z}+\tfrac{1}{2}\mu_c\frac{G_2(\nu_0)}{G_1(\nu_0)},\quad \tau=\frac{1}{2}\frac{g_0t}{T_c}\ ,$$

and

$$G_1(\nu_0)=-2\pi\frac{\partial}{\partial\nu_0}\left[1+i\frac{\partial}{\partial\nu_0}\right]\left[\frac{\sin(\frac{1}{2}\nu_0)}{\frac{1}{2}\nu_0}\exp(i\tfrac{1}{2}\nu_0)\right]\ ,$$
$$G_2(\nu_0)=\left[1-i\frac{\partial}{\partial\nu_0}\right]G_1(\nu_0),\quad G_3(\nu_0)=-i\frac{\partial}{\partial\nu_0}G_1(\nu_0),\quad G_4(\nu_0)=-\frac{\partial^2}{\partial\nu_0^2}G_1(\nu_0)\ . \quad (3)$$

The function $G_1(\nu_0)$ is the complex gain function while $G_{2,3,4}(\nu_0)$ are the higher-order corrections due to the finite length of the pulse.

Equation (2) has the same structure as that proposed by Elleaume for a linear undulator.[7] We rescale the coordinate ζ as

$$\xi=\left[\frac{G_1(\nu_0)}{\mu_c}\right]^{1/2}\{\zeta+[G_3(\nu_0)-\Theta]\}$$

and Eq. (2) becomes

$$\frac{\partial\tilde{E}(\xi,\tau)}{\partial\tau}=\hat{T}\tilde{E}(\xi,\tau)\ . \quad (4)$$

The operator $\hat{T}$ is of the form $\hat{T}=\delta\hat{I}+\Omega\hat{a}+\omega\hat{K}_0+\Omega_1(\hat{K}_++\hat{K}_-)$, i.e., it is a linear combination of the generators of the Heisenberg-Weyl group $\{\hat{a},\hat{a}^\dagger,\hat{I}\}$ and of the SU(1,1) group $\{\hat{K}_0,\hat{K}_+,\hat{K}_-\}$, namely,

$$\hat{a}=\frac{1}{\sqrt{2}}\left[\xi+\frac{\partial}{\partial\xi}\right],\quad \hat{a}^{\dagger}=\frac{1}{\sqrt{2}}\left[\xi-\frac{\partial}{\partial\xi}\right], \tag{5a}$$

($\hat{I}$ is the identity operator), with the commutation relation $[\hat{a}^{\dagger},\hat{a}]=\hat{I}$ and

$$\hat{K}_0=\tfrac{1}{4}(2\hat{a}^{\dagger}\hat{a}+1),\quad \hat{K}_+=\tfrac{1}{2}\hat{a}^{\dagger 2},\quad \hat{K}_-=\tfrac{1}{2}\hat{a}^2, \tag{5b}$$

which satisfy $[\hat{K}_0,\hat{K}_\pm]=\pm\hat{K}_\pm$, $[\hat{K}_+,\hat{K}_-]=-2\hat{K}_0$. The coefficients in the definition of the operator $\hat{T}$ are

$$\begin{aligned}
\delta&=G_1(\nu_0)\left[1+\frac{\mu_c^2}{8}\left[\frac{G_2(\nu_0)}{G_1(\nu_0)}\right]^2\right]\\
&\quad-\frac{\gamma_T}{g_0}-\frac{G_1(\nu_0)}{2}[G_3(\nu_0)-\Theta]^2,\\
\omega&=-\mu_c[1+G_1(\nu_0)G_4(\nu_0)],\\
\Omega&=\sqrt{2\mu_c G_1(\nu_0)}[G_3(\nu_0)-\Theta],\\
\Omega_1&=\frac{\mu_c}{2}[G_1(\nu_0)G_4(\nu_0)-1].
\end{aligned} \tag{6}$$

The problem of evaluating the space-time behavior of the optical field in an FEL in the long bunch limit has been formally reduced to that of studying the evolution of two-photon squeezed states. We can therefore apply the same techniques usually adopted to treat this kind of problem.[8,9] We expand $\tilde{E}(\xi,\tau)$ in terms of the harmonic-oscillator functions $u_n(\xi)=N_nH_n(\xi)e^{-(1/2)\xi^2}$,

$$\tilde{E}(\xi,\tau)=\sum_n C_n(\tau)u_n(\xi), \tag{7}$$

where H_n is the Hermite polynomial of order n. In writing Eq. (7) we have assumed that the laser field vanishes at a distance large compared with the dimensions of the electron pulse σ_z.

