Optical Stern-Gerlach effect

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A simple theory of the splitting of an atomic beam by monochromatic light is presented. It is argued that the components of an atomic beam split by the optical Stern-Gerlach effect do not correspond to the dressed states of the atom-field system.

I. INTRODUCTION

Although the splitting of an atomic beam by resonant light, the so-called optical Stern-Gerlach effect, has been of theoretical and experimental interest for over a decade,[1-5] no simple theory for arbitrary detuning has emerged and some questions of interpretation remain unanswered. The questions of interpretation derive from our experience with the splitting of atomic beams by static electric and magnetic fields. In these cases, the atomic energy levels invariably shift by different amounts when the field changes. When the change in the field results from a change in the position of the atom in an inhomogeneous field, the principle of virtual work tells us that the atom experiences different forces in the different states. In other words, the atom can exist in any of a number of eigenstates $|i\rangle$ of the operator representing the atom's internal energy, and the position-dependent eigenvalue $E_i(\mathbf{r})$ belonging to a particular state acts as a potential energy for the translational motion of the atom in that state. The beam splitting occurs when the forces $\mathbf{F}_i=-\nabla E_i$ are different for different states.

The above picture of beam splitting is so familiar from experience with the Stern-Gerlach effect and Stark beam splitting in an inhomogeneous electric field that it has often been surmised that the optical Stern-Gerlach effect must be exactly analogous, i.e., that there must exist "dressed states" of the atom-field system with energies $\varepsilon_i(\mathbf{r})$, and that the beam splitting is due to the different forces $\mathbf{F}_i=-\nabla\varepsilon_i$ acting on atoms in the different dressed states. It is the purpose of this paper to present a simple semiclassical theory of the optical Stern-Gerlach effect, which is valid for arbitrary detuning, and to show, by use of this theory, that the dressed-state interpretation is not always appropriate. With few exceptions, the force acting on an atom in a near-resonant electromagnetic wave is not derivable from a potential and therefore cannot be the gradient of any position-dependent energy (one exception to this rule is the exactly resonant standing wave, for which potential energies exist[4]). It should be emphasized that the results obtained here in no way decrease the utility of dressed states for the study of atomic motion in resonant radiation.[6] It simply means that the dressed states do not, in general, correspond to the split components of the atomic beam.

In the following section we present our theory of the optical Stern-Gerlach effect, which is restricted to the case where the atomic wave packet is small compared to the optical wavelength and spontaneous emission is negligible. In Sec. III some examples are given to illustrate the theory, and in Sec. IV the question of interpretation and the role of dressed states is discussed.

II. THEORY

Consider a two-level atom with position $\hat{\mathbf{R}}$, momentum $\hat{\mathbf{P}}$, internal energy $\hat{H}_0$, electric dipole moment $\hat{\boldsymbol{\mu}}$, and mass M. In the electric dipole approximation, the Hamiltonian describing the internal and translational motions of the atoms is

$$\hat{H}=\frac{\hat{P}^2}{2M}+\hat{H}_0-\hat{\boldsymbol{\mu}}\cdot\mathbf{E}(\hat{\mathbf{R}},t)\ , \tag{1}$$

where $\mathbf{E}(\mathbf{x},t)$ is the applied electric field. For simplicity, let the prescribed electric field be polarized in the fixed direction $\boldsymbol{\epsilon}$ at each point of space. Then $\mathbf{E}(\mathbf{x},t)=\boldsymbol{\epsilon}E(\mathbf{x},t)$, and the interaction term in (1) becomes $\hat{\mu}E(\hat{\mathbf{R}},t)$, where $\hat{\mu}$ is the component of $\hat{\boldsymbol{\mu}}$ in the direction of the electric field.

