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Squeezed-light generation in a medium governed by the nonlinear Schrödinger equation

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A first-order perturbation calculation is performed on a generalized nonlinear Schrödinger equation in order to investigate the degree of squeezing that can be obtained for a medium described by such a field theory. Expressions for the amount of squeezing in the normal and anomalous dispersion regimes are obtained. In particular, it is found that the amount of squeezing is greater in the anomalous dispersion region than in the normal dispersion region. By keeping fourth-order derivatives in the field, the squeezing when the pump is located at the zero-dispersion wavelength is also investigated.

Squeezed light, in which the quantum noise in one amplitude component is reduced below the vacuum noise level, has been generated using parametric processes in optical materials,[1-4] including optical fibers. The theory of squeezed-state generation in optical fibers has been discussed by Levenson *et al.*[5] and Shelby *et al.*,[6] in which the frequency dependence of the index of refraction was ignored. Here we study the effects of frequency-dependent dispersion in order to determine the bandwidth over which squeezing can occur. The approach taken is phenomenological, starting with the nonlinear Schrödinger equation which is often used to model light propagating through optical fibers.[7,8] When such a dispersive, nonlinear medium with a third-order susceptibility is pumped with a continuous-wave (cw) beam, squeezing results. We find that the degree of squeezing oscillates as a function of fiber length in the normal dispersion regime, whereas in the anomalous dispersion regime, it grows exponentially with fiber length. Similar results have been found by Drummond.[9]

By restricting ourselves to the nonlinear Schrödinger equation we neglect losses and Brillouin scattering processes which degrade the amount of squeezing that can be generated with real fibers. Shelby *et al.*[2] have, however, devised techniques for overcoming these problems. We also investigate the squeezing when the pump beam falls at a wavelength such that first-order dispersion is zero (ZDWL). Keeping higher-order dispersive terms, we obtain the somewhat surprising result that the squeezing is not affected by odd-order dispersive terms, but small values of even-order dispersion can significantly increase the frequency range of the squeezed light.

Our starting point is the generalized equation

$$i\frac{\partial A}{\partial z}+\beta\left(i\frac{\partial}{\partial \tau}\right)A+\frac{n_2\omega_0}{c}A^\dagger A^2=0 \ , \tag{1}$$

where $A(z,\tau)$ describes the evolution of the amplitude components of the electric field $E(z,\tau)$,

$$E(z,\tau)=A(z,\tau)e^{-i\omega_0\tau}+A^\dagger(z,\tau)e^{i\omega_0\tau} \ . \tag{2}$$

Here τ is related to the time t and position z along the medium via $\tau=t-\beta^{(1)}z$ ($\beta^{(1)}$ is the inverse of the group velocity), ω_0 is an optical carrier frequency, n_2 is the nonlinear index of refraction, and c is the velocity of light. The dispersion constants are given by the Taylor series expansion

$$\beta\left(i\frac{\partial}{\partial \tau}\right)=\sum_{n=2}^{\infty}\frac{\beta^{(n)}}{n!}\left(i\frac{\partial}{\partial\tau}\right)^n \ , \tag{3}$$

where $\beta^{(n)}=\partial^{(n)}\beta/\partial\omega^{(n)}$ evaluated at ω_0.

To a good approximation[10-13] the field amplitude operator for the light incident on the optical fiber can be written in the form

$$A(0,\tau)=\varepsilon_0\int d\omega\, b(0,\omega)e^{-i\omega\tau} \ , \tag{4}$$

where the $b(0,\omega)$ satisfy the usual boson commutation relations

$$[b(0,\omega),b^\dagger(0,\omega')]=\delta(\omega-\omega') \ ,$$
$$[b(0,\omega),b(0,\omega')]=0 \ , \tag{5}$$

and ε_0 converts the right-hand side to electric field units.

Equation (1) has the classical steady-state solution

$$A(z)=A_0e^{i\gamma z} \ , \tag{6}$$

where

$$\gamma=\frac{n_2\omega_0}{c}|A_0|^2 \ . \tag{7}$$

In the undepleted pump approximation, the field amplitude operator has the form

$$A(z,\tau)=A_0e^{i\gamma z}+a(z,\tau) \ , \tag{8}$$

where the quantum noise is

$$a(z,\tau)=\varepsilon_0\int d\omega\, b(z,\omega)e^{-i\omega\tau} \tag{9}$$

and is assumed to be small relative to the pump amplitude $|A_0|$.