We use the algebraic methods of Refs. 8 and 9 to obtain the analytical expression for the time-dependent coefficients $C_n(\tau)$. The algebraic structure of the operator $\hat{T}$ provides the following expression of the coefficients $C_m(\tau)=\sum_n S_{m,n}C_n(\tau_0)$ where $C_n(\tau_0)$ are the initial conditions and the elements of the "scattering matrix" S are

$$\begin{aligned}
S_{m,n}(\tau,\tau_0)&=\exp[\delta(\tau-\tau_0)+\tfrac{1}{4}\omega(\tau-\tau_0)(2n+1)]\\
&\quad\times\mathscr{H}^{-(n+m+1)/2}\mathscr{F}\mathscr{S}_{m,n}.
\end{aligned} \tag{8}$$

Details of the derivation of Eq. (8) and the definition of the auxiliary functions $\mathscr{F},\mathscr{G},\mathscr{P},\mathscr{R}$ are presented in the Appendix. The matrix elements $\mathscr{S}_{nm}$ are

$$\mathscr{S}_{m,n}(\tau,\tau_0)=\begin{cases}
\sqrt{n!m!}\,\mathscr{P}^{n-m}\displaystyle\sum_{r=0}^{[m/2]}\frac{(-1)^r}{r!}\left[\frac{\mathscr{F}}{2}\right]^r\left[\frac{\mathscr{G}}{2}\right]^r\sum_{s=[m-n/2]}^{r}\left[\frac{s\mathscr{P}^2}{\mathscr{G}}\right]^s\frac{1}{(r-s)!}\frac{1}{(n-2r+2s)!}\\
\qquad\times L_m^{n-m+2s}(\mathscr{P}\mathscr{R})\quad\text{for } m\le n,\\
(-1)^{m-n}\left[\dfrac{n!}{m!}\right]^{1/2}\mathscr{R}^{(m-n)}\displaystyle\sum_{r=0}^{[n/2]}\frac{(-1)^r}{r!}\left[\frac{\mathscr{F}}{2}\right]^r\left[\frac{\mathscr{G}}{2}\right]^r\frac{1}{(n-2r)!}\\
\qquad\times\displaystyle\sum_{s=[(n-m)/2]}^{r}(-1)^s\frac{(n-2s)!}{(r-s)!}\left[\frac{2\mathscr{R}^2}{\mathscr{F}}\right]^2 L_{n-2s}^{m-n+2s}(\mathscr{P}\mathscr{R})\quad\text{for } m\ge n,
\end{cases} \tag{9}$$

where $L_n^m(\mathscr{P}\mathscr{R})$ is the associate Laguerre polynomial. The above result is the most general solution to the problem of the FEL pulse propagation in the limit of long electron pulse.

In first order in μ_c and neglecting intermode coupling, the scattering matrix becomes diagonal $S_{m,n}(\tau,\tau_0)\approx e^{\omega_n(\tau-\tau_0)}\delta_{m,n}$ where ω_n, the growth-rate eigenfrequency, is

$$\omega_n(\Theta,\mu_c)=\delta+(2n+1)\omega+2\frac{\Omega_1\Omega^2}{Q^2}. \tag{10}$$

Consequently, the optical electric field reads

$$E(\zeta,\tau)=\sum_{n=0}^{\infty}C_n(\tau_0)e^{\omega_n(\tau-\tau_0)}u_n(\zeta). \tag{11}$$

This is a superposition of "modes" with the property that the longitudinal profile shape remains unchanged during the interaction and with their growth rate determined by Eq. (10). These "modes" can be identified with the SM introduced in Ref. 1; in particular, Eq. (11) coincides with the results found by Elleaume in Ref. 7.

The first two terms of Eq. (10) describe the free-field energy with the usual level spacing of a harmonic oscillator; the last term is due to the interaction part of the "Hamiltonian." Equation (10) describes both the dispersive and absorptive behavior of SM's and their degeneracy is removed by the μ_c parameter. Furthermore, the SM gain exhibits a quadratic dependence on Θ that qualitatively reproduces the gain-cavity detuning plots of Refs. 1 and 5 around the maximum of the gain function. We stress that the result in Eq. (10) is in agreement, for the first supermode, with the one obtained in Ref. 5, where the threshold regime of a storage-ring FEL was analyzed. In Fig. 1 we plot the gain of the SM's for several values of μ_c.