From the Hamiltonian (1) there follow the Heisenberg equations of motion for $\hat{\mathbf{R}}$ and $\hat{\mathbf{P}}$,

$$\dot{\hat{\mathbf{R}}}=(i\hbar)^{-1}[\hat{\mathbf{R}},\hat{H}]=\hat{\mathbf{P}}/M\ , \tag{2a}$$

$$\dot{\hat{\mathbf{P}}}=(i\hbar)^{-1}[\hat{\mathbf{P}},\hat{H}]=\hat{\mu}\nabla E(\hat{\mathbf{R}},t)\ . \tag{2b}$$

We are interested in the case where the atomic wave packet is small compared to the wavelength of the applied field. This being the case, the field gradient ∇E is essentially constant across the wave packet, and can be evaluated at the centroid $\mathbf{r}$ of the packet ($\mathbf{r}=\langle\hat{\mathbf{R}}\rangle$). Thus (2b) becomes

$$\dot{\hat{\mathbf{P}}}=\hat{\mu}\nabla E(t)\ , \tag{3}$$

where $\nabla E(t)=\nabla E(\mathbf{r}(t),t)$. The integral of this equation from 0 to t,

$$\Delta\hat{\mathbf{P}}=\int_0^t dt'\hat{\mu}(t')\nabla E(t')\ , \tag{4}$$

is the operator representing the change of atomic momentum in this time interval. Note that the dipole moment operator $\hat{\mu}$ depends only on the internal degrees of freedom of the atom, and, for a two-level atom, may be represented by a 2×2 matrix. The integral in (4) is then also a 2×2 matrix. Hence there are two eigenvalues $\Delta\mathscr{P}^i_\pm$ for each component $\Delta\hat{P}^i$ of $\Delta\hat{\mathbf{P}}$. These are the allowed values of this component of momentum transfer in the time interval $[0,t]$. Associated with the eigenvalues $\Delta\mathscr{P}^i_\pm$ are the eigenstates $|\pm\rangle^i$ of $\Delta\hat{P}^i$. If we suppose that the atom starts out in its ground state $|1\rangle$ at time 0, and this is the initial time of the Heisenberg picture, then the probability P^i_+ (P^i_-) for momentum transfer $\Delta\mathscr{P}^i_+$ ($\Delta\mathscr{P}^i_-$) is $|\langle 1|+\rangle^i|^2$ ($|\langle 1|-\rangle^i|^2$). In this way Eq. (4) determines the deflections $\Delta\mathscr{P}^i_\pm$ of a narrow atomic beam and the probabilities $P^i_\pm$ of these deflections. That is, (4) provides a complete description of the optical Stern-Gerlach effect.

In order to apply Eq. (4), we need the trajectory $\mathbf{r}(t)$ of the centroid of the atomic wave packet [since $\nabla E(t)=\nabla E(\mathbf{r}(t),t)$] and the atomic dipole moment $\hat{\mu}(t)$. The first of these is obtained from the expectation values of Eqs. (2) with $\nabla E(\hat{\mathbf{R}},t)$ replaced by $\nabla E(\mathbf{r}(t),t)$. After eliminating $\langle\hat{\mathbf{P}}\rangle$ between these equations, we have

$$M\ddot{\mathbf{r}}=\langle\hat{\mu}\rangle\nabla E(\mathbf{r},t)\ , \tag{5}$$

which may be solved for $\mathbf{r}(t)$ once $\langle\hat{\mu}(t)\rangle$ and the initial conditions $\mathbf{r}(0)$ and $\dot{\mathbf{r}}(0)$ are known.

To obtain the dipole momentum $\hat{\mu}(t)$ and its expectation value, we return to Eq. (1) and note that the part of the Hamiltonian describing the internal motion of the atom is

$$\hat{H}'=\hat{H}_0-\hat{\mu}E(t)\ , \tag{6}$$

where $E(t)=E(\mathbf{r}(t),t)$, i.e., the internal motion is driven by the electric field at the position $\mathbf{r}(t)$ of the moving atom. Let $|1\rangle$ and $|2\rangle$ be the eigenstates of the unperturbed Hamiltonian $\hat{H}_0$ and $E_1=-\frac{1}{2}\hbar\omega_0$ and $E_2=\frac{1}{2}\hbar\omega_0$ the unperturbed energy eigenvalues, where ω_0 is the Bohr transition frequency. Then, with operators $\hat{S}$, $\hat{S}^\dagger$, and $\hat{\sigma}_3$ defined as

$$\begin{aligned}\hat{S}&=|1\rangle\langle 2|\ ,\\ \hat{S}^\dagger&=|2\rangle\langle 1|\ ,\\ \hat{\sigma}_3&=|2\rangle\langle 2|-|1\rangle\langle 1|\ ,\end{aligned} \tag{7}$$