Substituting Eq. (8) into Eq. (1) we obtain the linearized equation

$$i\frac{\partial a}{\partial z}+\beta\left(i\frac{\partial}{\partial\tau}\right)a+2\gamma a+\gamma e^{i2(\theta+\gamma z)}a^\dagger=0 \ , \tag{10}$$

where the pump phase θ is defined by $A_0 = |A_0| e^{i\theta}$. In the frequency domain this becomes

$$i\frac{\partial b(z,\omega)}{\partial z} + \beta(\omega) b(z,\omega) + 2\gamma b(z,\omega) + \gamma e^{i2(\theta+\gamma z)} b^\dagger(z,-\omega) = 0 \ , \tag{11}$$

or introducing

$$c(z,\omega) = e^{-i(\theta+\gamma z)} b(z,\omega) \ . \tag{12}$$

Equation (11) and its Hermitian conjugate can be written together in matrix form as

$$i\frac{\partial}{\partial z}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} c(z,\omega) \\ c^\dagger(z,-\omega) \end{bmatrix} + \begin{bmatrix} \beta(\omega)+\gamma & \gamma \\ \gamma & \beta(-\omega)+\gamma \end{bmatrix}\begin{bmatrix} c(z,\omega) \\ c^\dagger(z,-\omega) \end{bmatrix} = 0 \ . \tag{13}$$

We look for solutions of the form

$$\begin{bmatrix} c(z,\omega) \\ c^\dagger(z,-\omega) \end{bmatrix} = e^{i\lambda z}\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} \ . \tag{14}$$

The eigenvalues λ are obtained from the characteristic equation

$$\det\begin{vmatrix} -\lambda+\beta(\omega)+\gamma & \gamma \\ \gamma & \lambda+\beta(-\omega)+\gamma \end{vmatrix} = 0 \ , \tag{15}$$

and have the form

$$\lambda_\pm = \beta_a(\omega) \pm \sqrt{\beta_s^2(\omega) + 2\gamma\beta_s(\omega)} \quad \text{if } \beta_s^2(\omega)+2\gamma\beta_s(\omega) \geq 0 \ , \tag{16}$$

or

$$\lambda_\pm = \beta_a(\omega) \pm i\sqrt{|\beta_s^2(\omega) + 2\gamma\beta_s(\omega)|} \quad \text{if } \beta_s^2(\omega)+2\gamma\beta_s(\omega) \leq 0 \ , \tag{17}$$

where

$$\beta_s(\omega) = \frac{\beta(\omega)+\beta(-\omega)}{2} \ , \tag{18}$$

$$\beta_a(\omega) = \frac{\beta(\omega)-\beta(-\omega)}{2} \ . \tag{19}$$

The calculations will now be carried through for the case given by Eq. (16). The corresponding calculations for the case starting with Eq. (17) follow similarly but only the results will be given.

Introducing

$$B(\omega) = \sqrt{|\beta_s^2(\omega) + 2\gamma\beta_s(\omega)|} \ , \tag{20}$$

the solution of Eq. (13), given the initial condition, $c(0,\omega)$ is

$$c(z,\omega) = e^{i\beta_a(\omega)}\left\{\cos[B(\omega)z] + \frac{i[\beta_s(\omega)+\gamma]}{B(\omega)}\sin[B(\omega)z]\right\} c(0,\omega) + i\frac{e^{i\beta_a(\omega)}\gamma}{B(\omega)}\sin[B(\omega)z]c^\dagger(0,-\omega) \ . \tag{21}$$

So, using Eq. (12) one sees that the boson creation and annihilation operators evolve according to

$$b(z,\omega) = \mu(z,\omega) b(0,\omega) + \nu(z,\omega) b^\dagger(0,-\omega) \ , \tag{22}$$

where

$$\mu(z,\omega) = e^{i[\beta_a(\omega)+\gamma z]}\left\{\cos[B(\omega)z] + \frac{i[\beta_s(\omega)+\gamma]}{B(\omega)} \times \sin[B(\omega)z]\right\} \ , \tag{23}$$

$$\nu(z,\omega) = i\frac{e^{i[\beta_a(\omega)+\gamma z+2\theta]}}{B(\omega)}\gamma\sin[B(\omega)z] \ . \tag{24}$$