We can apply this result to the analysis of single-passage experiments characterized by $\mu_c\ll 1$ such as those by Newnam *et al.*[10] and Bizarri *et al.*[11] The problems with these kinds of FEL devices arise only in con-

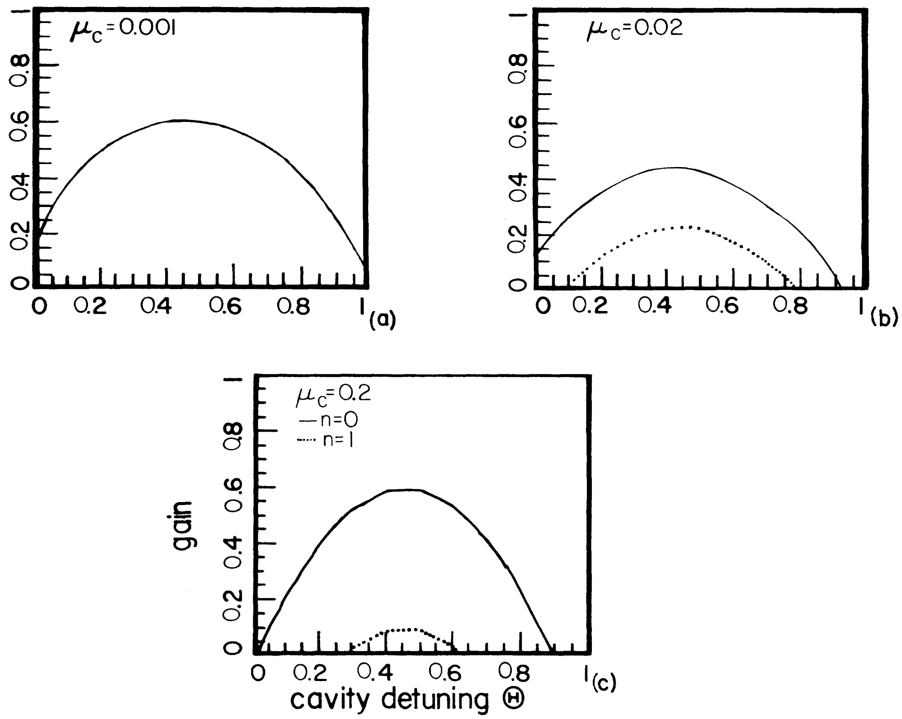

FIG. 1. Gain of a helical FEL vs the cavity detuning parameter Θ for different coupling parameters μ_c. (a) $g_0=1$, $Q=0.24$, $\mu_\epsilon=0$; (b) $g_0=1$, $Q=0.24$, $\mu_\epsilon=0.1$; (c) $g_0=0.8$, $Q=0.01$, $\mu_\epsilon=0$.

nection with the inhomogeneous broadening induced by the energy spread and emittance. The inclusion of these effects in the above formalism is almost straightforward; assuming indeed an e beam with a Gaussian energy distribution and a Lorentzian emittance distribution, we redefine $G_1(\nu_0)$ according to

$$G_1(\nu_0,\mu_\epsilon,\mu_x,\mu_y)=-2\pi\frac{\partial}{\partial\nu_0}\left[1+i\frac{\partial}{\partial\nu_0}\right]\times\int_0^1 d\xi\,\frac{e^{i\nu_0\xi-(1/2)\mu_\epsilon^2\xi^2}}{(1+i\mu_x\xi)(1+i\mu_y\xi)}\,, \tag{12}$$

where $\mu_{\epsilon,x,y}$ are the inhomogeneous broadening parameters due to the energy spread and emittance, respectively;[3] we define

$$\mu_\epsilon=\frac{2\sigma_\epsilon\pi}{\left[\dfrac{\Delta\omega}{\omega}\right]_0}$$

where σ_ϵ is the rms energy spread and $\mu_{x,y}$ is given by

$$\mu_{x,y}=\frac{\sqrt{2\,|\,h_{x,y}\,|}\,\dfrac{\pi K}{(1+K^2)}\,\dfrac{\gamma\epsilon_{x,y}}{\lambda_w}}{\left[\dfrac{\Delta\omega}{\omega}\right]_0}\,,$$

where λ_w is the period of the wiggler, $h_{x,y}$ are the sextupolar terms of the wiggler and $\epsilon_{x,y}$ are the x and y emittance of the electron beam.