the unperturbed Hamiltonian takes the form $\hat{H}_0=\frac{1}{2}\hbar\omega_0\hat{\sigma}_3$ and the dipole moment operator reads

$$\hat{\mu}=\mu(\hat{S}+\hat{S}^\dagger)\ , \tag{8}$$

where $\mu=\langle 1|\hat{\mu}|2\rangle$ is the dipole transition moment, here assumed to be real. Consequently, the Hamiltonian for the internal motion (6) becomes

$$\hat{H}'=\tfrac{1}{2}\hbar\omega_0\hat{\sigma}_3-\mu E(t)(\hat{S}+\hat{S}^\dagger)\ . \tag{9}$$

From their definitions (7) the operators $\hat{S}$, $\hat{S}^\dagger$, and $\hat{\sigma}_3$ are found to have commutators

$$\begin{aligned}[\hat{S},\hat{\sigma}_3]&=2\hat{S}\ ,\\ [\hat{S},\hat{S}^\dagger]&=-\hat{\sigma}_3\ ,\\ [\hat{\sigma}_3,\hat{S}^\dagger]&=2\hat{S}^\dagger\ ,\end{aligned} \tag{10}$$

and these, together with the Hamiltonian (9), lead to the Heisenberg equations of motion for $\hat{S}$ and $\hat{\sigma}_3$:

$$\dot{\hat{S}}=-i\omega_0\hat{S}-i\mu E(t)\hat{\sigma}_3/\hbar\ , \tag{11a}$$

$$\dot{\hat{\sigma}}_3=2i\mu E(t)(\hat{S}^\dagger-\hat{S})/\hbar\ . \tag{11b}$$

The equation for $\hat{S}^\dagger$ is the Hermitian conjugate of (11a).

We now specialize to the case of a monochromatic field

$$\begin{aligned}E(\mathbf{x},t)&=\mathscr{E}(\mathbf{x})\cos[\theta(\mathbf{x})+\omega t]\\ &=\tfrac{1}{2}\mathscr{E}e^{i(\theta+\omega t)}+\text{c.c.}\ ,\end{aligned} \tag{12}$$

with arbitrary amplitude $\mathscr{E}(\mathbf{x})$ and phase $\theta(\mathbf{x})$. Because of the first term on the right-hand side in (11a), the operators $\hat{S}$ and $\hat{S}^\dagger$ are rapidly oscillating functions of time. For a monochromatic field whose frequency is near the atomic transition frequency ($\omega\approx\omega_0$), it is more convenient to work with the slowly changing variables $\hat{\sigma}$ and $\hat{\sigma}^\dagger$ defined as

$$\begin{aligned}\hat{S}&=\hat{\sigma}e^{-i(\theta+\omega t)}\ ,\\ \hat{S}^\dagger&=\hat{\sigma}^\dagger e^{i(\theta+\omega t)}\ .\end{aligned} \tag{13}$$

Upon substituting (8) and the gradient of (12) into (4) and using (13), we find that the integrand in (4) contains some slowly varying terms and terms that oscillate at approximately twice the optical frequency. The latter are negligible because they integrate to zero over the very short time interval π/ω. Keeping only the slowly varying terms, we have

$$\Delta\hat{\mathbf{P}}=\tfrac{1}{2}\mu\int_0^t[\nabla\mathscr{E}(\hat{\sigma}+\hat{\sigma}^\dagger)+i\mathscr{E}\nabla\theta(\hat{\sigma}-\hat{\sigma}^+)]dt'\ . \tag{14}$$

Next we substitute (12) and (13) into Eqs. (11), and again find slowly varying and rapidly oscillating terms. The rapidly oscillating terms are ineffective because they average to zero over a short period of time. In the rotating-wave approximation these terms are discarded and Eqs. (11) become

$$\dot{\hat{\sigma}}=i(\Delta+\dot{\theta})\hat{\sigma}-\tfrac{1}{2}i\Omega\hat{\sigma}_3\ , \tag{15a}$$

$$\dot{\hat{\sigma}}_3=i\Omega(\hat{\sigma}^\dagger-\hat{\sigma})\ , \tag{15b}$$

where $\Delta=\omega-\omega_0$ is the detuning between the field frequency ω and the atomic frequency ω_0, $\Omega=\mu\mathscr{E}/\hbar$ is the on-resonance Rabi frequency of the two-level atom, and $\dot{\theta}=d\theta(\mathbf{r}(t))/dt=\nabla\theta\cdot\dot{\mathbf{r}}$.