It is useful to note that since $\beta_s(\omega)$ and $B(\omega)$ are even functions of ω while $\beta_a(\omega)$ is odd, we have the following symmetries (these symmetries are, in fact, required by unitarity):[11-13]

$$|\mu(\omega)|^2 = |\mu(-\omega)|^2 \ , \quad |\nu(\omega)|^2 = |\nu(-\omega)|^2 \ , \tag{25}$$

$$\mu(z,\omega)\nu(z,-\omega) = \mu(z,-\omega)\nu(z,\omega) \ . \tag{26}$$

The squeezing or noise reduction in one amplitude component of an electromagnetic field can be detected with a homodyne detector[13-16] in which an intense local oscillator light at the optical carrier frequency ω_0 is made to interfere with the signal light on the surface of a photodetector. In the case under consideration the intense carrier or pump beam with amplitude A_0 can serve as the local oscillator, provided a method, such as used by Shelby *et al.*,[2] is devised for shifting the carrier phase relative to quantum noise $a(z,\tau)$.

Generally, a unit quantum efficiency homodyne detector[13-16] measures the operator

$$I = \frac{1}{\sqrt{2}}\int_{-\Delta\omega}^{\Delta\omega} d\omega [b(z,\omega)e^{i\phi}e^{-i\omega\tau} + b^\dagger(z,\omega)e^{-i\phi}e^{i\omega\tau}] \ , \tag{27}$$

where ϕ is the local oscillator phase.

We will now evaluate the expectation value of I for the case when at frequencies other then the carrier frequency only vacuum fluctuations enter the fiber, that is, the state vector $|0\rangle$, is defined by

$$b(0,\omega)|0\rangle = 0 \text{ for all } \omega \ . \tag{28}$$

Further, $|0\rangle$ is normalized such that

$$\langle 0|0\rangle = 1 \ . \tag{29}$$

From Eqs. (22) and (27) it is evident that I is linear in $b(0,\omega)$ and $b^\dagger(0,\omega)$. Hence,

$$\langle 0|I|0\rangle = 0 \ , \tag{30}$$

that is, the quantum noise has a zero mean value. The second moment is

$$\langle 0|I^2|0\rangle=\int_0^{\Delta\omega} d\omega\{|\mu(\omega)|^2+|\nu(\omega)|^2 +2\,\mathrm{Re}[\mu(z,\omega)\nu(z,-\omega)e^{i2\phi}]\} . \quad (31)$$

The power spectrum of the quantum noise is thus[13]

$$S(\omega)=|\mu(\omega)|^2+|\nu(\omega)|^2 +2\,\mathrm{Re}[\mu(z,\omega)\nu(z,-\omega)]e^{i2\phi} . \quad (32)$$

Squeezing occurs at frequency ω if there is a local oscillator phase ϕ for which

$$S(\omega)<1 .$$

The noise power at frequency ω is maximized or minimized by choosing the local oscillator phase so that

$$\mathrm{Re}[\mu(z,\omega)\nu(z,-\omega)e^{i2\phi}]=\pm|\mu(\omega)||\nu(\omega)|, \quad (33)$$

that is,

$$S_{\max,\min}(\omega)=[|\mu(\omega)|\pm|\nu(\omega)|]^2 . \quad (34)$$

For the case of normal dispersion $[\beta_s^2(\omega)+2\gamma\beta_s(\omega)\geq 0]$, $|\mu(\omega)|$ and $|\nu(\omega)|$ are obtained, from Eqs. (23) and (21), as

$$|\mu(z,\omega)|=\left[1+\frac{\gamma^2}{B^2(\omega)}\sin^2[B(\omega)z]\right]^{1/2} ,$$
$$|\nu(z,\omega)|=\frac{\gamma}{B(\omega)}|\sin[B(\omega)z]| . \quad (35)$$

For the case of anomalous dispersion $[\beta_s^2(\omega)+2\gamma\beta_s(\omega)\leq 0]$, $|\mu(\omega)|$ and $|\nu(\omega)|$ are given by

$$|\mu(\omega)|=\left[1+\frac{\gamma^2}{B^2(\omega)}\sinh^2[B(\omega)z]\right]^{1/2} ,$$
$$|\nu(\omega)|=\frac{\gamma}{B(\omega)}\sinh[B(\omega)z] . \quad (36)$$