The formalism we have discussed in this work is sufficiently general to be easily extended to different FEL configurations. In particular, Eq. (2) applies to single-pass electron-beam sources, i.e., linear, induction, microtron, electrostatic accelerators; however, including the self-consistent degradation of the electron beam, we could also describe a storage-ring FEL. A tapered undulator FEL could also be treated with this formalism; the results remain almost unchanged, only the G functions defined in Eq. (3) have to be modified. Finally, Eq. (2) cannot describe the start-up of the laser since it has been derived omitting the spontaneous emission contribution; however, including a source term in Eq. (1) it can be used to study the start-up phenomena of the free-electron laser. Some of these problems will be the subject of a forthcoming paper.

ACKNOWLEDGMENTS

This work was partially supported by a travel grant R.G. 85/0366 extended to us by the NATO (North Atlantic Treaty Organization) Collaborative Research Program. (J.C.G.) also acknowledges support from ONR (Office of Naval Research) under Contract No. N00014-86-K-0692.

APPENDIX

A solution of Eq. (4) can be obtained defining a nonunitary evolution operator

$$\tilde{E}(\xi,\tau)=\hat{U}(\tau,\tau_0)\tilde{E}(\xi,\tau_0) \tag{A1}$$

that satisfies the equation

$$\frac{\partial\hat{U}(\tau,\tau_0)}{\partial\tau}=\hat{T}\hat{U}(\tau,\tau_0) . \tag{A2}$$

The non-Hermicity of the Hamiltonian $\hat{T}$ and the consequent nonunitarity of $\hat{U}$ is due to the gain (absorption) process described by Eq. (4).

An exact solution of the operators equation (A2) can be obtained noticing that $\hat{T}$ is a linear combination of the generators of both SU(1,1) and the Weyl group. Using the Wei-Norman method, we can write the evolution operator $\hat{U}$ in the following ordered form:

$$\hat{U}(\tau,\tau_0)=e^{\delta(\tau-\tau_0)+s(\tau)}e^{2[h(\tau)+(1/2)\omega(\tau-\tau_0)]\hat{K}_0} \times e^{g(\tau)\hat{K}_+}e^{-f(\tau)\hat{K}_-}e^{p(\tau)\hat{a}^+}e^{-r(\tau)\hat{a}} . \tag{A3}$$

The various functions appearing in the exponents must satisfy the system of differential equations:

$$\begin{aligned}\frac{dh(\tau)}{d\tau}&=-g(\tau)\frac{df(\tau)}{d\tau} ,\\ \frac{dg(\tau)}{d\tau}&=-\Omega_1 e^{-2h(\tau)+\omega(\tau-\tau_0)}-g(\tau)\frac{dh(\tau)}{d\tau} ,\\ \frac{df(\tau)}{d\tau}&=\Omega_1 e^{2h(\tau)-\omega(\tau-\tau_0)} ,\end{aligned} \tag{A4}$$

with the initial conditions $h(\tau_0)=g(\tau_0)=f(\tau_0)=0$. We also define

$$\begin{aligned}p(\tau)&=\Omega\int_{\tau_0}^{\tau}d\tau' g(\tau')e^{h(\tau')} ,\\ r(\tau)&=-\Omega\int_{\tau_0}^{\tau}d\tau'[1+f(\tau')g(\tau')]e^{h(\tau')} ,\\ s(\tau)&=-\int_{\tau_0}^{\tau}d\tau' p(\tau')\frac{dr(\tau')}{d\tau'} .\end{aligned} \tag{A5}$$

Using the harmonic-oscillator eigenstates, we can compute the evolution operator matrix elements $S_{m,n}=\langle m\mid\hat{U}\mid n\rangle$ which is given in Eq. (8) in the main text. The explicit forms of the functions entering in this equation are obtained solving Eq. (A4) in combination with Eq. (A5); we obtain

$$\begin{aligned}h(\tau)&=-\tfrac{1}{2}\omega(\tau-\tau_0)-\ln[\alpha(\tau)] ,\\ g(\tau)&=2\frac{\Omega_1}{Q}\alpha(\tau)\sinh[\tfrac{1}{2}Q(\tau-\tau_0)] ,\\ f(\tau)&=-2\frac{\Omega_1}{Q}[\alpha(\tau)]^{-1}\sinh[\tfrac{1}{2}Q(\tau-\tau_0)] ,\\ \alpha(\tau)&=\cosh[\tfrac{1}{2}Q(\tau-\tau_0)]-\frac{\omega}{Q}\sinh[\tfrac{1}{2}Q(\tau-\tau_0)] ,\end{aligned}$$