As a final step in the derivation of our working equations, we reexpress Eqs. (14) and (15) in terms of the Hermitian operators

$$\begin{aligned}\hat{\sigma}_1&=\hat{\sigma}+\hat{\sigma}^\dagger\ ,\\ \hat{\sigma}_2&=i(\hat{\sigma}-\hat{\sigma}^\dagger)\ ,\end{aligned} \tag{16}$$

which, together with $\hat{\sigma}_3$, obey the Pauli algebra

$$\hat{\sigma}_j\hat{\sigma}_K=\delta_{jK}+i\epsilon_{jKL}\hat{\sigma}_L\ . \tag{17}$$

The result is the set of operator equations

$$\Delta\hat{\mathbf{P}}=\tfrac{1}{2}\hbar\int_0^t(\nabla\Omega\hat{\sigma}_1+\Omega\nabla\theta\hat{\sigma}_2)dt'\ , \tag{18}$$

$$\dot{\hat{\sigma}}_1=(\Delta+\dot{\theta})\hat{\sigma}_2\ , \tag{19a}$$

$$\dot{\hat{\sigma}}_2=-(\Delta+\dot{\theta})\hat{\sigma}_1+\Omega\hat{\sigma}_3\ , \tag{19b}$$

$$\dot{\hat{\sigma}}_3 = -\Omega\hat{\sigma}_2 , \tag{19c}$$

upon which all subsequent results are based.

Equations (18) and (19) are solved as follows. Given the centroid motion $\mathbf{r}(t)$, the coefficients in (19), $\Omega = \mu\mathscr{E}(\mathbf{r})/\hbar$ and $\dot{\theta} = \nabla\theta(\mathbf{r})\cdot\dot{\mathbf{r}}$, are known functions of time. Thus these equations can be solved for $\hat{\sigma}_1(t)$, $\hat{\sigma}_2(t)$, and $\hat{\sigma}_3(t)$ in terms of the initial values of these operators,

$$\begin{aligned} \hat{\sigma}_1(0) &= |1\rangle\langle 2| + |2\rangle\langle 1| , \\ \hat{\sigma}_2(0) &= i(|1\rangle\langle 2| - |2\rangle\langle 1|) , \\ \hat{\sigma}_3(0) &= |2\rangle\langle 2| - |1\rangle\langle 1| . \end{aligned} \tag{20}$$

The solution proceeds exactly as if the $\hat{\sigma}_i$'s were c numbers because Eqs. (19) are linear. Each of the operators $\hat{\sigma}_i(t)$ is a linear combination of the initial operators (20). It follows that the integral in (18) is also a linear combination of $\hat{\sigma}_i(0)$. Thus each component of (18) has the form

$$\Delta\hat{P} = \alpha\hat{\sigma}_1(0) + \beta\hat{\sigma}_2(0) + \gamma\hat{\sigma}_3(0) , \tag{21}$$

with eigenvalues

$$\Delta\mathscr{P}_\pm = \pm(\alpha^2+\beta^2+\gamma^2)^{1/2} \tag{22}$$

and eigenvectors

$$\begin{aligned} |+\rangle &= \cos\left[\frac{\phi}{2}\right] e^{-i\eta/2}|1\rangle + \sin\left[\frac{\phi}{2}\right] e^{i\eta/2}|2\rangle , \\ |-\rangle &= -\sin\left[\frac{\phi}{2}\right] e^{-i\eta/2}|1\rangle + \cos\left[\frac{\phi}{2}\right] e^{i\eta/2}|2\rangle , \end{aligned} \tag{23}$$