From Eqs. (34)–(36) it is evident that $S_{\max,\min}(\omega)$ depends only on $\beta_s(\omega)$, that is, only even order terms in the dispersion contribute to the squeezing. These expressions are general. However, we now apply them to a specific example in optical fiber communications. We consider a conventional optical fiber,[17] in which the ZDWL ($\beta^{(2)}=0$) occurs at 1.3 μm. We consider three regions: (a) large anomalous dispersion dominated by $\beta^{(2)}<0$($\sim$1.5 μm), (b) large normal dispersion dominated by $\beta^{(2)}>0$($\sim$1 μm), and (c) near the ZDWL ($\sim$1.3 μm), where $\beta^{(4)}$ contributes to the squeezing. From published[17] results, we will calculate $\beta^{(3)}$ and $\beta^{(4)}$. At 1.5 μm, $\beta^{(2)}=-20$ ps^2/km, and for illustration purposes we use $\gamma=3$ corresponding to 200 mW of pump power and a propagation distance $z=0.5$ km. The numerical computation for $S_{\min}$ versus frequency is given in Fig. 1. The solid line corresponds to the squeezing for the anomalous region. At $\omega=0$ the noise reduction is about 90%, and varies as γz. The dashed line corresponds to the squeezing occurring in the normal region at 1 μm, where $\beta^{(2)}=20$ ps^2/km. All other parameters are the same as before. The features of the squeezing in the normal dispersion region are qualitatively similar to those of the anomalous region except that the frequency range in the normal case is

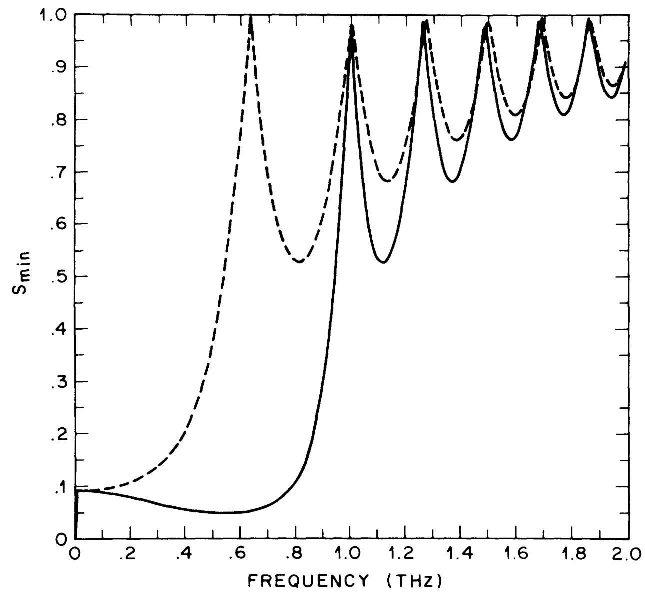

FIG. 1. Solid (dashed) line is the squeezing as a function of frequency for anomalous (normal) dispersion.

less that of the anomalous region. In the normal region, the squeezing reaches 85% at $\sim$0.3 THz, whereas in the anomalous region this value is extended out to $\sim$0.8 THz, which is nearly three times the squeezing bandwidth obtained at 1.0 μm. This results from the nonlinear and dispersive interaction. The nonlinearity broadens the spectrum through self-phase modulation (SPM).[18] Squeezing results from the interaction of a coherent state with its conjugate. In the normal region, the dispersion acts to separate the frequency components created by SPM; whereas the opposite occurs in the anomalous region resulting in greater squeezing bandwidth. Details will be presented in a later paper.[19] When a pulsed pump beam is used this interaction of nonlinearity and anomalous dispersion can give rise to solitons.[7]