with

$$Q=(\omega^2-4\Omega_1^2)^{1/2}\equiv 2\mu_c[G_1(\nu_0)G_4(\nu_0)]^{1/2} .$$

The auxiliary functions $p(\tau),r(\tau),s(\tau)$ are given by

$$\begin{aligned}p(\tau)&=\tfrac{1}{2}\Omega_1\Omega\left[\frac{\sinh[\frac{1}{4}Q(\tau-\tau_0)]}{\frac{1}{4}Q}\right]^2 ,\\ r(\tau)&=-\Omega\frac{\sinh[\frac{1}{4}Q(\tau-\tau_0)]}{\frac{1}{4}Q}\left[\cosh[\tfrac{1}{4}Q(\tau-\tau_0)]+\frac{\omega}{Q}\frac{\sinh[\frac{1}{4}Q(\tau-\tau_0)]}{\frac{1}{4}Q}\right] ,\\ s(\tau)&=2\Omega_1\left[\frac{\Omega}{Q}\right]^2+4\Omega_1\frac{\Omega^2}{Q^3}\sinh[\tfrac{1}{2}Q(\tau-\tau_0)](\cosh[\tfrac{1}{2}Q(\tau-\tau_0)]+\frac{\omega}{Q}\sinh[\tfrac{1}{2}Q(\tau-\tau_0)])\\ &\quad-4\sinh[\tfrac{1}{4}Q(\tau-\tau_0)](\cosh[\tfrac{1}{4}Q(\tau-\tau_0)]+\frac{\omega}{Q}\sinh[\tfrac{1}{4}Q(\tau-\tau_0)]) .\end{aligned} \tag{A6}$$

Finally, for convenience we have also introduced the functions

$$\begin{aligned}&\mathscr{I}(\tau,\tau_0)=e^{s(\tau)},\ \mathscr{H}(\tau,\tau_0)=e^{-h(\tau)},\ \mathscr{F}(\tau,\tau_0)=f(\tau)e^{-h(\tau)},\ \mathscr{G}(\tau,\tau_0)=g(\tau)e^{h(\tau)} ,\\ &\mathscr{P}(\tau,\tau_0)=\mathscr{H}^{-1/2},\ \mathscr{R}(\tau,\tau_0)=r(\tau)\mathscr{H}^{1/2} .\end{aligned} \tag{A7}$$

[1]G. Dattoli, A. Marino, and A. Renieri, Opt. Commun. **35**, 407 (1980); G. Dattoli, A. Marino, A. Renieri, and F. Romanelli, IEEE J. Quantum Electron. **QE-17**, 1371 (1981).

[2]W. B. Colson and A. Renieri, J. Phys. (Paris) **44C**, 11 (1983); J. Goldstein, Proc. Soc. Photo-Opt. Instrum. Eng. **453**, 2 (1984); M. N. Rosenbluth, H. V. Wong, and B. N. Moore, *ibid.* **453**, 25 (1984).

[3]G. Dattoli and A. Renieri, *Experimental and Theoretical Aspects of the Free Electron Laser*, Vol. IV of *Laser Handbook*, edited by M. L. Stitch and M. S. Bass (North-Holland, New York, 1985), p. 1.

[4]G. Dattoli and A. Renieri, Nuovo Cimento B **61**, 153 (1981); G. Dattoli, J. Gallardo, and A. Torre, Phys. Rev. A **31**, 3755 (1985).

[5]G. Dattoli and A. Renieri, Nuovo Cimento B **59**, 1 (1980).

[6]P. Elleaume, J. M. Ortega, M. Billardon, C. Bazin, M. Bergher, M. Velghe, Y. Petroff, D. A. G. Deacon, K. E. Robinson, and J. M. J. Madey, J. Phys. (Paris) **45**, 989 (1984).

[7]P. Elleaume, IEEE J. Quantum Electron. **QE-21**, 1012 (1985).

[8]F. Ciocci, G. Dattoli, A. Renieri, and A. Torre, Phys. Rep. **141**, 1 (1986).

[9]G. Dattoli, A. Torre, and R. Caloi, Phys. Rev. A **33**, 2789 (1986); B. L. Schumaker and C. M. Caves, Phys. Rev. **31**, 3093 (1985).

[10]B. E. Newnam *et al.*, J. Opt. Soc. Am. **B1**, 505 (1985).

[11]U. Bizarri *et al.*, Nucl. Instrum. Methods **A250**, 254 (1986).