where $\tan\phi = (\alpha^2+\beta^2)^{1/2}/\gamma$ $(0 \le \phi \le \pi)$ and $\tan\eta = \beta/\alpha$. If the atom is in its ground state $|1\rangle$, the probabilities to be in states $|+\rangle$ and $|-\rangle$ are

$$\begin{aligned} P_+ &= \cos^2\left[\frac{\phi}{2}\right] = \frac{1}{2}\left[1 + \frac{\gamma}{(\alpha^2+\beta^2+\gamma^2)^{1/2}}\right] , \\ P_- &= \sin^2\left[\frac{\phi}{2}\right] = \frac{1}{2}\left[1 - \frac{\gamma}{(\alpha^2+\beta^2+\gamma^2)^{1/2}}\right] , \end{aligned} \tag{24}$$

respectively.

III. EXAMPLES

A. Resonant standing wave

Consider first the well-understood case of a general standing wave,

$$E(\mathbf{x},t) = \mathscr{E}(\mathbf{x})\cos(\omega t) \tag{25}$$

(not necessarily a plane standing wave), whose frequency is exactly resonant with the atomic transition ($\Delta = 0$). For a standing wave, the phase $\theta(\mathbf{x})$, as defined by (12), is zero and Eqs. (19) reduce to

$$\begin{aligned} \dot{\hat{\sigma}}_1 &= 0 , \\ \dot{\hat{\sigma}}_1 &= \Omega\hat{\sigma}_3 , \\ \dot{\hat{\sigma}}_3 &= -\Omega\hat{\sigma}_2 . \end{aligned} \tag{26}$$

Observe that $\hat{\sigma}_1$ is constant in this case $[\hat{\sigma}_1(t) = \hat{\sigma}_1(0)]$, and Eq.(18) becomes simply

$$\Delta\hat{\mathbf{P}} = \tfrac{1}{2}\hbar\hat{\sigma}_1(0)\int_0^t \nabla\Omega(t')dt' . \tag{27}$$

The eigenstates of this operator [the eigenstates of $\hat{\sigma}_1(0)$] are

$$|\pm\rangle = \frac{1}{\sqrt{2}}(|1\rangle \pm |2\rangle) , \tag{28}$$

and the eigenvalues

$$\Delta\mathscr{P}_\pm = \pm\tfrac{1}{2}\hbar\int_0^t \nabla\Omega(t')dt' . \tag{29}$$

Here an atom initially in the ground state has probabilities $P_+ = P_- = \frac{1}{2}$ of being in one or the other of the states $|\pm\rangle$. Accordingly, an atomic beam is split into two equally populated components.

B. Resonant plane running wave

For this case the wave amplitude has the form

$$E(\mathbf{x},t) = \mathscr{E}\cos(\mathbf{K}\cdot\mathbf{x} - \omega t) , \tag{30}$$

with $\mathscr{E}$ constant and phase $\theta = -\mathbf{K}\cdot\mathbf{x}$. consequently, The Rabi frequency $\Omega = \mu\mathscr{E}/\hbar$ is constant in space and time, and the phase derivative, $\theta = -\mathbf{K}\cdot\dot{\mathbf{r}}$, is the Doppler shift of the wave frequency due to motion of the atom. By a resonant running wave we mean a wave whose Doppler-shifted frequency is exactly resonant with the atomic transition ($\omega - \mathbf{K}\cdot\dot{\mathbf{r}} = \omega_0$ or $\Delta = \mathbf{K}\cdot\dot{\mathbf{r}}$). This being the case, Eqs. (19) become

$$\begin{aligned} \dot{\hat{\sigma}}_1 &= 0 , \\ \dot{\hat{\sigma}}_2 &= \Omega\hat{\sigma}_3 , \\ \dot{\hat{\sigma}}_3 &= -\Omega\hat{\sigma}_2 . \end{aligned} \tag{31}$$

These equations are identical to Eqs. (26) for a resonant standing wave, but now the momentum transfer operator, Eq. (18), depends on $\hat{\sigma}_2$ instead of $\hat{\sigma}_1$,

$$\Delta\hat{\mathbf{P}} = -\tfrac{1}{2}\hbar\mathbf{K}\Omega\int_0^t \hat{\sigma}_2(t')dt' . \tag{32}$$