We now examine squeezing near the ZDWL (1.3 μm). This region's significance arises from its proposed use in future optical communication systems. Our analysis shows that $\beta^{(3)}$ will not contribute to squeezing; therefore we examine $\beta^{(4)}$. The effects of other perturbations[20] will be presented elsewhere.[21] A value of $\beta^{(4)}=-2\times 10^{-4}$ ps^4/km was extrapolated from delay measurements.[17] Initially we choose a pump beam exactly located at the ZDWL ($\beta^{(2)}=0$). Figure 2 shows the squeezing at or near the ZDWL in which all other parameters are the same as in Fig. 1. The solid line corresponds to the exact ZDWL case. We find that the spectrum of the squeezing is greatly enhanced with respect to Fig. 1. The 85% squeezing value occurs at $\sim$30 Thz, nearly 40 times that of the anomalous region shown in Fig. 1. The $\beta^{(4)}$ term brings in the fourth power of the frequency, whereas $\beta^{(2)}$ requires only the second power of the frequency. Thus, even small values of $\beta^{(4)}$ can significantly influence the squeezing. However, in practice it is very difficult to have the pump beam exactly at the ZDWL; therefore we examine a region about the ZDWL and choose realistic values

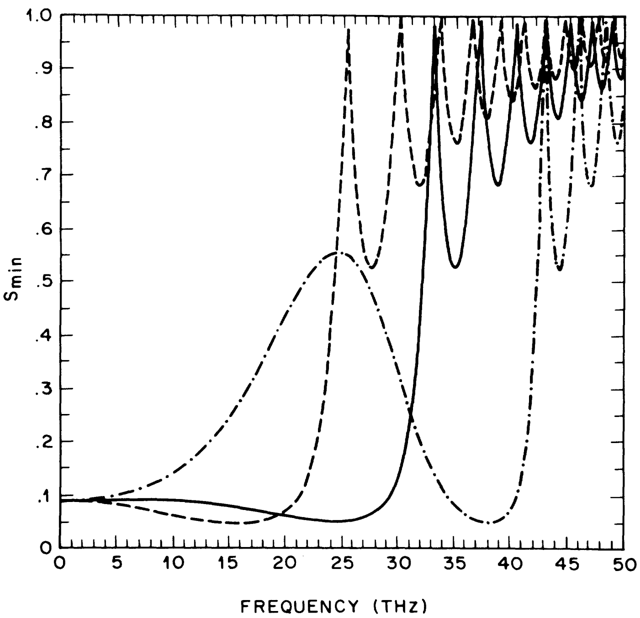

FIG. 2. Squeezing as a function of frequency. Solid line: $\beta^{(2)}=0$, $\beta^{(4)}=-2\times10^{-4}$ ps^4/km. Dashed line: $\beta^{(2)}=-0.02$ ps^2/km, $\beta^{(4)}=-2\times10^{-4}$ ps^4/km. Dot-dashed line: $\beta^{(2)}=+0.02$ ps^2/km, $\beta^{(4)}=-2\times10^{-4}$ ps^4/km.

of $\beta^{(2)}=\pm0.02$ ps^2/km. The dashed line in Fig. 2 corresponds to $\beta^{(2)}=-0.02$ ps^2/km and $\beta^{(4)}=-0.0002$ ps^4/km. As can be seen from Eq. (20), $\beta^{(2)}$ dominates the squeezing, yet $\beta^{(4)}$ plays a significant role. In Fig. 2 the qualitative features are similar to those in Fig. 1. However, the 85% squeezing occurs at $\sim$22 THz less than the $\sim$30 THz for $\beta^{(4)}$ alone because the effects of $\beta^{(4)}$ have tempered those of $\beta^{(2)}$. The dotted line corresponds to $\beta^{(2)}=0.02$ ps^2/km and $\beta^{(4)}=-0.0002$ ps^4/km. The qualitative shape of the squeezing is strikingly different from previous results. At low frequencies $\lesssim$15 THz, $\beta^{(2)}$ dominates, but at higher frequencies $\gtrsim$20 THz, the effects of $\beta^{(4)}$ are significant and increase the squeezing at $\sim$38 THz to $\sim$90%, which is greater than the squeezing at 0 THz. The squeezing at $\sim$55% extends all the way out to $\sim$45 THz. The details of the interaction of this perturbation on squeezing will be presented elsewhere.[19]

In conclusion, we have calculated the noise squeezing for physical systems governed by the nonlinear Schrödinger equation. We then applied them to an optical fiber communication system, finding a greater frequency spectrum for squeezing in the anomalous dispersion region than in the normal dispersion region. Somewhat surprisingly we found that odd-order dispersive terms had no effect on squeezing. However, small values of higher-order even terms had significant effects on squeezing.

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