The solution of Eqs. (31) is

$$\hat{\sigma}_1(t) = \hat{\sigma}_1(0) , \tag{33a}$$

$$\hat{\sigma}_2(t) = \cos(\Omega t)\hat{\sigma}_2(0) + \sin(\Omega t)\hat{\sigma}_3(0) , \tag{33b}$$

$$\hat{\sigma}_3(t) = -\sin(\Omega t)\hat{\sigma}_2(0) + \cos(\Omega t)\hat{\sigma}_3(0) . \tag{33c}$$

Use of (33b) in (32) yields the momentum-transfer operator

$$\Delta\hat{\mathbf{P}} = -\tfrac{1}{2}\hbar\mathbf{K}\{\sin(\Omega t)\hat{\sigma}_2(0) + [1-\cos(\Omega t)]\hat{\sigma}_3(0)\} , \tag{34}$$

whose eigenvalues are

$$\Delta\mathscr{P}_\pm = \pm\hbar\mathbf{K}\sin(\Omega t/2) . \tag{35}$$

For an atom initially in its ground state, the probabilities of these momentum transfers are

$$P_\pm(t) = \frac{1}{2}\left[1 \pm \sin\left[\frac{\Omega t}{2}\right]\right] , \tag{36}$$

which are time dependent. This means that the eigenstates of momentum transfer are not stationary states, there are transitions between them. The expectation value of momentum transfer,

$$\begin{aligned}\langle \Delta\hat{\mathbf{P}}\rangle &= \Delta\mathscr{P}_+ P_+ + \Delta\mathscr{P}_- P_- \\ &= \hbar\mathbf{K}\sin^2(\Omega t/2)\ ,\end{aligned} \tag{37}$$

oscillates between 0 and $\hbar\mathbf{K}$ at the Rabi frequency as the atom absorbs from and emits into the applied field.

C. Standing wave off resonance

The field in this case is that of Eq. (25). The phase is still zero, but the detuning is not, and Eqs. (19) read

$$\begin{aligned}\dot{\hat{\sigma}}_1 &= \Delta\hat{\sigma}_2\ ,\\ \dot{\hat{\sigma}}_2 &= -\Delta\hat{\sigma}_1+\Omega\hat{\sigma}_3\ ,\\ \dot{\hat{\sigma}}_3 &= -\Omega\hat{\sigma}_2\ .\end{aligned} \tag{38}$$

To keep things simple, we assume that the centroid $\mathbf{r}(t)$ moves perpendicular to the field gradient $\nabla\mathscr{E}$, so that the Rabi frequency Ω remains constant in time at the moving atom. This is the case, for example, when an atomic beam propagates parallel to the planes of maximum intensity in a plane standing wave. For Δ and Ω constant, the solution of Eqs. (38) is as follows:

$$\begin{bmatrix}\hat{\sigma}_1(t)\\ \hat{\sigma}_2(t)\\ \hat{\sigma}_3(t)\end{bmatrix} = \frac{1}{R^2}\begin{bmatrix}\Delta & 0 & \Omega\\ 0 & R & 0\\ -\Omega & 0 & \Delta\end{bmatrix}\begin{bmatrix}\cos(Rt) & \sin(Rt) & 0\\ -\sin(Rt) & \cos(Rt) & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\Delta & 0 & -\Omega\\ 0 & R & 0\\ \Omega & 0 & \Delta\end{bmatrix}\begin{bmatrix}\hat{\sigma}_1(0)\\ \hat{\sigma}_2(0)\\ \hat{\sigma}_3(0)\end{bmatrix}, \tag{39}$$

and, in particular,

$$\hat{\sigma}_1(t) = \frac{\Omega^2+\Delta^2\cos(Rt)}{R^2}\hat{\sigma}_1(0) + \frac{\Delta}{R}\sin(Rt)\hat{\sigma}_2(0) + \frac{\Delta\Omega[1-\cos(Rt)]}{R^2}\hat{\sigma}_3(0)\ , \tag{40}$$

where $R=(\Omega^2+\Delta^2)^{1/2}$. The momentum transfer in time t is

$$\Delta\hat{\mathbf{P}} = \tfrac{1}{2}\hbar\nabla\Omega\int_0^t \hat{\sigma}_1(t')dt' = \frac{\hbar\nabla\Omega}{2R^2}\left[\left[\Omega^2 t + \frac{\Delta^2\sin(Rt)}{R}\right]\hat{\sigma}_1(0) + \Delta[1-\cos(Rt)]\hat{\sigma}_2(0) + \Delta\Omega\left[t-\frac{\sin(Rt)}{R}\right]\hat{\sigma}_3(0)\right]. \tag{41}$$

From the general formulas (22) and (24), we find that the allowed values of momentum transfer [the eigenvalues of (41)] are

$$\Delta\mathscr{P}_\pm = \pm\frac{\hbar\nabla\Omega}{2R^2}[\Omega^2R^2t^2+4\Delta^2\sin^2(Rt/2)]^{1/2}\ , \tag{42}$$

and the probabilities of these values are

$$P_\pm = \frac{1}{2}\left[1\pm\frac{\Delta\Omega[Rt-\sin(Rt)]}{R(\Omega^2R^2t^2+4\Delta^2\sin^2(Rt/2))^{1/2}}\right]. \tag{43}$$

Here the beam splitting begins as if the detuning were zero. For $Rt \ll 1$, the momentum transfers and probabilities are $\Delta\mathscr{P}_1 = \pm\frac{1}{2}\hbar\nabla\Omega t$ and $P_\pm=\frac{1}{2}$, respectively. These are the same as those obtained in the on-resonance case. But for longer times ($Rt \gg 1$), the eigenvalues (42) become

$$\Delta\mathscr{P}_\pm = \pm\frac{\hbar\nabla\Omega^2 t}{4(\Omega^2+\Delta^2)^{1/2}}\ , \tag{44}$$

and the probabilities (43)

$$P_\pm = \frac{1}{2}\left[1\pm\frac{\Delta}{(\Omega^2+\Delta^2)^{1/2}}\right]. \tag{45}$$

Note that, for $\Delta\to\infty$, $P_+\to 1$ and $P_-\to 0$; and, for $\Delta\to-\infty$, $P_+\to 0$ and $P_-\to 1$. In other words, for large detuning ($|\Delta|\gg\Omega$) the atomic beam does not split at all, but rather is deflected with a single momentum transfer ($\Delta\mathscr{P}_+$ for $\Delta\to\infty$ or $\Delta\mathscr{P}_-$ for $\Delta\to-\infty$).

IV. DISCUSSION

A question of importance for the interpretation of the optical Stern-Gerlach effect is whether the forces acting on the beam components are derivable from potentials equal to dressed-state energies. To answer this question we start with the Hamiltonian describing the interaction of a two-level atom with a single quantized field mode in the rotating-wave approximation,

$$\hat{H} = \tfrac{1}{2}\hbar\omega_0\hat{\sigma}_3 + \hbar\omega\hat{a}^\dagger\hat{a} + \tfrac{1}{2}i\hbar\Omega_1(\hat{a}^\dagger\hat{\sigma}-\hat{\sigma}^\dagger\hat{a})\ , \tag{46}$$

where $\hat{a}^\dagger$ and $\hat{a}$ are the photon creation and annihilation operators for the mode. In (46) we have taken the one-photon Rabi frequency Ω_1 to be real. This is equivalent to assuming that the phase of the field is zero, i.e., the mode under consideration is a standing wave. This special case is sufficient for the purpose at hand.

Suppose the atom is initially in its ground state $|1\rangle_A$ and the field contains exactly n photons, i.e., the field state is $|n\rangle_F$. Then the initial atom-field state is the direct product $|F\rangle = |1\rangle_A|n\rangle_F$. Because of the rotating-wave approximation, the Hamiltonian (46) causes transitions from this state to only one other state, namely the state $|A\rangle = |2\rangle_A|n-1\rangle_F$ with the atom excited and $n-1$ photons in the field. Thus the system state vector remains of the form

$$|4\rangle = f\,|F\rangle + a\,|A\rangle \tag{47}$$

for all time (here, f is the amplitude for all of the energy to be in the field and a the amplitude for the atom to be excited). Using the Hamiltonian (46), we obtain the following Schrödinger equation for amplitudes f and a:

$$\begin{aligned} i\hbar\dot{f} &= H_{11}f + H_{12}a \ , \\ i\hbar\dot{a} &= H_{21}f + H_{22}a \ , \end{aligned} \tag{48}$$

where the 2×2 Hamiltonian matrix is

$$H_{ij} = \begin{bmatrix} \hbar\omega n + \frac{1}{2}\hbar\Delta & \frac{1}{2}i\hbar\sqrt{n}\,\Omega_1 \\ -\frac{1}{2}i\hbar\sqrt{n}\,\Omega_1 & \hbar\omega n - \frac{1}{2}\hbar\Delta \end{bmatrix} . \tag{49}$$

The dressed states are the eigenstates of this Hamiltonian,

$$\begin{aligned} |-\rangle &= \cos\left[\frac{\phi}{2}\right]|F\rangle + \sin\left[\frac{\phi}{2}\right]|A\rangle \ , \\ |+\rangle &= -\sin\left[\frac{\phi}{2}\right]|F\rangle + \cos\left[\frac{\phi}{2}\right]|A\rangle \ , \end{aligned} \tag{50}$$

where $\tan\phi = -\Omega/\Delta$, and the dressed-state energies are the eigenvalues

$$\varepsilon_{\pm} = \hbar\omega n \mp \tfrac{1}{2}\hbar(\Omega^2+\Delta^2)^{1/2} \ , \tag{51}$$

in which $\Omega = \sqrt{n}\,\Omega_1$ is the n-photon Rabi frequency and $\Delta = \omega - \omega_0$ the detuning. With the Rabi frequency position dependent, the negative gradients of the dressed-state energies are

$$\mathbf{F}_{\pm} = -\nabla\varepsilon_{\pm} = \pm\frac{\hbar\nabla\Omega^2}{4(\Omega^2+\Delta^2)^{1/2}} \ . \tag{52}$$

Because there are no transitions between dressed states, an atom in one of these states remains in that state indefinitely, and, if the force were given by (52), the momentum transfer in time t, for Ω and $\nabla\Omega$ constant, would be

$$\Delta\mathscr{P}_{\pm} = \pm\frac{\hbar\nabla\Omega^2 t}{4(\Omega^2+\Delta^2)^{1/2}} \ , \tag{53}$$

and the probabilities for these transfers would be found from Eqs. (50) to be

$$P_{\pm} = \frac{1}{2}\left[1 \pm \frac{\Delta}{(\Omega^2+\Delta^2)^{1/2}}\right] . \tag{54}$$

We are now in a position to compare the actual deflections derived above with those derived from the dressed-state hypothesis. First, it is clear from (52) and (53) that, for exact resonance ($\Delta=0$), both approaches yield the same deflections, $\Delta\mathscr{P}_{\pm} = \pm\frac{1}{2}\hbar\nabla\Omega t$, and the same probabilities $P_{\pm}=\frac{1}{2}$. But when the detuning is nonzero, the results are different. Only in the limit of long interaction time ($Rt \gg 1$) do the correct results, (42) and (43), agree with the predictions of the dressed-state formalism. We conclude that the forces acting on the split components of an atomic beam in the optical Stern-Gerlach effect are not, in general, gradients of dressed-state energies. Moreover, since the probabilities for the eigenstates of momentum transfer (43) are time dependent, these eigenstates are not stationary, and therefore cannot be the dressed states. This conclusion also follows from Eq. (18), which is the integral of the equation of motion

$$\hat{\mathbf{F}} = \dot{\hat{\mathbf{P}}} = \tfrac{1}{2}\hbar[\nabla\Omega\hat{\sigma}_1 + \Omega\nabla\theta\hat{\sigma}_2] \ . \tag{55}$$

When both terms in the force are nonzero, the force is not derivable from a potential. Hence the force acting on the atom cannot be the gradient of a dressed-state energy. In short, the picture of the optical Stern-Gerlach effect as a splitting of dressed states is inappropriate.

Although some off-resonance beam-definition experiments have been performed, such as the standing-wave experiment of Gould, Ruff, and Pritchard,[7] we know of no experiment which tests the regime $Rt \ll 1$, where the dressed-atom theory is predicted to fail.

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